Survey of India

THE TIDES

REVISED BY

Major C. M. THOMPSON, I. A.

DEPUTY SUPERINTENDENT, SURVEY OF INDIA

This pamphlet forms part V of the Hand-book of Professional Instructions (Third Edition) for the Geodetic Branch.

PUBLISHED BY ORDER OF

COLONEL COMMANDANT E. A. TANDY, R.E.

SURVEYOR GENERAL OF INDIA

PRINTED AT THE GEODETIC BRANCH OFFICE
SURVEY OF INDIA, DEHRA DUN, 1926.

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PREFACE

THE tide predicting machine belonging to the Indian Government having been transferred from the National Physical Laboratory, Teddington to the Trigonometrical Survey Head-quarters at Dehra Dun in 1921, this revision of the Tidal Chapter of the Handbook of professional instructions for the Trigonometrical Branch, Survey of India, has been taken up with a view to provide a manual which may assist an observer who may have to undertake the general duties connected with the tidal party, as well as the Harmonic Analysis and prediction of tides in accordance with the methods at present in use in the department.

In a preliminary report presented at the Geophysical Discussion, *vide* the British Association report 1918, it was stated that the tide tables, as at present produced, appear to be adequate for practical needs. This was based on the facts that the practically important constituents can be determined fairly accurately, and that Harmonic prediction presents no theoretical difficulties like those of Harmonic Analysis. The investigations of Dr. A. T. Doodson of the Liverpool Tidal Institute show, however, that the published tables of Harmonic predictions are also very unsatisfactory.

(1). About half the error in short period tides he attributes to the inadequate treatment of shallow water effects.

(2). The remaining half is due to tidal constituents, not included in Darwin's schedules of 1883, and whose origin is not definitely known.

(3). Time devoted to modification of Harmonic constants by repeated analyses would be better spent in analysing for new constituents.

The British Association report for 1920 states "that the principle of Harmonic analysis is part of the theory of the small oscillations of a dynamical system, and its application becomes less accurate as the range of a tide becomes a larger fraction of the depth of the water, or as the tidal currents become greater. It yet remains to be found to what precise extent the purely astronomical tide at any station may be expressed as a series of a reasonable number of Harmonic constituents. When this has been done, and the methods of analysis and prediction refined, so as to give predictions correct to this extent, a hopeful investigation may be made into the residual astronomical tide and the whole of the meteorological disturbance."

Modern investigations are thus more especially directed towards improving the predictions for Riverain ports and estuaries, having shallow foreshores, where at present the results are more liable to error, and to the study of meteorological effects. It is likely therefore that changes and improvements in the present methods may be introduced in the course of time.

My acknowledgments are due to Dr. J. DE GRAAFF HUNTER, M. A., Sc.D., F. Inst. P., more especially for his explanations regarding the application of corrections to Riverain port predictions and the multiple contact method of running the Tidal machine; and to Capt. E. A. GLENNIE, D.S.O., R.E. for his preliminary notes on the erection and running of the same. I also take this opportunity of thanking Mr. D. H. LUXA, the tidal assistant, for helping throughout with the preparation and proof reading of the book Mr. R. B. MATHUR, B.A. for the preparation of diagrams and correction of the tables; Computer M. CHATTERJI of the Computing Office, for assisting in the compilation of the portion relating to Riverain predictions as well as in proof reading; and Mr. SHYAM NARAYAN, B.Sc., Asstt. Supervisor of the Printing Office, Dehra Dūn, for his general superintendence of the printing.

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12th April 1926.

Survey of India

THE TIDES

CHAPTER I

Theory and Computation

1. From early times the rise and fall of the level of the ocean twice, or nearly so in the 24 hours was attributed to the influence of the moon and the sun, but this phenomenon was not satisfactorily accounted for till Newton in 1687 expounded his theory of gravitation which explained the cause of the tide-generating forces.

The tide-generating force of a satellite at any point tends to accelerate a body relatively to the earth as if acted on by a force depending on the difference in magnitude and direction between the moon's attraction at that point and at the earth's centre.

The tide-generating force varies directly as the mass of the tide-generating body and inversely as the cube of the distance. Hence the sun, though 25½ million times larger than the moon is 389 times as distant and therefore produces a tide-generating force less than one half of the moon's. In order to explain the tidal phenomena two general elementary theories were evolved, and though neither is actually adequate to explain the actual tides, they are important as forming the basis of investigation.

These theories are known as the equilibrium and canal theories.

2. *The equilibrium theory.*—The figure of equilibrium of the ocean under the tide-generating force of the moon, supposing that it were at each moment allowed to assume that position of rest which it would attain if infinite time were allowed, is a prolate ellipsoid of revolution with the major axis directed to the moon. The same applies to the sun. When the tide-generating forces of both are superimposed, we would find, under this theory, that spring or high tides would occur at syzygies (conjunction and opposition) and neap or low tides at quadratures (moon's quarters). From syzygy to quadrature the tide would be found to 'prime' or occur earlier than under the moon's influence alone. From quadrature to syzygy the tide would be found to 'lag' or occur later than under the moon's influence alone.

The uncorrected equilibrium theory is useful as a working hypothesis in tidal work, as it enables us to infer suitable forms of expression for the tidal disturbances, knowing the law of the forces to which they are due.

3. *The corrected equilibrium theory* differs from the former in assuming the earth not wholly covered by water and so the surface even of a deep sea cannot actually coincide with the spheroid of the uncorrected theory, but will be parallel to it, the distance therefrom, at any given point, varying with time. It applies roughly to small deep bodies of water.

4. *The canal theory.*—If the earth's rotation is taken into account and the moon's tide-generating force is conceived as creating waves in a shallow equatorial canal under 13 miles deep, the moon would outstrip the waves generated, and the latter would, if allowed to settle into steady oscillation, cause low water directly under the moon and at a point opposite to it, or an inversion of the tides that would be expected from the equilibrium theory.

The consideration of the motion of a free wave in non-equatorial canals parallel to the equator led to the conclusion that, whilst the tides of a shallow ocean are inverted at the equator and direct towards the pole, in a canal somewhere between higher latitudes and the equator, there would theoretically be quasi-infinite tides with an enormous range of rise and fall.

5. The above theories did not suffice to solve the problem, which was of great complexity. The orbits of the earth and moon are not circular, nor do they lie in the same plane as the equator. Thus the positions of the sun and moon relatively to the earth are continually varying in distance, declination and right ascension. Consequently the level of the ocean is subject to momentary variations in the dynamical actions of the disturbing bodies and these cause a variety of tides, which recur periodically, some in long and others in short periods.

After Newton the early workers on tidal theory were Bernoulli, Maclaurin, Euler and Cavalleri, who all contributed essays to the Académie des Sciences at Paris in 1738, but little advance in tidal theory was made till Laplace took up the question about the year 1774.

Problem of the tide covering the globe to a uniform extent.

6. The very difficult mathematical problem of the tide covering the globe to a uniform extent was first successfully attacked by Laplace. He showed that, whilst the tides of a shallow ocean are inverted at the equator, they are direct towards the poles, but that the other indication of the canal theory regarding quasi-infinite tides was wrong. He found instead that theoretically the tidal variation of level vanishes in some latitude between the equator and pole. At this circle the water

flows north and south and to and fro between east and west, but never to raise or depress the level of the sea. It is not true to say that there is no tide at this circle of "evanescent" tide, for there are tidal currents without rise or fall. When the ocean was considered as divided into parallel canals, the north and south currents which prevent the tides becoming very great as indicated by the canal theory, were thereby obliterated. (This problem has since been more fully investigated by later mathematicians, notably Mr. S. S. Hough in 1897-98, *vide* the article "Tides" in the Encyclopædia Britannica, Eleventh Edition).

Laplace's method of fictitious satellites and periodic law.

7. Laplace also treated the problem of the tides by assuming a number of fictitious satellites of various masses, moving in circular orbits with uniform motion, so arranged that the sum of their tidal forces was exactly the same as that due to the real sun and moon in their proper orbits. Among these satellites were the mean sun and moon, and others which correct the motions of the mean sun and moon for declination, parallax and other inequalities in their motions.

He moreover enunciated the most important principle that the state of oscillation of a system of bodies in which the primitive conditions of movement have disappeared through friction is co-periodic with the forces acting on the system.

He thus was able to resolve the tidal forces by considering them as having simple circular motion of periods dependent on the fictitious satellites above-mentioned and to discuss the tides at any port by a combination of theory and observation.

The foundations of Harmonic Analysis were laid by Laplace, for he enunciated the principle of forced oscillations; he introduced tidal bodies having uniform motions; he showed how to develope the tide-producing potential in a series of periodic terms and pointed out the more important harmonic constituents of the astronomical tide; he developed the method of least squares sufficiently to make it applicable to the determination of the coefficients of a sine and cosine function of an angle and its harmonics. But he did not attempt an analysis of equidistant ordinates based upon this knowledge, nor did he completely develope the tide-producing potential.

The theory now adopted regarding tides is substantially the same, but presented in an entirely different manner, the fictitious satellites being discarded, and the results being developed directly.

The connection between the method of Laplace and the modern method of development may be traced by remembering that the proper motion of each one of Laplace's fictitious satellites is at once derivable from the argument or angle under the sign of cosine which is associated with the partial tide in the later discussion, and Lord Kelvin considered it helpful to retain this connecting link between the earlier and modern theories. The method of Laplace also serves to explain and illustrate certain points connected with the nomenclature of terms used at present.

Subsequent to Laplace and up to about the year 1866 the principal investigators of tidal theory were Sir J. Lubbock senior, Whewell and Airy.

8. Up to this point the methods of comparison of tidal theory and tidal observation had been synthetic, *i.e.* they had merely considered the tidal wave as a whole, moreover the methods had consisted of merely averaging the times and heights of high and low-waters in selected sets or groups, without paying attention to the heights and times recorded at times other than those of high or low water.

Harmonic Analysis

9. In 1866 Sir William Thomson (afterwards Lord Kelvin) took up the investigation. Tidal theory was still very far from representing the actual state of the tides. Observations showed in fact that the irregular distribution of land and water and the varying depths of the ocean in various places combined with meteorological conditions produced irregularities in the oscillation of the sea of such complexity that the rigorous solution of the problem by synthetic methods seemed unlikely. This state of affairs led Sir William Thomson to abandon the attempt at mathematical synthesis, and to resolve the complex whole into a number of separate parts, by means of the method of Harmonic Analysis.

The methods evolved were afterwards perfected by the investigations of Mr. E. Roberts, F. R. A. S. and Prof. G. H. Darwin.

In 1882, 1883 and 1885 Prof. G. H. Darwin, (afterwards Sir George Darwin) presented reports at the British Association meetings which have ever since formed the standard manual on the subject. The methods evolved by Darwin are known as the British Association methods and are those in use in the Survey of India at the present day.* They are dependent on a spherical Harmonic Analysis of the tide-generating potential, whereby the theory was developed to a higher degree of accuracy than had been previously attained. Details of the method of development are given in G. T. Survey Volume XVI.

* A still fuller development of the tide-generating potential has been made by Prof. A. T. Doodson, D. Sc. of the Tidal Institute of the University of Liverpool *vide* the proceedings of the Royal Society Vol. 100 of 1921, which includes further constituents not included by Prof. Darwin in his schedules. This development includes all terms whose coefficients, relatively to the greatest coefficient, are greater than 0·00010. This development is considered sufficient to more than cover the needs of research work.

10. A brief account of the method is given here.

If we expand the potential in such a manner as to obtain the complete expression for a tide in terms of a series of simple harmonic functions of the time, the principle of forced oscillations allows us to conclude that the oscillations of the sea will be of the same periods and types as the several terms of the potential, but with amplitudes and phases, which can only be deduced by observation.

The portion of the tide following any .period strictly, can be analysed by Fourier's theorem into one or more simple cosine terms whose angles or arguments, (which are proportional to time), go through 360° and multiples thereof, in the given periodic time. Either the process or the result is spoken of as a Harmonic Analysis.

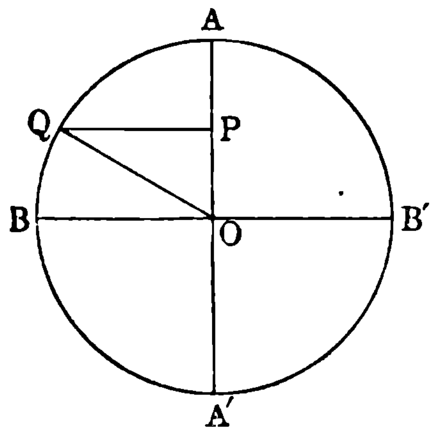
The height of the water at any place may be expressed as the sum of a certain number of simple harmonic functions of the time of which the periods are known, being the periods of certain constituents of the sun's and moon's motions. Any such harmonic term is called a tidal component or tide. The expression for it in ordinary analytical notation is $A \cos nt + B \sin nt$, or $R \cos (nt - \zeta)$, if $A = R \cos \zeta$ and $B = R \sin \zeta$ where t denotes time measured in any unit from any era, n the corresponding angular velocity (a quantity such that $\frac{2\pi}{n}$ is the period of the function), R and ζ the amplitude and epoch respectively, and A and B, coefficients, immediately determined from observation by Harmonic Analysis, which consists virtually in the method of least squares, applied to deduce the most probable values of these coefficients from the observations.

The oscillations of the sea will not however necessarily be of the simple harmonic form, and accordingly overtides of double and triple frequency will have to be introduced in order to represent the motion according to Fourier's method (compare para 37). Such overtides occur particularly in rivers and estuaries where the water is shallow and the waves become distorted. Compound tides also occur, which are explained in para 38.

The method of Harmonic Analysis thus considers each tide as a simple harmonic oscillation and each tide is designated by a separate initial letter or combination of letters selected more or less arbitrarily as explained in para 39 *et seq.*

11. *Simple Harmonic motion.*—When a point Q moves uniformly in a circle, the perpendicular QP drawn from its position at any instant to a fixed diameter AA′ of the circle, intersects the diameter in a point P,

whose position changes by a simple harmonic motion. The amplitude or semi-range of the motion is OA or OA′ in the figure.



Now each partial component tide may similarly be represented by a circle of known diameter; and if we suppose a point to move uniformly right round the circumference of this circle, so as to make a complete revolution in the time which is this tide's period, then the height of a point above or below the horizontal diameter BB′ of the circle at any moment, represents the height of the partial tide at that moment.

The velocity of rotation of a tide rests primarily on certain considerations—combinations of the angular velocities of the earth's rotation round its axis, the moon's rotation round the earth, the earth's round the sun, and the progression of the moon's perigee, which are decided *a priori* from theoretical considerations.

These preliminary angular velocities correspond to the arguments of the fictitious stars of Laplace's method.

Speed, phase, amplitude, and epoch.

12 The uniform hourly change in the angle of any component is called its speed, the value of its angle reckoned from its high water at any instant is called its phase, its constant semi-range is called its amplitude, and the epoch or lag is the angle in degrees, which the arm OQ (vide figure) revolving uniformly in the period of the particular tide has to run through till high water of this constituent from a certain instant or era of reckoning, dependent in the case of each particular tide on the astronomical cause to which it is attributable.

Otherwise expressed, the epoch may be defined as the constant angular retard of the maximum of any component behind its astronomical cause or fictitious satellite's transit, as assigned under Laplace's method.

Thus if n denote the periodic speed of a tide and ζ its epoch, its time of high water is $\frac{\zeta}{n}$ hours after the transit of the fictitious satellite.

The epoch thus enables us to ascertain the point which the tide has reached at any given moment during its movement over the circumference of the circle.

Tidal constants.

13. The amplitude and epoch form a pair of *tidal constants* referring to the particular place where the observations were made.

Composition of simple harmonic motions.

14. As the component tides into which the tidal wave is resolved by the method of Harmonic Analysis follow the simple harmonic law, the resultant variation of level, when their motion is compounded, affords an example of the composition of simple harmonic motions in one line.

Thus the momentarily varying level of the surface of the ocean is supposed to be the resultant of a large number of tides, each of which is perfectly independent of all the others, and has its own amplitude and period of revolution, which remain constant throughout all time. Occasionally several of the most important tides are in conjunction, and then the range between high and low water is a maximum, as at spring tides; at others, some tides are in opposition to others, and then the tidal range is a minimum, as occurs at neap tides.

If therefore we are able to find by means of continuous observations of the varying level of the sea, through obliterating the effects of extraneous tides, the amplitude and epoch of each of the several tides, of which the height of the sea level is the resultant, and also know the velocities of rotation of these constituent tides, we are in a position to be able to compute and predict the height of the sea level, at any future moment, at the station where the observations were taken, on which our calculations are based.

Harmonic curves.

15. We may exhibit graphically any case of single simple or compound simple harmonic motion in one line by curves, in which the abscissæ represent the intervals of time and the ordinates the corresponding distances of the moving point from its mean position. In the case of a single simple harmonic motion, the corresponding curve would be that described by the point P in the figure on page 6, if, while Q maintained its uniform circular motion, the circle were to move with uniform velocity in any direction perpendicular to OA. The curve in this case would be the simple harmonic curve of sines (or cosines).

At places where the semidiurnal component tides are much larger than the diurnal, quarter diurnals etc., the tidal curve will be found to approximate towards a sine (or cosine) curve whose period is about 12 lunar hours.

The actual tidal curves are usually more complex, particularly in the tropics where the inequality in the diurnal tide is large, and also its range.

Prediction.

16. The harmonic analysis of tidal observations consists in the dissection of the aggregate tidal wave into a number of partial constituent waves of which the amplitude and epoch are determined, and prediction involves the recomposition or synthesis of these waves. In the synthetic process, the partial waves have to be recompounded in their proper relative positions which are determined by the positions of the moon and sun at the time chosen for the commencement of the prediction. This can be effected by a laborious process of mathematical computation, uniting each group of tides (*e.g.* diurnal, semidiurnal etc.,) into single waves and finally into a resultant wave. The task of forming a general tide table by these means consists in the determination of all the possible periods and heights of the resultant wave and the tabulation of the heights and intervals after the moon's passage of its high and low water.

Tide-predicting machine.

17. The synthesis is however more easily carried out by means of the tide-predicting machine, which was invented by Lord Kelvin in 1872, and constructed under the supervision and design of Mr. E. Roberts, F.R.A.S. by Messrs. Légé and Co. of Paris.

This machine effects the composition of simple harmonic motions in one line and traces the tidal curves on a diagram. By measurements of the abscissæ and ordinates from the latter the times and heights of the tides at any time can be read.

A fuller description of this machine and the method of its employment are given in Chapter II.

Explanations of certain terms used in tidal work.

18. A *tide* may be defined as the daily rising and falling of the waters of the ocean.

An *astronomical tide* is a tide due to the attractions of the sun and moon.

A *meteorological tide* may be defined as the regular alternation of sea level due to the wind, changes of atmospheric pressure, and other meteorological causes, *e.g.* land and sea breezes by day and night, the melting of snows, and annual variability in rainfall and evaporation which produce such apparently periodic changes of level in river estuaries, that they partake of the tidal character. These causes give rise to daily, semi-annual and annual meteorological tides.

19. The terms "*ebb and flow*" in tidal rivers are not synonymous with "rise and fall" for at the moment of high water the current is most rapid up stream and at low water most rapid down stream. Hence the tidal current "flows" long after high water has passed and "ebbs" for a long time after low water and when the water level is rising. As a consequence the tide in rivers rises quicker than it falls, and a shorter time elapses between low and high water than between high and low water.

When an estuary contracts considerably, the range of a tide becomes greatly magnified as it narrows. The augmentation of the height is due to the concentration of the energy of the motion of a large mass of water into a narrow space.

20. *Bore or Eagre.*—The heading back of the sea water by the natural current of the river, and the progressive change of the shape of a wave in shallow water, which gradually steepens in front, while its rear slope becomes more gradual, is supposed to cause the phenomenon known as a bore or eagre, in which the tide rises with such rapidity that the wave assumes the form of a wall of water and advances in this form. The following Indian rivers have bores :—the Ganges, the Hooghly, Brahmaputra and Indus.

The phenomenon also occurs in rivers in other countries, as well as in narrow arms of the sea, *e.g.* the Amazon, Tsientang, Garonne, Dordogne, Seine, Trent, Severn, Wye rivers and the Solway Frith, arms and bays of the Bay of Fundy.

21. *Seiches* are short-period oscillations, (usually from about 10 to 60 minutes), existing at times in lakes or land-locked bays. They represent oscillations in which usually the whole body of the liquid swings to and fro. They are caused by sudden changes in atmospheric pressure, or winds which sweep over its surface, etc. The period of such a seiche is

$$\frac{\text{twice length of lake}}{\sqrt{gh}}$$

where g denotes the acceleration of gravity, and h, the depth of the lake or bay. Seiches may not always be uni-nodal as supposed above, nor does the nodal line always run transversely to the body of water.

An account of seiches is given in Darwin's Tides. The phenomenon has been observed on lakes such as Lake Constance and Lake Geneva etc., and on bays in India, compare p. 337 of the U. S. Coast and Geodetic Survey Report for 1897. Such a seiche with a period of about 15 minutes exists in Madras harbour.

22. *Establishment of a port.*—For a very rough determination of the time of high water, it is sufficient to add the solar time of high water on the days of full and change of moon, called the "vulgar establishment of the port" to the time of the moon's passage over the meridian, either visibly above or invisibly below the horizon.

23. *Corrected Establishment of a port.*—Spring tide occurs from 1 to 3 days after the full and change of moon. It is more important to know what occurs at spring tide than at new and full moon, so that the term "corrected establishment of a port" is used to denote the interval elapsing at a spring tide between the moon's transit and high water. The difference between the ordinary and corrected establishments is of small amount.

24 *Spring rise, Neap range, Neap rise.*—The average height at spring tide between high and low water marks is called the 'spring rise'. That at neap tides however is called 'the neap range'. The term 'neap rise' is used to mean the average height between high water of neap tides and the low water of spring tides. The term "rise" thus refers to the rise above level of low water at spring tides, both in the cases of spring and neap tides.

25. *Age of the tide.*—The average interval elapsing between the full or change of moon and spring tide is called "the age of the tide," as it represents the age or interval which the tide is assumed to take to reach a place after the moon's meridian passage at full or change of moon.

26. *The mean establishment of a port.*—The mean establishment comprising the mean of all the luni-tidal intervals is that generally utilised, as its value is not dependent on the age of the tide, as is the value of the vulgar establishment, and the corrections to the mean value on account of age are not variable in the same manner as those which have to be applied to the vulgar establishment. The mean establishment can be computed by taking the mean of the luni-tidal intervals for a whole lunation *i.e.* from one new moon to the next one succeeding it. The computation is carried out as follows :—

Select a whole month's tidal registerations preferably in mid-year. Find out from the Nautical Almanac the date and time of the new moon for the particular month selected. This is given in astronomical time each day commencing and ending at noon*. The tidal registrations are generally in civil time, each day commencing and ending at midnight. Convert astronomical to civil time for the values of the transits and make out a table as fallows :—

Date	Time of high water in local mean time.	Time of moon's transit at Greenwich in Greenwich mean time.	Luni-tidal interval.

It should be noted that the time of high water will always be after that of the moon's transit and hence the former will be the greater.

The mean of all the luni-tidal intervals is then taken out.

This mean is referred to the time of moon's transit at Greenwich. It is now necessary to correct this so as to refer to the local time of transit over the meridian of the place. Now the moon moves away from the sun, so that its distance increases by about 48 minutes in 24 hours so that roughly a correction of about minus 2 minutes is necessary for every hour of longitude east of Greenwich.

This correction having been applied, the resulting value is the mean establishment of the port.

Values of the mean establishment of each port are published in the tide tables derived from several sets of results, computed as above explained.

Predictions for Riverain ports.

27. In the case of predictions for Riverain ports, it has been found better to refer the tides not to the moon's transits immediately preceding, but to transits varying from 21 to 40 hours previously. Monthly mean values of the intervals are thus obtained between moon's transit and high and low water, and charts of these are plotted corresponding to each hour. The values corresponding to the transits read from these charts undergo further corrections for declination, parallax, as well as for readings taken from a curve representing the diurnal tide.

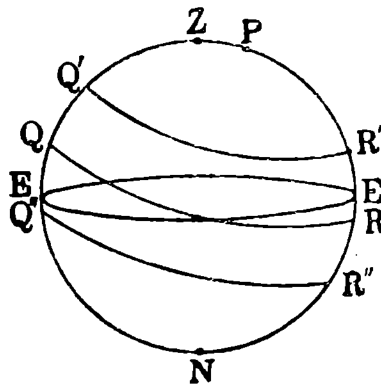
In this method the tides are referred to transits of the moon *at Greenwich* in order to simplify the computations, and to obviate the necessity of finding *the local times* of transit.

* From 1925 onwards the times styled G. M. T. are reckoned from midnight, as in civil usage, so as to correspond with the civil date, instead of from noon, this change having been notified by the Nautical Almanac.

The method of Riverain prediction is fully explained in Chapter I para 109 *et seq*.

28. *Diurnal Inequality.*—Although this term may be applied to the times and heights of high water, yet its application has more immediate reference to the height, as the rise and fall in feet is the important question to the pilot.

The reason for the diurnal inequality is best explained by means of a diagram.



Taking the equilibrium theory as a working hypothesis, the lunar tide must be highest when the moon is nearest the zenith or nadir, Z and N in the diagram. Hence high tide takes place at the moon's upper or lower transits, when its zenith and nadir distance are least.

But for a place in north latitude, when the moon's declination is north, it describes a small circle Q′R′, and its least zenith distance ZQ′ is less than its least nadir distance NR′. Hence there is a difference of height between alternate superior and inferior high waters, the superior high water meaning that which occurs when the moon is above the horizon, and the inferior when it is below. Thus the diurnal inequality vanishes, when the moon is on the equator EE′ and is at its maximum, when the declination is greatest. But there is a more marked diurnal inequality in alternate low waters, and, as the difference amounts sometimes to nearly one-half of the extreme range of the spring tides in the tropics, the importance of having tide-tables which recognise these inequalities of both high and low waters cannot be exaggerated.

29. *Datum of soundings or Plane of reference.*—In England the diurnal inequality is so small as to be relatively unimportant, and in 1878 in the Admiralty tide-tables, times were only predicted for the time and mean height of the high waters, while the zero of the tables was given as the mean height of the low water of ordinary spring tides.

The observations at Karachi made it obvious that the Admiralty method was insufficient for our inter-tropical tide, as it ignored the diurnal inequality, and a datum of reference, based on means of low waters, which varied by as much as 5 to 8 feet would not be satisfactory, as many low waters would fall below the zero and become negative.

The British Admiralty refer their soundings and tide-tables, as already stated, to the mean low water mark of ordinary spring tides.

In India however, on account of the diurnal inequality, the datum of reference was taken as below the mean sea-level by the sum of the semi-ranges of the tides M_2, S_2, K_1, and O_1 (vide para 42). This datum is called the Indian "spring low-water mark." This differs from the Admiralty datum of soundings which we have to adopt in our tide-tables in order to agree with their charts.

As, in spite of the adoption of the above datum, we still continue to obtain negative results for low water heights, it has been suggested for consideration by the Hydrographer to the Admiralty, London S.W. that we should adopt a still lower datum of reference viz :—a datum obtained by deducting from mean sea-level the sum of the semi-ranges of the tides M_2, S_2, K_1, O_1 and also Sa.

The objects of tidal research.

30. The results obtained from tidal observations are both of scientific and practical utility. Though we have little to do with the scientific side of the subject at present, it may not be out of place to mention that a better knowledge of the laws of the tides is expected to lead to an evaluation of the mass of the moon, to more definite information regarding the rigidity of the earth, to an approximation of the depth of the sea from the observed velocities of tidal waves, and to information regarding the retardation of the earth's velocity due to tidal friction, etc.

31. *Tidal friction.*—In the evolution of worlds, tidal action has had a very important influence. It has long been recognised that, in the case of the earth, the tidal wave must act as a kind of friction brake, gradually retarding the rate of rotation. But any such change in the rate of rotation of the primary body must be accompanied by changes in the distance and time of revolution of the moon. Calculating backwards Prof. G. H. Darwin has found that originally the moon must have been nearer the earth. Previous to its existence as a separate satellite it was part and parcel of the earth and its origin as a satellite is believed to have been due to the tidal action of the sun acting upon a fluid or semi-fluid body, whose period of rotation was nearly equal to

its natural period of oscillation. Thus the properly timed impulses acted so as to produce large tidal distortions, which finally resulted in the separation of the body into two parts. The tidal action of the earth has long ago compelled the latter to rotate upon its axis in exactly the same time as it revolves in its orbit. There can be little doubt that before the moon had cooled down to its present unchangeable condition, very large tides must have been generated in it, and these would act as friction brakes, so long as the period of the moon's axial rotation was shorter than its time of orbital revolution. There is evidence of Jupiter's satellites having the same peculiarity, and it is now believed that Venus and Mercury have their day equal to their year. This would be attributable to the fact that the greater proximity of the sun to the two inferior planets would produce correspondingly larger tides with a correspondingly quicker reduction of the axial to the orbital period. Similar to the tides, as they exist in the fluid portion of the earth, there are tidal stresses in the solid crust, and it has been considered that they may possibly be a factor in the production of earthquakes. Prof. G. H. Darwin has considered the question as to whether there is any evidence of a slight yielding of the crust under the influence of tidal stresses. He finds evidence of a slight yielding showing that the earth has an effective rigidity about equal to steel.

GENERAL DESCRIPTION OF THE TIDES.

32. As already explained in para 28, there are diurnal inequalities in the tides owing to the varying declination of the sun and moon. These phenomena can be represented by supposing a diurnal tide high only once in a lunar day of $24^h\ 50^m$ combined with a semi-diurnal tide high twice in this period; also a diurnal tide high once in a solar day of 24^h combined with a semi-diurnal tide high twice in this period.

The moon's declination goes through a complete cycle of changes in a lunar month. Thus for half a month the moon's declination will be north and for half a month south of the equator, and, at corresponding periods, will have the same value but be of opposite sign. Hence it will describe diurnal circles equidistant from the equator at intervals of a fortnight, and the tides so formed have the same heights. This can be represented by supposing a fortnightly tide of the proper height combined with the diurnal and semi-diurnal ones. In just the same way the smaller tides, caused by the sun, can be artificially represented by combining a diurnal and semi-diurnal tide, (the solar day being used), and a semi-annual tide.

It may likewise be shown that according as the moon is in apogee or perigee, there will be a lesser or greater heaping up of water round the equator and sinking at the poles, and so there would be a monthly tide from this cause called the lunar monthly elliptic, and for similar reasons there would be an annual solar tide caused by parallax, as the earth was nearer or further from the sun.

There are also meteorological tides, as already explained in para 18, of daily, semi-annual or annual periods.

33. Hence the chief tidal constituents in most localities are :—

1. Lunar and solar semi-diurnal tides.
2. Lunar and solar diurnal tides.
3. Lunar fortnightly and solar semi-annual tides.
4. Lunar monthly and solar annual tides.

34. *Short period tides.*—Of these all tides which recur in periods of or about a day in duration or any aliquot part of the quasi-diurnal period are classified as short period tides.

Long period tides.—The remainder of periods of a fortnight, a month, 6 months, a year, etc. are classified as long period tides. One exceptional tide, the 19-yearly is due to the rotation of the moon's nodes in 18·6 years *vide* end of para 41.

It has been reasoned also that there should be a very minute tide due to forced nutation and precession of a probable period of about 430 days, the effects of which may be detectable by careful observation (*vide* Darwin's Tides page 231, and Christie's "Latitude variation tide").

35. As certain terms concerning the relative movements of the moon, the sun and the earth enter into tidal work and theory, the following explanation of these terms is here given.

Inequalities in the relative motion of the sun, moon and earth.

The moon's distance from the earth is not the same in different parts of its orbit. The form of its orbit is an ellipse with the earth in one of the foci. This ellipse is however continually distorted by various inequalities, chiefly due to the sun's attractive energy contending with that of the earth for mastery over its satellite.

The lunar orbit is inclined at to the earth's orbit or ecliptic at an angle of 5° 8′ 40″. The points where the two orbits intersect are called the nodes, and the line joining them the line of nodes. The point of the lunar orbit nearest the earth is the perigee and that most distant the apogee, and the line joining them the line of apsides.

Both the line of nodes and the line of apsides change their places, the former turning completely round in 18·6 years, the latter in nearly 9 years. These motions which are due to the sun's disturbing influence take place in opposite directions, the line of apsides revolving with the orbital motion, the line of nodes against it. Thus the form of the moon's orbit, and the varying nature of the forces governing it, as well as the perturbing influence of the sun cause variations in the moon's velocity. These are allowed for usually, by taking as a foundation the mean or average angular velocity, and considering its variations under the title of inequalities.

36. The tides which arise from these inequalities are classified as elliptic, declinational, variational or evectional.

Elliptic declinational, variational and evectional tides.

The term elliptic is applied to those tides which depend on the quicker or slower motion in different parts of their elliptic orbits of the sun, moon and earth, and the consequent increase or decrease of the disturbing forces as the bodies approach or recede from each other (*i.e.* the parallax).

The term declinational is applied to those tides which are caused by the changing declinations of the disturbing bodies.

The term variational is applied to those tides caused by the moon's variation, which is an inequality in the moon's motion arising from changes in direction and amount of the sun's disturbing force upon it, as it moves in its own orbit.

The term evectional is applied to those tides which arise from the moon's evection, an inequality in its motion depending on the position of the axis of the moon's orbit and the line of nodes with regard to the sun, the effects of which are complicated.

The diurnal and semi-diurnal tides have inequalities caused by parallax, and therefore there are elliptic diurnal and semi-diurnal tides. The solar annual is also an elliptic tide. The semi-diurnal tides also have inequalities depending on the varying declinations of the sun and moon, and there are therefore declinational semi-diurnal tides. The lunar semi-diurnal tide also has inequalities depending on the evection and variation perturbations of the moon, and there are therefore lunar evectional and variational semi-diurnal tides.

Overtides.

37. Out in the ocean the principal lunar semi-diurnal tide is a simple wave, but when it runs into shallow water at the coast line, and still more so

in an estuary, it changes its shape. The low water lasts longer than high water in rivers, as already mentioned in para 19, and the time which elapses from low to high water is generally shorter than that from high to low water. The wave is no longer simple, and this has to be considered as consisting of the fundamental lunar semi-diurnal wave with a period of 12 hrs. 50 mins., of the first overtide or octave with a period of 6 hrs. 25 mins., and of the second overtide or twelfth with a period of 4 hrs. 17 mins., and of the third overtide or double octave with a period of 3 hrs. 13 mins., etc.

Hence in shallow water, as hereafter explained in para 43, we find the component tides M_2, S_2, N_2; etc. accompanied by the overtides M_4, S_4, N_4 ; M_6, S_6, N_6 ; etc.

The analogy with musical notes is here complete, for a musical note of any quality is built up of a fundamental, together with its octave and twelfth, which are called overtones. So also the distorted tidal wave in a river is regarded as consisting of a simple fundamental tide with overtides of one-half and one-third the length and period of the fundamental wave.

In estuaries the first overtide of the lunar semidiurnal tide is often of great importance and even the second is considerable, the third is usually small and the fourth and higher overtides imperceptible. In the same way overtides must be introduced to represent the change of form of the principal solar semi-diurnal tide, but it is not usually found necessary to consider them in the case of the less important partial tides. The octave and the twelfth may legitimately be classed as due to the attractions of the sun and the moon, although arising indirectly from the distortion caused by the shallowness of the water.

Compound Tides.

38. In shallow water there may be sensible compound tides, viz:— the lunar and solar tides will give rise to tides having for their arguments the sum and difference of the arguments, or multiples of the arguments of the original tides. For instance as explained hereafter in para 41 the original tides $2\gamma - 2\sigma$ or M and $2\gamma - 2\eta$ or S, the sum of which gives $4\gamma - 2\sigma - 2\eta$, a quarter-diurnal tide, and the difference $2\sigma - 2\eta$ a synodic * fortnightly tide. There appears also to be a shallow water semi-diurnal tide of sensible magnitude, whose argument is,

$$4(\gamma - \sigma) - 2(\gamma - \eta) \text{ or } 2\gamma - 4\sigma + 2\eta$$

* The term synodic is applied to the tides dependent on the sun and moon being in conjunction.

but this is also the argument of the variation tide, and this will account for any perturbations in the value of that tide (vide table in para 42).

These compound tides are sometimes called Helmholtz tides from their analogy with his theory of compound sounds.

39. The following description extracted from the British Association report of 1920 gives a full account of the Harmonic Tidal constituents that have been derived. It may be mentioned that the nomenclature of the tides has been carried out partly by the use of English, and, partly, by the use of Greek letters. The former were accepted as denoting some of the fictitious satellites of Laplace's theory, *eg.* M, the mean Moon, S the mean Sun, etc.; the latter from γ, the earth's velocity of rotation, σ the mean motion of the Moon, η, the mean motion of the Sun, ϖ, the mean motion of Lunar Perigee. Some were arbitrarily named.

40. *Harmonic Tidal Constituents.*—The gravitational forces generating the tides are derivable from a potential which is everywhere proportional to what the height of the tide would be if water covered the whole earth and had lost its inertia without losing its gravitational properties.

Such a tide—the equilibrium tide—may be calculated by adding the amounts by which a certain pair of nearly spherical surfaces of revolution project above the mean water level. Each of these surfaces encloses a volume equal to that of the earth, and is slightly variable in shape. They move so that their axes, while always passing through the centre of the earth, also always pass through the centres of the sun and moon respectively. The tides due to either of these spheroids may be expressed as a series of constituents, each of which varies harmonically in a period determined by astronomical data. From dynamical principles it follows that to each of these constituents there will correspond a similar constituent in the actual tides, that is, a constituent varying harmonically in the same period.

To find, in the actual tides at any station, the amplitude of each of these constituents, together with the lag of its phase behind that of the corresponding constituent of the generating potential, is the object of the harmonic analysis of tidal observations.

41. Let us consider the speeds of the constituents of lunar origin; we have to examine the motion, relative to any point on the earth's surface, of the spheroid whose axis passes always through the moon.

The pole of this spheroid which is nearer the moon is a little further from the earth's centre than is the opposite pole, while the whole departure from sphericity depends on the distance of the moon.

Let γ denote the angular speed of the earth's rotation and σ the mean motion of the moon.

If the moon moved with constant angular speed in the plane of the equator and at a constant distance from the earth, we should have, at any station, high water occurring regularly at intervals of $\pi/(\gamma-\sigma)$ with a maximum range of tide at the equator. The rise and fall of the water would not be quite simply harmonic, but could be resolved, with sufficient accuracy, into a harmonic constituent of speed $2\ (\gamma-\sigma)$, of amplitude inversely proportional to the cube of the moon's distance, and two much smaller constituents of speeds $(\gamma-\sigma)$, $3(\gamma-\sigma)$, and of amplitudes inversely proportional to the fourth power of the moon's distance. The fact that the moon does not move as here supposed causes many modifications, but it is only on the constituent of speed $2\ (\gamma-\sigma)$ that their effect need be considered.

Let us suppose the moon to move in the equator, but take into account the elliptic, evectional and variational inequalities in her distance and motion. These inequalities have speeds

$$\sigma - \varpi,\ 2\ (\sigma-\varpi),\ (\sigma-2\eta+\varpi),\ 2\ (\sigma-\eta)$$

where ϖ denotes the mean motion of the lunar perigee and η that of the sun. The effect of each is to make the moon's sidereal motion increase and decrease with the reciprocal of her distance, and thus to make the period of the tides increase and decrease with their range. The effect of the first order elliptic inequality and the evectional inequality is the introduction of new harmonic constituents of speeds

$$2\ (\gamma-\sigma) \pm (\sigma-\varpi),\ 2\ (\gamma-\sigma) \pm (\sigma-2\eta+\varpi)$$

of which, for the reason just given, the greater are those of speeds

$$2\ (\gamma-\sigma) - (\sigma-\varpi),\ 2\ (\gamma-\sigma) - (\sigma-2\eta+\varpi).$$

The effect of the second order elliptic and evectional inequalities is sufficiently represented by the introduction of new harmonic constituents of speeds—

$$2\ (\gamma-\sigma)-2\ (\sigma-\varpi),\ 2\ (\gamma-\sigma)-2\ (\sigma-\eta).$$

The daily mean level of the water depends slightly on the departure from sphericity of the spheroid, so that we have long-period elliptic, evectional, and variational constituents of speeds,

$$\sigma - \varpi,\ \sigma \cdot 2\eta + \varpi,\ 2\ (\sigma-\eta)$$

respectively.

If the moon moved with constant angular speed in a parallel of latitude other than the equator, consecutive high tides would be unequal except at the equator, and we should require the introduction of a new constituent of speed $\gamma-\sigma$, with an amplitude vanishing at the equator.

Also, the amplitude of the constituent speed $2(\gamma-\sigma)$ would be less than when the moon was in the equator.

But since the declination of the moon changes, the diurnal constituent requires modification. If its amplitude could be regarded as changing harmonically with speed σ, it would be replaced by two harmonic constituents of equal amplitudes and speeds $\gamma-\sigma\pm\sigma$.

Owing to the fact that this is not quite so, the amplitude of the constituent of speed $(\gamma-2\sigma)$ is a little greater than that of speed γ, and there is another smaller constituent of speed $\gamma+2\sigma$.

Again, introducing the first order elliptic inequality we get new harmonic constituents of speeds $(\gamma-2\sigma)\pm(\sigma-\varpi)$, $\gamma\pm(\sigma-\varpi)$, of which those of speeds $(\gamma-\sigma\pm\varpi)$ are regarded as forming a single constituent of speed $(\gamma-\sigma)$ and slowly varying amplitude. The second order elliptic, the evectional and variational inequalities give rise to new constituents of speeds $(\gamma-2\sigma)-2(\sigma-\varpi)$, $(\gamma-2\sigma)-(\sigma-2\eta+\varpi)$, $(\gamma-2\sigma)-2(\sigma-\eta)$.

Also, the changing declination of the moon causes the amplitudes of the semi-diurnal constituents to vary, but it is sufficiently accurate to take mean values in all cases except that of speed $2(\gamma-\sigma)$.

As the effect is to make the speed and range of the tide increase or decrease together, we get a new constituent of speed $2(\gamma-\sigma)+2\sigma$.

Again, the changing declination of the moon introduces the principal variation in daily mean level, in the form of a constituent of speed 2σ which with the first order elliptic inequality gives two more of speeds $2\sigma\pm(\sigma-\varpi)$.

The amplitudes of all the constituents depending on the inclination of the moon's orbit to the equator vary with the position of the node on the ecliptic. As the monthly mean level also depends on the inclination of the moon's orbit to the equator, we have a small constituent with a speed $\dot{N}$ equal to that of revolution of the moon's nodes. The speeds of the constituents of solar origin may be similarly determined, but only the declinational and first order elliptic effects on the primary constituent need be considered.

List of constituents.

42. On collecting the results we have the following tables. The constituents of the same species have similar geographical distributions of generating potential; they are arranged in decreasing order of magnitude. The symbols are the same as those in general use, with the exception of σ_1, which is now introduced for the first time. The corresponding amplitude in the generating potential is larger than that of some of the constituents given in the

other species. All the constituents given in the tables have, according to Darwin, larger amplitudes in the generating potential than any omitted.

Semi-Diurnal Species.

Symbol.	Name.	Speed.	Speed in degrees per ms. hour.
M_2	Principal lunar ...	$2(\gamma-\sigma)$	28·9841042
S_2	Principal solar ...	$2(\gamma-\eta)$	30·0000000
N_2	Larger lunar elliptic ...	$2\gamma-3\sigma+\varpi$	28·4397296
K_2	Luni-solar declinational ...	2γ	30·0821372
ν_2	Larger lunar evectional ...	$2\gamma-3\sigma+2\eta-\varpi$	28·5125830
L_2	Smaller lunar elliptic ...	$2\gamma-\sigma-\varpi$	29·5284788
T_2	Solar elliptic ...	$2\gamma-3\eta$	29·9589314
$2N_2$	2nd order lunar elliptic ...	$2\gamma-4\sigma+2\varpi$	27·8953548
μ_2	Lunar variational ...	$2\gamma-4\sigma+2\eta$	27·9682084
λ_2	Smaller lunar evectional ...	$2\gamma-\sigma-2\eta+\varpi$	29·4556254

Principal Diurnal Species.

Symbol.	Name.	Speed.	Speed in degrees per ms. hour.
K_1	Luni-solar declinational ...	γ	15·0410686
O_1	Larger lunar declinational	$\gamma-2\sigma$	13·9430356
P_1	Larger solar declinational	$\gamma-2\eta$	14·9589314
Q_1	Lunar elliptic ...	$\gamma-3\sigma+\varpi$	13·3986609
J_1	Supplementary lunar elliptic	$\gamma+\sigma-\varpi$	15·5854433
OO_1	Second order lunar ...	$\gamma+2\sigma$	16·1391016
ρ_1	Lunar evectional ...	$\gamma-3\sigma+2\eta-\varpi$	13·4715144
$2Q_1$	2nd order lunar elliptic ...	$\gamma-4\sigma+2\varpi$	12·8542862
σ_1	Lunar variational ...	$\gamma-4\sigma+2\eta$	12·9271398

Long Period Species.

Symbol.	Name.	Speed.	Speed in degrees per ms. hour.
Mf	Lunar fortnightly	2σ	1·0980330
Mm	Lunar monthly ...	$\sigma - \varpi$	0·5443747
Ssa	Solar semi-annual ...	2η	0·0821372
—	Nineteen yearly ...	$\dot{N}$	19·34 per annum
—	Ter-mensual ...	$3\sigma - \varpi$ ...	1·6424077
—	Monthly evectional	$\sigma - 2\eta + \varpi$ ...	0·4715211
MSf	Fortnightly variational ...	$2(\sigma - \eta)$	1·0158958
Sa	Solar annual ...	η	0·0410686

Note 1.—The speeds in degrees per ms. hour have been added to the tables given in the B. A. report and the nomenclature of tides made to agree with that in use on the tide-predicting machine and in the computations.

Note 2.—The subscript figures after the initial letter or letters in the symbols applied to a tide refer to the period of the tide. Thus M_1 is a lunar diurnal tide, M_2 a lunar semi-diurnal tide, M_4 a lunar quarter-diurnal overtide.

$(2M_2K_3)_3$ is a compound ter-diurnal tide, M_2N a compound quarter-diurnal tide, etc.

Besides the above there is the constituent M_1 of speed $\gamma - \sigma$ which consists partly of that of variable amplitude of the principal diurnal species and partly of that of amplitude inversely proportional to the fourth power of the moon's distance. There is also the ter-diurnal constituent M_3 of speed $3(\gamma - \sigma)$ and amplitude inversely proportional to the fourth power of the moon's distance.

SHALLOW WATER CONSTITUENTS.

43. In shallow water a harmonic constituent is accompanied by others having for their phases multiples of the phase of the primary constituent. Also, two harmonic constituents are accompanied by two others, having for their phases the sum and difference of the phases of the primary constituents. Some of these shallow water constituents have speeds the same as those of certain primary constituents, (*vide* the description of overtides and compound tides paras 37 & 38.)

In the following tables only those shallow water constituents are mentioned which it has been the custom to consider hitherto.

Primary Constituents affected by Shallow Water Constituents.

Symbol.	Primary constituent of shallow water effect.	Speed.	Speed in degrees per ms. hour.
M_2	K_1 O_1 ...	$\gamma+\gamma - 2\sigma$ ...	28·9841042
L_2	N_2 M_4 ...	$4(\gamma-\sigma)-(2\gamma-3\sigma+\varpi)$	29·5284788
μ_2, 2SM	S_2 M_4 ...	$4(\gamma-\sigma)-2(\gamma-\eta)$ ...	27·9682084
K_1	M_2 O_1 ...	$2(\gamma-\sigma)-(\gamma-2\sigma)$	15·0410686
O_1	M_2 K_1 ...	$2(\gamma-\sigma)-\gamma$ ...	13·9430356
P_1	K_1 S_2 ...	$2(\gamma-\eta)-\gamma$ ...	14·9589314
Q_1	K_1 N_2 ...	$(2\gamma-3\sigma+\varpi)-\gamma$ ...	13·3986609
Mf	K_1 O_1 ...	$\gamma-(\gamma-2\sigma)$ ...	1·0980330
Mm	M_2 N_2 ...	$2(\gamma-\sigma)-(2\gamma-3\sigma+\varpi)$	0·5443747
MSf	M_2 S_2 ...	$2(\gamma-\eta)-2(\gamma-\sigma)$ ...	1·0158958
M_1	O_1 N_2 ...	$(2\gamma-3\sigma+\varpi)-(\gamma-2\sigma)$	14·4920521

Other Shallow Water Constituents.

Symbol.	Primary constituent of shallow water effect.	Speed.	Speed in degrees per ms. hour.
M_4	M_2 ...	$4(\gamma-\sigma)$ ...	57·9682082
M_6	M_2 ...	$6(\gamma-\sigma)$ ...	86·9523126
M_8	M_2 ...	$8(\gamma-\sigma)$ ...	115·9364164
S_4	S_2 ...	$4(\gamma-\eta)$ ...	60·0000000
S_6	S_2 ..	$6(\gamma-\eta)$ ...	90·0000000
$(MS)_4$	M_2 S_2 ...	$4\gamma-2\sigma-2\eta$ ...	58·9841042
$(M_2K_1)_3$	M_2 K_1 M_4 O_1 ...	$3\gamma-2\sigma$ ...	44·0251728
$(2M_2K_1)_3$	M_2 O_1 M_4 K_1 ...	$3\gamma-4\sigma$ ...	42·9271398
$(S_2K_1)_3$	S_2 K_1 ...	$3\gamma-2\eta$ ...	45·0410686
$(M_2N)_4$	M_2 N_2 ...	$4\gamma-5\sigma+\varpi$.	57·4238338
$(2SM)_2$	S_4 M_2	$2\gamma+2\sigma-4\eta$ ...	31·0158958

METEOROLOGICAL CONSTITUENTS.

44. The observed values of Ssa and Sa are largely of meteorological origin, as also those of S_1 of speed $\gamma-\eta$ or 15·0000000 degrees per ms. hour.

45. Only 24 of the tides above described are taken account of by the tide-predicting machine, viz :—

M_2, S_2, K_1, O, N, P, K_2, μ, ν, L, T, Q, J, MS, 2SM, η or Sa, 2η or Ssa, M_4, M_6, S_1, 2N, M_2N, $2M_2K_1$ and M_2K_1, of which about 12 represent the true astronomical tide. This number of components is sufficient to represent the tide at all seaports, but the number of components is not sufficient to represent the tide at Riverain ports which have to be dealt with by the method of computing the semidiurnal tide, and correcting it by means of a curve run on the machine for the diurnal tide, as explained in Chapter I para 109.

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HARMONIC ANALYSIS OF TIDAL OBSERVATIONS.

47. Before proceeding to an account of the method in which the observations are manipulated numerically, it will be advisable to give a brief sketch of some of the properties of harmonic curves, their connection with the tidal observations and the means of determining the various constants.

A curve of this form may be represented by the equation

Harmonic curves.

$$y = a \cos (nx + b).$$

The curve is periodic, that is to say, after a certain period it takes its original form: for if $x + \frac{2\pi}{n}$ is put for x, y again becomes

$$a \cos (nx + b).$$

If $\frac{2\pi}{n}$ is put equal to λ, then the quantity λ is called the 'wave-length' of the curve; for it is the distance along the axis of x between two successive equal and similarly placed ordinates. The constant a is called the 'amplitude' of the curve because its value is that of the greatest displacement. The angle $nx + b$ or $\frac{2\pi}{\lambda} x + b$ is called the 'phase' of the curve: the constant b is therefore known if the phase is given for any value of x.

Any two curves of equal wave-length may be combined into another of the same wave-length. For the equation

$$y = a_1 \cos \left(\frac{2\pi}{\lambda} x + b_1\right) + a_2 \cos \left(\frac{2\pi}{\lambda} x + b_2\right)$$

$$= (a_1 \cos b_1 + a_2 \cos b\ ) \cos \frac{2\pi}{\lambda} x - (a_1 \sin b_1 + a_2 \sin b_2) \sin \frac{2\pi}{\lambda} x$$

$$= A \cos\left(\frac{2\pi}{\lambda} x + B\right),$$

where $$A^2 = a_1^2 + a_2^2 + 2\, a_1 a_2 \cos(b_1 - b_2),$$

and $$\tan B = \frac{a_1 \sin b_1 + a_2 \sin b_2}{a_1 \cos b_1 + a_2 \cos b_2}$$

represents a harmonic curve of the same wave-length as the two components. Similarly for any number of curves of the same wave-length.

Two or more curves of different wave-lengths cannot however be combined into a single harmonic curve; but if the wave-lengths are commensurable the resultant curve is periodic. For let

$$y = a_1 \cos\left(\frac{2\pi}{\lambda_1} x + b_1\right) + a_2 \cos\left(\frac{2\pi}{\lambda_2} x + b_2\right) + a_3 \cos\left(\frac{2\pi}{\lambda_3} x + b_3\right) + \&c.,$$

and let λ be the least common multiple of λ_1, λ_2, λ_3, &c., so that their actual values are $\frac{\lambda}{m_1}$, $\frac{\lambda}{m_2}$, $\frac{\lambda}{m_3}$, &c., where m_1, m_2, m_3, &c., are integers; then $y = a_1 \cos\left(\frac{2\pi}{\lambda} m_1 x + b_1\right) + a_2 \cos\left(\frac{2\pi}{\lambda} m_2 x + b_2\right)$ + &c., and if $x + \lambda$ is put for x the value of y is unaltered, so that the resulting curve is a periodic curve of wave-length λ.

Connection between harmonic curves and tidal observations

48. Such are a few of the properties of harmonic curves, and the next thing to be done is to point out the connection between harmonic curves and tidal observations.

In a *Report of a Committee for the Harmonic Analysis of Tidal Observations* for the British Association, 1883, Professor G. H. Darwin has deduced an expression for the height of the tide at any time: each term, which is of the harmonic form $R \cos(nt - \zeta)$, arises from some specific cause in the elaboration of the equilibrium theory of tides and is regarded as a separate tide due to this cause. Thus there are as many tides as there are terms in the series, and the height of each simple tide is equal to a constant, R, multiplied by the cosine of a certain angle $nt - \zeta$ called the 'argument', which is partly made up of a simple function of the time and partly dependent on the position of the sun or moon or both.

The maximum value of the cosine being unity, the constant, R, gives the greatest height above the mean of the particular tide, that is, the 'semi-range' or 'amplitude'.

The part of the argument which is a function of the time is of the form nt, so that n represents the rate at which the argument increases: it is called the 'speed' of the tide and is reckoned in mean solar hours. Also since the tide's maximum occurs when the remainder of the argument, *viz.*, ζ is equal to nt, it follows that $\frac{\zeta}{n}$ gives the time which must elapse from the beginning of the observations till the time of the first high-water of the tide: ζ is therefore called the 'epoch'.

Determination of constants R and ζ.

49. For the purpose of arithmetical calculation the form R cos $(nt-\zeta)$ in which the tide is presented, is not convenient and it is therefore expanded into

$$A \cos nt + B \sin nt,$$

so that $$R^2 = A^2 + B^2 \quad \text{and} \quad \tan \zeta = \frac{B}{A};$$

and the immediate object of the numerical reductions is to find the A's and B's from which the R's and ζ's are at once obtained by means of the above equations. It now remains to explain how the A's and B's are determined from the observations.

The expression deduced by Darwin gives for the height of the tide at any time,

$$h = A_0 + \Sigma\,(A \cos nt) + \Sigma\,(B \sin nt).$$

Now among these n's will be found n_1, $2n_1$, $3n_1$, etc.; n_2, $2n_2$, $3n_2$; and so on, and it would be practically impossible to determine the corresponding A's and B's in a direct manner. It has, however, been found possible, by a method of manipulation of the observed quantities which will be explained below, to separate the terms containing n_1 from all the others, and then the problem presents no difficulty. As will be explained, it is reduced to the question of determining the constants from a series of equations of the form

$$h = A_0 + A_1 \cos nt + B_1 \sin nt + A_2 \cos 2nt + B_2 \sin 2nt + \&c.,$$

where t has any integral value from 0 to $\frac{2\pi}{n}$, so that nt goes through its variations from 0 to 2π.

Now it is clear, if r and s be any two integers, and the summation extends from $t = 0$ to $t = \frac{2\pi}{n}$, that

$$\Sigma \cos rnt = 0, \text{ and } \Sigma \sin rnt = 0,$$

since to each positive value, there is a corresponding negative value of the cosine or sine.

$$\text{Also } \Sigma \cos^2 rnt = \tfrac{1}{2}\,\Sigma\,(1 + \cos 2rnt) = \frac{\pi}{n};$$

and $\Sigma \sin^2 rnt = \frac{1}{2} \Sigma (1 - \cos 2\, rnt) = \frac{\pi}{n}$;

$\Sigma \cos rnt \cos snt = \frac{1}{2} \Sigma \cos (r+s)\, nt + \frac{1}{2} \Sigma \cos (r-s)\, nt = 0$, and similarly

$\Sigma \cos rnt \sin snt$ and $\Sigma \sin rnt \sin snt$ are each equal to zero.

Consequently for determining the constants there are the following equations :—

$$\Sigma\, h = \frac{2\pi}{n} A_0, \qquad \text{or } A_0 = \frac{n}{2\pi} \Sigma\, h;$$

$$\Sigma\, h \cos rnt = \frac{\pi}{n} A_r, \quad \text{or } A_r = \frac{n}{\pi} \Sigma\, h \cos rnt;$$

$$\Sigma\, h \sin rnt = \frac{\pi}{n} B_r, \quad \text{or } B_r = \frac{n}{\pi} \Sigma\, h \sin rnt.$$

The A's and B's being now determined, the R's and ζ's are calculated from the two formulæ given above.

The method of determining the A's and B's described above is only applicable to the short-period tides : the means of determining them for the long-period tides will be described hereafter.

Determination of constants H and κ

50. But in order that a comparison of the records of different years may be made, it is necessary to exhibit the height of the tide in yet a different form ; for when it is represented by $R \cos (nt - \zeta)$, it is clear that ζ may have any value from 0 to 360° and that the results of the analysis of successive years of observations will not be comparable with each other.

Such being the case, let it be supposed that the results of the analysis are presented in a number of terms of the form

$$f H \cos (V + u - \kappa).$$

Here V is a linear function of the moon's and the sun's mean longitudes, the mean longitudes of the moon's and the sun's perigees, and the local mean solar time at the place of observation reduced to angle at 15° per hour. V therefore increases uniformly with the time and its rate of increase per mean solar hour is the n of the first method, or the ' speed ' of the tide.

It is supposed that u stands for a certain function of the longitude of the node of the lunar orbit, at an epoch half a year later than 0^h of the first day. Strictly speaking, u should be taken as this same function of the longitude of the moon's node, varying as the node moves ; but as the variation is but small in the course of a year, u may be treated as a constant and put equal to an average value for the year, which average value is taken as the true value of u at exactly mid-year.

Together $V + u$ constitute the whole 'argument'* according to the equilibrium theory of tides, with the sea covering the whole earth; and it therefore follows that $\frac{\kappa}{n}$ is the lagging of the tide which arises from kinetic action, friction of the water, imperfect elasticity of the earth, and the distribution of the land.

It is also supposed that H is the mean value in British feet of the semi-range of the particular tide in question; and f is a numerical factor of augmentation or diminution due to the variability of the obliquity of the lunar orbit: the method of determining it is fully explained in, and its values tabulated at the end of Professor Darwin's Report and also in the Auxiliary Tables appended.

It is obvious then, that if the tidal observations are consistent from year to year, H and κ should come out the same from each year's reductions: and it is only when the results are presented in such a form as this, that it will be possible to judge whether the harmonic analysis is giving satisfactory results.

The determination of H and κ from R and ζ is made as follows:—
Clearly $H = \frac{R}{f}$ and is at once found; also $nt - \zeta$ is identical with $V + u - \kappa$, so that if V_0 be the value of V at 0^h of the first day, that is when $t = 0$, then,

$$-\zeta = V_0 + u - \kappa;$$

so that

$$\kappa = \zeta + V_0 + u.$$

Thus the rule for the determination of κ is:—add to the value of ζ the value of the 'argument'* at 0^h of the first day.

The above statement of procedure applies to nearly all the tides except K_1, & K_2, which have their origins jointly in the tide-generating forces of the sun and the moon, and also the tides L & M_1, which are rendered complex from the fact that the tidal analysis only extends over a year. These tides require special treatment as explained in Part I pages 54 to 58 of Volume 16 G. T. Survey. For the various tides both those requiring special treatment as well as those ordinarily treated the tables Nos. 1 to 20 have been provided to facilitate the computation of which a sample is given, from which the procedure may be readily followed.

The tides are divided for purposes of calculation into short period and long period tides as explained previously. The short

* See Schedules [B, i], [B, ii], [B, iii] and C of Professor Darwin's Report.

period tides are still further sub-divided into semi-diurnal and diurnal tides. The former have periods equal or nearly equal to 12 mean solar hours and the latter have periods equal or nearly equal to 24 mean solar hours. Besides these there are the 'overtides' and 'compound' tides whose origin has already been explained. The long period tides have periods of about a fortnight, a month, half a year or a year, as the case may be. A table giving a list of the tides with speeds, initial arguments and factors for reduction, which will be found of assistance, is given below.

51. The symbols in the table have the following meanings :—

List of tides.

γ = earth's angular velocity of rotation.

σ = mean motion of the moon.

η = " " " sun.

ϖ = " " " lunar perigee.

h = sun's mean longitude.

p = mean longitude of the moon's perigee.

ν = right ascension of the 'intersection' or descending node of the equator on the lunar orbit.

s = moon's mean longitude.

ξ = longitude 'in the moon's orbit' of the 'intersection.'

I = obliquity of the lunar orbit to the equator.

ω = " " ecliptic.

i = inclination of the moon's orbit to the ecliptic.

P = longitude of the moon's perigee at mid-year measured from the 'intersection.'

The letters with the zero subscript represent the values of the corresponding functions at 0^h of the 1st day of the year of observation.

LIST OF TIDES

LIST OF

Initial	Speed in symbols and in m.s. hours		Name of tide
S_1	$\gamma-\eta$	15°	Principal Solar Diurnal
S_2	$2(\gamma-\eta)$	30	,, ,, Semi-Diurnal
$*S_4$	$4(\gamma-\eta)$	60	Principal Solar Series Over-Tides
$*S_6$	$6(\gamma-\eta)$	90	
$*S_8$	$8(\gamma-\eta)$	120	
P_1	$\gamma-2\eta$	14·9589314	Larger Solar (Declinational) Diurnal
T_2	$2\gamma-3\eta$	29·9589314	Solar Elliptic
$*M_1$	$\gamma-\sigma-\varpi$ and $\gamma-\sigma+\varpi$	14·4920521	Principal Lunar Diurnal
M_2	$2(\gamma-\sigma)$	28·9841042	,, ,, Semi-Diurnal
$*M_3$	$3(\gamma-\sigma)$	43·4761563	Principal Lunar Series Over-Tides
M_4	$4(\gamma-\sigma)$	57·9682082	
M_6	$6(\gamma-\sigma)$	86·9523126	
$*M_8$	$8(\gamma-\sigma)$	115·9364164	
K_1	γ	15·0410686	Luni-Solar Diurnal (Declinational)
K_2	2γ	30·0821372	,, ,, Semi-Diurnal ,,
N_2	$2\gamma-3\sigma+\varpi$	28·4397296	Larger Lunar Elliptic
$2N_2$	$2\gamma-4\sigma+2\varpi$	27·8953548	Lunar Elliptic, 2nd order
L_2	$2\gamma-\sigma-\varpi$ and $2\gamma-\sigma+\varpi$	29·5284788	Smaller Lunar Elliptic
ν_2	$2\gamma-3\sigma-\varpi+2\eta$	28·5125830	Larger Lunar Evectional
O_1	$\gamma-2\sigma$	13·9430356	Lunar (Declinational) Diurnal
Q_1	$\gamma-3\sigma+\varpi$	13·3986609	Lunar Elliptic Diurnal
J_1	$\gamma+\sigma-\varpi$	15·5854433	Supplementary Lunar Elliptic diurnal
$(MS)_4$	$4\gamma-2\sigma-2\eta$	58·9841042	Compound Tides
μ_2 or 2MS	$2\gamma-4\sigma+2\eta$	27·9682084	
$(2SM)_2$	$2\gamma+2\sigma-4\eta$	31·0158958	
$(M_2K_1)_3$	$3\gamma-2\sigma$	44·0251728	
$(2M_2K_1)_3$	$3\gamma-4\sigma$	42·9271398	
$(M_2N)_4$	$4\gamma-5\sigma+\varpi$	57·4238338	
*MSf	$2\sigma-2\eta$	1·0158958	Variational Fortnightly
*Mm	$\sigma-\varpi$	0·5443747	Lunar Monthly
*Mf	2σ	1·0980330	,, Fortnightly
Sa	η	0·0410686	Solar Annual
Ssa	2η	0·0821372	,, Semi-Annual

* Not used for purposes of prediction.

TIDES

Initial Argument $= V_0 + u$	Factor for reduction $= \frac{1}{f}$	Initial
Zero	Unity	S_1
"	"	S_2
"	"	S_4
"	"	S_6
"	"	S_8
$-h_0 + \frac{1}{2}\pi$	"	P_1
$-(h_0 - p_1)$	"	T_2
$(h_0 - \nu) - (s_0 - \xi) + Q - \frac{1}{2}\pi$ where $\tan Q = \frac{1}{3} \tan P$	Fac. O $\div \sqrt{\frac{5}{2} + \frac{3}{2} \cos 2P}$	M_1
$2(h_0 - \nu) - 2(s_0 - \xi)$	$\left(\frac{\cos \frac{1}{2}\omega \cos \frac{1}{2} i}{\cos \frac{1}{2} I}\right)^4$	M_2
$\frac{3}{2}$ Arg. M_2	(Fac. $M_2)^{\frac{3}{2}}$	M_3
2 Arg. M_2	(Fac. $M_2)^2$	M_4
3 Arg M_2	(Fac. $M_2)^3$	M_6
4 Arg. M_2	(Fac. $M_2)^4$	M_8
$h_0 - \nu' - \frac{1}{2}\pi$ where $\tan \nu' = \frac{\sin \nu}{\cos \nu + \cdot 464\, k}$	$\frac{1 \cdot 46407\, k}{\sqrt{1 + (\cdot 464 k)^2 + \cdot 928\, k \cos \nu}}$ where $k = \frac{\sin 2\omega(1 - \frac{3}{2}\sin^2 i)}{\sin 2I}$	K_1
$2h_0 - 2\nu''$ where $\tan 2\nu'' = \frac{\sin 2\nu}{\cos 2\nu + \cdot 464\, k}$	$\frac{1 \cdot 46407\, k}{\sqrt{1 + (\cdot 464 k)^2 + \cdot 928\, k \cos 2\nu}}$ where $k = \frac{\sin^2 \omega(1 - \frac{3}{2}\sin^2 i)}{\sin^2 I}$	K_2
Arg. $M_2 - (s_0 - p_0)$	Fac. M_2	N_2
Arg. $N_2 - (s_0 - p_0)$	Fac. M_2	$2N_2$
Arg. $M_2 + (s_0 - p_0) - R + \pi$ where $\tan R = \frac{\sin 2P}{\frac{1}{6}\cot^2 \frac{1}{2} I - \cos 2P}$	Fac. $M_2 \div \sqrt{1 - 12 \tan^2 \frac{1}{2} I \cos 2P}$	L_2
Arg. $M_2 + (s_0 - p_0) + 2h_0 - 2s_0$	Fac. M_2	ν_2
$(h_0 + \nu) - 2(s_0 - \xi) + \frac{1}{2}\pi$	$\frac{\sin \omega \cos^2 \frac{1}{2}\varpi \cos^4 \frac{1}{2} i}{\sin I \cos^2 \frac{1}{2} I}$	O_1
Arg. $O_1 - (s_0 - p_0)$	Fac. O_1	Q_1
$(h_0 - \nu) + (s_0 - p_0) - \frac{1}{2}\pi$	$\frac{\sin 2\omega(1 - \frac{3}{2}\sin^2 i)}{\sin 2I}$	J_1
Arg. M_2	Fac. M_2	$(MS)_4$
Arg. M_4	Fac. M_4	μ_2 or 2MS
$2\pi -$ Arg. M_2	Fac. M_2	$(2SM)_2$
Arg. M_2 + Arg. K_1	Fac. M_2 × Fac. K_1	$(M_2K_1)_3$
Arg. M_4 − Arg. K_1	Fac. M_4 × Fac. K_1	$(2M_2K_1)_3$
Arg. M_2 + Arg. N_2	Fac. M_2 × Fac. N_2	$(M_2N)_4$
$2\pi -$ Arg. M_2	Fac. M_2	MSf
$s_0 - p_0$	$\frac{(1 - \frac{3}{2}\sin^2 \omega)(1 - \frac{3}{2}\sin^2 i)}{(1 - \frac{3}{2}\sin^2 I)}$	Mm
$2(s_0 - \xi)$	$\frac{\sin^2 \omega \cos^4 \frac{1}{2} i}{\sin^2 I}$	Mf
h_0	Unity	Sa
$2h_0$	"	Ssa

52. The tide-gauge gives a graphical record of the height of the water above some known datum for every instant of time. The first operation performed on the tidal record is the measurement, in feet and decimals, of the height of the water above the true zero of the gauge (the height of which relatively to the datum is known) at every mean solar hour. The period chosen for analysis is about one year, and the first measurement corresponds to noon, but it has been found inconvenient hitherto to have the same initial noon at the several ports.

Description of numerical harmonic analysis for short period tides.

It would seem, at first sight, preferable to take the measurements at each mean lunar hour; but the whole of the actual process in use is based on measurements taken at the mean solar hours, and a change to lunar time would involve a great deal of fresh labour and expense.

If T be the period of any one of the diurnal tides, or the double period of any one of the semi-diurnal tides, it approximates more or less nearly to 24 mean solar hours, and if it be divided into 24 equal parts, each part may be spoken of as a T-hour, while for brevity mean solar time will be referred to as S-time.

Suppose, now, that there are two clocks, each marked with 360° or 24 hours, and that the hand of the first or S-clock goes round once in 24 S-hours and that of the second or T-clock goes round once in 24 T-hours; and suppose that the two clocks are started at 0° or 0^h at noon of the initial day. For the sake of distinctness, imagine that a T-hour is longer than an S-hour so that the T-clock goes slower than the S-clock.

The measurements of the tide-curve give the height of the water exactly at each S-hour; and it is required from these data to determine the height of the water at each T-hour. For this end, it is necessary to count T-time; but this must be done with reference to S-time and, moreover, the time must always be specified as an integral number of hours.

Beginning, then, with 0^h of the first day, it is necessary to count 0, 1, 2, &c., as the T-hand comes up to its hour marks. But as the S-hand gains on the T-hand, there will come a time when the T-hand, being exactly at the p hour-mark, the S-hand is nearly as far as $p + \frac{1}{2}$. When, however, the T-hand has advanced to the $p + 1$ hour-mark, the S-hand will be a little beyond $p + 1 + \frac{1}{2}$; that is to say, a little less than half-an-hour before $p + 2$. Counting, then, the T-hours in S-time, it is necessary to jump from p to $p + 2$. The counting will go on continuously for a number of hours nearly equal to $2p$, and then another number will be dropped, and so on throughout the whole year. If the T-hand went faster than the S-hand, it is obvious that one number would be repeated at two successive hours instead of one being dropped. Each such process may be described as a 'change'.

Now if there is a sheet marked for entries of heights of water according to T-hours from results measured at S-hours, the S-measurements must be entered continuously up to p: then comes a 'change' and the dropping of one of the S-series, after which the entry goes on continuously until another 'change' when another is dropped, and so on.

Since a 'change' occurs at the time when a T-hour falls almost exactly between two S-hours, it will be more accurate to insert the two S-entries which fall on each side of the truth. If this be done the whole of the S-series of measurements is entered on the T-sheet. Similarly if it is the T-hand which goes faster than the S-hand a gap may be left in the T-series instead of duplicating an entry. For the analysis of the T-tide there is, therefore, prepared a sheet arranged in rows and columns: each row corresponds to one T-day and the columns are marked 0^h, 1^h, ... 23^h; the 0^h's may be called T-noons. A dot is put in each space for entry, and where there is a 'change' two dots are put if there is to be a double entry, and a bar if there is to be no entry; black vertical lines mark the end of each S-day. These black lines will, of course, fall into slightly irregular diagonal lines across the page, being steeper and steeper, the more nearly T-time approaches to S-time. They slope downwards from right to left if the T-hour is longer than the S-hour, and the other way in the opposite case. The 'changes' also run diagonally with a slope in the opposite direction to that of the black lines when the T-hour is longer than the S-hour, and in the same direction in the opposite case.

A sample is annexed of parts of pages drawn up for the entries of the M-series and J-series of tides, in the former of which T-time is mean lunar time.

M—Series

	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	S-hour.
1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2^d 0^h
2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3 1
3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4 2
4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5 2
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	6 3
				&c					&c				&c				&c				&c				
67	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	70 7
68	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	71 8
69	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	72 9
70	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	73 10
71	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	74 11
Sum	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	No. of days
No.	73	73	74	74	73	73	73	73	75	73	73	73	74.	74	73	73	73	75	74	73	73	74	73	74	73^d 12^h

J—Series

	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	S-hour.
1	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	.	.	.	1^d 22^h
2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	2 21
3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	.	.	3 20
4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	4 19
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5 19
				&c					&c				&c				&c				&c				
72	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	.	.	.	70 6
73	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	71 5
74	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	.	.	.	72 4
75	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	73 3
76	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	74 3
Sum	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	No. of days
No.	73	74	73	73	74	73	73	74	73	73	73	74	73	72	74	74	71	74	73	73	73	72	74	73	73^d 4^h

Since the first day is numbered 1 and the first hour 0^h, it follows that to find the number of hour values entered in the form from 0^h of the first day, it is necessary to subtract 1 from the number of the day and to add 1 to the number of hour.

For each class of tide there are five pages similar to the annexed examples, giving in all about 370 values for the height of the water at each of the 24 special hours: the number of values for each hour varies slightly according as more or less 'changes' fall into each column.

The numbers entered in each column are summed on each of the five pages; the five sets of results are now summed and the results divided each by the proper divisor for its column, thus giving a mean value for that column. In this way 24 numbers are found which give the mean height of the water at each of the 24 special hours.

Eliminating effects of other tides.

53. Now it is obvious that if the heights be taken for each mean solar hour for a very long period, and the 24 averages taken, grouped and subjected to analysis as above, the result will exactly express the particular tide under investigation and the influences of all the other tides will be eliminated, because of the great number of the periods included in the summation; for although the other constituents would no doubt influence each individual height, yet in that period they will have been in such a number of different positions all round the circle of revolution, that in the averagings their force would be eliminated.

The elimination can however never be quite complete, as the period of analysis only extends over about a year, and the selection of certain periods not quite equal to a year for analysis will theoretically minimise the effects of the various tides on each particular tide.

For, suppose that the expression for the height of the water is

$$A_1 \cos n_1 t + B_1 \sin n_1 t + A_2 \cos n_2 t + B_2 \sin n_2 t,$$

where n_2 is nearly equal to n_1, and that it is required to eliminate the n_2-tide so as to be left only with the n_1-tide.

The expression may then be put equal to

$$\{ A_1 + A_2 \cos (n_1 - n_2) t - B_2 \sin (n_1 - n_2) t \} \cos n_1 t$$
$$+ \{ B_1 + A_2 \sin (n_1 - n_2) t + B_2 \cos (n_1 - n_2) t \} \sin n_1 t;$$

which shows that the tide may be regarded as oscillating with a speed n_1, but with slowly varying range. Now, in the column appertaining to any hour in the form, $n_1 t$ is a multiple of 15°, if n_1 be a diurnal; and of 30°, if n_1 be a semi-diurnal tide. Consider the column headed 'p-hour'; then $n_1 t = 15°p$ for diurnals and $30°p$ for semi-diurnals.

Hence the sum of all the entries, of which suppose there are q, in the column numbered p-hours, is for diurnal tides,

$$\cos 15^\circ p \left\{ A_1 q + A_2 \left[\cos (n_1 - n_2) \frac{15p}{n_1} + \cos (n_1 - n_2) \left(\frac{2\pi}{n_1} + \frac{15p}{n_1} \right) \right.\right.$$
$$\left.\left. + \cos (n_1 - n_2) \left(2\frac{2\pi}{n_1} + \frac{15p}{n_1} \right) \right] + B_2 [\&c.] \right\} + \sin 15^\circ p \left\{ \&c. \right\} \quad . . (a)$$

and for semi-diurnal tides, the arguments of all the circular functions in the expression (a) are to be doubled.

Now such a number of terms is to be chosen, that the series by which A_2 and B_2 are multiplied may vanish. This is exactly the case, if the series is exactly re-entrant and is nearly the case, if nearly re-entrant.

The condition is exactly satisfied, if

$$(n_1 - n_2)\, q \frac{2\pi}{n_1} = 2\pi\, r \text{ for diurnal tides,}$$

or

$$(n_1 - n_2)\, q \frac{4\pi}{n_1} = 2\pi\, r \text{ for semi-diurnal tides,}$$

where r is either a positive or negative integer.

That is to say, if

$$(n_1 - n_2)\, q = n_1\, r \text{ for diurnal tides,}$$

or

$$(n_1 - n_2)\, q = \tfrac{1}{2}\, n_1 r \text{ for semi-diurnal tides.}$$

It is not worth while attempting to eliminate the effects of the semi-diurnal tides on the diurnal tides and *vice versâ*, because the periods could only differ by the fraction of a day, and owing to the incommensurability of the speeds, it is impossible to avoid being wrong to that amount.

It is of course impossible to choose for each tide n_1, a period which shall minimise the effects of more than one of the tides of short-period n_2, in vitiating the values of the mean semi-ranges of the tide n_1; and accordingly the periods have been chosen so as to minimise the effect of the principal solar semi-diurnal tide S_2 on the principal lunar semi-diurnal tide M_2 and of the M_2 tide upon the other semi-diurnal tides: in the case of the diurnal tides, the periods are chosen to minimise the effects of either O or K_1.

In choosing the period for reduction 369 days 3 hours was taken for the S and M tides, as being to the nearest hour, 12½ lunations or 25 periods of spring and neap tides, and therefore giving the least possible influence of the mean lunar and solar semi-diurnal tides, each on the sets of averages used in the calculation of the other.

For the L & N semi-diurnal tides a period of 358 days 6 hours was chosen as being a period containing an integral number of periods of these tides, and eliminating as far as possible the chief or semi-diurnal lunar tide. Similarly the period for the ν tide was found to be 349 days 22 hours, and for the J & Q tides 370 days 5 hours. These 5 tidal constituents are all elliptic.

For the declinational tidal constituents the periods found similarly to those above are all so nearly equal, that the same period, *viz* :—369 days 3 hours, has been used for the whole of them.

Tabulating these periods we have

Tides named for brevity	Periods
S, M, O, K, P, μ, T, MS, 2SM 2N, M_2N, M_2K_1, $2M_2K_1$, ...	369 days 3 hours
J, Q, ...	370 days 5 hours
L, N, ...	358 days 6 hours
ν, ...	349 days 22 hours

These periods are those actually used in the computations and some of them do not exactly agree with those given in the theoretical explanation on pages 67-73 Part 1 Volume 16 G. T. S., though in the sample computations given on pages 80-279 of the same Volume, the correct periods, as given above, are adopted.

Returning now to the general notation and considering the 24 mean values, each pertaining to the 24 T-hours, it may be supposed that all the tides, excepting the T-tide, are adequately eliminated, and, in fact, a computation of the necessary corrections for the absence of complete elimination, which is given in Tidal Report of 1872, of the Tidal Committee of the B. A. under the presidency of Sir William Thomson, shows that this is the case.

Augmenting factors for correcting heights.

54. Now it is obvious that any one of the 24 values does not give true height of the T-tide at the T-hour, but gives the average height of water, as due to the T-tide, estimated over half a T-hour before and half a T-hour after that hour. A correction must therefore be determined on this account.

The required expression for the height of the tide at any T-hour is $h = A_1 \cos \theta + B_1 \sin \theta. +$ &c, &c. $+ A_r \cos r\,\theta + B_r \sin r\,\theta +$ &c.

But the results of analysis give instead of this the mean of all the h's between the limits $\theta + \frac{a}{2}$ and $\theta - \frac{a}{2}$.

That is $ha = \Sigma h$ between these limits

$= \&c. + \Sigma A_r \cos r\theta + \Sigma B_r \sin r\theta + \&c.$ between these limits,

or $$ha = \&c. + A_r \frac{2}{r} \sin \frac{ra}{2} \cos r\theta + B_r \frac{2}{r} \sin \frac{ra}{2} \sin r\theta + \&c.$$

whence $$h = \&c. + \frac{\sin \frac{ra}{2}}{\frac{ra}{2}} A_r \cos r\theta + \frac{\sin \frac{ra}{2}}{\frac{ra}{2}} B_r \sin r\theta + \&c.$$

Consequently the coefficients that express the oscillation which goes through its period r times in 24 T-hours, must be augmented by the factor $\frac{\frac{ra}{2}}{\sin \frac{ra}{2}}$ to give the true A_r and B_r.

Remembering that a is 15° and putting for r, 1, 2, 3, &c., in succession, the augmenting factors for the diurnal, semi-diurnal, terdiurnal oscillation, &c., become

$$\frac{7 \cdot 5\pi}{180 \sin 7^\circ 30'}; \quad \frac{15\pi}{180 \sin 15^\circ}; \quad \frac{22 \cdot 5\pi}{180 \sin 22^\circ 30'}; \ \&c.$$

Thus, the augmenting factors are :—

for A_1, B_1	1·00286
A_2, B_2	1·01152
A_3, B_3	1·02617
A_4, B_4	1·04720
A_6, B_6	1.11072
A_8, B_8	1·20920

In the reduction of S-series of tides, the numbers treated are the actual heights of the water exactly at the S-hours, and therefore no augmenting factor is requisite.

Determination of A's and B's.

55. If now t denotes T-times expressed in hours and n is 15°, the height h, as expressed by the averaging process explained above, is given by the formula

$$h = A_0 + A_1 \cos nt + B_1 \sin nt + A_2 \cos 2nt + B_2 \sin 2nt + \&c.,$$

where t is 0, 1, 2 . . . . 23.

Then, if Σ is the sum of the series of 24 terms found by giving t its 24 values, as before shown,

$$A_0 = \tfrac{1}{24} \Sigma h; \ A = \tfrac{1}{12} \Sigma h \cos nt; \ B_1 = \tfrac{1}{12} \Sigma h \sin nt;$$

$A_2 = \frac{1}{12} \Sigma \cos 2\,nt$; $B_2 = \frac{1}{12} \Sigma h \sin 2nt$; &c., &c.
Also, since $n = 15°$ and t is an integer, all the cosines and sines involved are equal to one of the following :—0 ; $\pm$ sin 15° ; $\pm$ sin 30° ; $\pm$ sin 45° ; $\pm$ sin 60° ; $\pm$ sin 75° ; $\pm$ 1. These are denoted in the computation forms by 0, $\pm S_1$, $\pm S_2$, $\pm S_3$, $\pm S_4$, $\pm S_5$, $\pm$ 1.

This enables the forms to be arranged in the neat tabular form on pages 1, 2 and 3 of the Analysis of Short-Period Tides, a specimen of which is given in Form No. VI, where the 24 hourly values to be submitted to analysis are written continuously down columns I and II. The subsequent operations are sufficiently indicated by the headings to the columns and it will be found on examination that the results are in reality the sums of the several series given above.

The A's and B's are thus deduced, and then $R = \sqrt{A^2 + B^2}$: it must be multiplied by the augmenting factor, already evaluated, to obtain the augmented R which, when multiplied by $\frac{1}{f}$, gives H. Next the angle whose tangent is $\frac{B}{A}$ gives ζ, which must be added to the appropriate $V_0 + u$ to find κ. The form used will be found in the specimen of the reductions : it also serves for the final treatment of the long period tides, except that there is no augmenting factor, and that the increase of n for $11\frac{1}{2}$ hours has to be added to ξ.

Long-period tides.

56. For the purpose of determining the tides of long period, it is necessary to eliminate the oscillations of water-level arising from the tides of short period. As the quickest of the tides of long period has a period of many days, the height of the water at one instant for each day gives sufficient data. Thus there will, in a year's observations, be 365 heights to be submitted to harmonic analysis. In leap-years, the last day's observation must be dropped, because the treatment is adapted for analysing 365 values.

In finding the value of the height of the water for each day, the algebraical mean of 24 consecutive hourly values, beginning with the height at noon, is taken : the result will then apply to the middle instant of the period 0^h to 23^h, that is to say to $11^h\ 30^m$ at night.

Clearance for short-period tides.

57. The formation of a daily mean does not obliterate the tidal oscillations of short period, because none of the tides, excepting those of the principal solar series, have commensurable periods in mean solar time.

A correction, or 'clearance of the daily mean', should therefore be applied for all the important tides of short period, excepting for the solar tides.

Let R cos $(nt - \zeta)$ be the expression for one of the tides of short period, as evaluated by the harmonic analysis for the same year: and let a be the value of $nt - \xi$ at any noon. Then the 24 consecutive hourly heights of the water due to this tide, beginning with that noon, are:—

$$R \cos a;\ R \cos (n+a);\ R \cos (2n+a)\ \ .\ .\ .\ \ R \cos (23n+a);$$

Now the sum of these 24 quantities is $R\frac{\sin 12n}{\sin \frac{1}{2}n}\cos(a + 11\frac{1}{2}n)$, so that the 'clearance of the daily mean' is $-\frac{1}{24}R\frac{\sin 12n}{\sin \frac{1}{2}n}\cos(a + 11\frac{1}{2}n)$, and is additive.

It has been found, practically, that only three tides of short period *viz.* M_2, N, O, exercise any appreciable effect, so that clearances for them have to be applied. It was formerly the custom to compute the clearness for these three tides, for every day in the year, as above, and to correct the daily means accordingly: but the procedure now is different, and a single correction, for each short-period tide, is applied to each of the final equations, instead of to each daily mean. The process will be explained more fully below.

The late Professor J. C. Adams suggested an alternative plan in which the tide-predicting machine may be used for the evaluation of the sums of the clearances. It is evident that $R \cos (a + 11\frac{1}{2}n)$ is the height of the tide n at $11^h\ 30^m$, and the same is true for each such tide. Hence if the tide-predicting machine is used to run off a year of fictitious tides with the semi-range of each tide equal to $\frac{1}{24}\frac{\sin 12n}{\sin \frac{1}{2}n}$ of its true semi-range, and with all the solar series and the annual and semi-annual tides put at zero, the height given at each $11^h\ 30^m$ in the year is the sum for each day of all the clearances to be subtracted.

Uncleared equations.

58. The mean of the 365 quantities is now taken to give the mean height of the water for the year; and it is evident that, even if the daily means are uncleared from the effects of the short-period tides, as is the case in practice, this yearly mean cannot be sensibly vitiated.

The yearly mean height is next subtracted from each of the 365 daily means, and 365 quantities, δh, are found giving the mean daily height of the water above the mean yearly height.

These quantities are to be the subject of harmonic analysis: and the tides chosen for evaluation are those which have been denoted above as Mm, Mf, MSf, Sa and Ssa.

Let

$$\begin{aligned}\delta h = \; & \mathrm{A}\cos(\sigma-\varpi)t + \mathrm{B}\sin(\sigma-\varpi)t \\ & + \mathrm{C}\cos 2\sigma t + \mathrm{D}\sin 2\sigma t \\ & + \mathrm{C}'\cos 2(\sigma-\eta)t + \mathrm{D}'\sin 2(\sigma-\eta)t \\ & + \mathrm{E}\cos \eta t + \mathrm{F}\sin \eta t \\ & + \mathrm{G}\cos 2\eta t + \mathrm{H}\sin 2\eta t,\end{aligned}$$

where t is time measured from the first $11^{\mathrm{h}}\ 30^{\mathrm{m}}$.

Then a little manipulation, for which the reader is referred to Professor Darwin's Report of 1883, gives the following equations:—

	Coefficient of A	Coefficient of B	Coefficient of C	Coefficient of D	Coefficient of C	Coefficient of D	Coefficient of E	Coefficient of F	Coefficient of G	Coefficient of H
$\Sigma\delta h \times \cos(\sigma-\varpi)t =$	+183·05	+ 2·14	+ 0·73	+ 4·29	+ 0·77	+ 5·04	+ 4·88	− 0·34	+ 4·96	− 0·69
,, $\times \sin(\sigma-\varpi)t =$	+ 2·14	+181·95	− 4·15	+ 1·02	− 4·90	+ 1·07	+ 3·80	+ 0·34	+ 3·88	+ 0·69
,, $\times \cos 2\sigma t =$	+ 0·73	− 4·15	+183·18	+ 0·88	+ 0·61	+ 0·92	− 1·50	− 0·10	− 1·51	− 0·19
,, $\times \sin 2\sigma t =$	+ 4·29	+ 1·02	+ 0·88	+181·82	+ 0·92	− 0·75	+ 3·05	− 0·08	+ 3·06	− 0·17
,, $\times \cos 2(\sigma-\eta)t =$	+ 0·77	− 4·90	+ 0·61	+ 0·92	+183·19	+ 0·97	− 1·68	− 0·11	− 1·70	− 0·23
,, $\times \sin 2(\sigma-\eta) =$	+ 5·04	+ 1·07	+ 0·92	− 0·75	+ 0·97	+181·81	+ 3·25	− 0·10	+ 3·27	− 0·23
,, $\times \cos \eta t =$	+ 4·88	+ 3·80	− 1·50	+ 3·05	− 1·68	+ 3·25	+182·43	+ 0·00	− 0·14	+ 0·00
,, $\times \sin \eta t =$	+ 0·34	+ 0·34	− 0·10	− 0·08	− 0·11	− 0·10	+ 0·00	+182·57	+ 0·00	+ 0·00
,, $\times \cos 2\eta t =$	+ 4·96	+ 3·88	− 1·51	+ 3·06	− 1·70	+ 3·27	− 0·14	+ 0·00	+182·43	+ 0·00
,, $\times \sin 2\eta t =$	− 0·69	+ 0·69	− 0·19	− 0·17	− 0·23	− 0·23	+ 0·00	+ 0·00	+ 0·00	+182·57

If the daily means have been cleared by the use of the tide-predicting machine as above described, these ten equations are ready for solution, but if not, then before solution of the final equations, corrections for clearance must be applied to the left hand sides.

Numerical determination of clearances.

59. The left-hand sides of these equations must now be cleared from the effects of the three tides of short period. This is done in the following manner:—

It has been shown before that the 'clearance' is

$$-\tfrac{1}{24}\,\mathrm{R}\frac{\sin 12n}{\sin \frac{1}{2}n}\cos\left(nt-\zeta+11\tfrac{1}{2}n\right).$$

The proper clearances therefore to be applied to the left-hand sides of the first and second equations will be:—

$$-\tfrac{1}{24}\Sigma\ R\frac{\sin 12n}{\sin \frac{1}{2}n}\cos\left(nt-\zeta+11\tfrac{1}{2}n\right)\cos\left(\sigma-\varpi\right)t,$$

and
$$-\tfrac{1}{24}\Sigma\ R\frac{\sin 12n}{\sin \frac{1}{2}n}\cos\left(nt-\zeta+11\tfrac{1}{2}n\right)\sin\left(\sigma-\varpi\right)t,$$

the summation extending over 365 days.

Writing $R\cos\zeta = A$ and $R\sin\zeta = B$, Professor Darwin in his report of 1883 has deduced these clearances in the forms

$$X_1\ A + X_2\ B \text{ and } Y_1\ A + Y_2\ B,$$

where the X's apply to the left-hand sides containing a cosine and the Y's to those containing a sine, and the A's and B's are taken from the analysis of the corresponding short-period tides. Simple formulæ are also giving for computing the X's and Y's; their values are shown in the annexed table, and are to be applied with the signs there given :—

		$\sigma-\varpi$	2σ	$2(\sigma-\eta)$	η	2η
M_2 $n=2(\gamma-\sigma)$	X_1	−0·05557	+0·00302	+ 5·7393	−0·10410	−0·10465
	X_2	−0·17036	−0·03773	− 2·9228	−0·07525	−0·07546
	Y_1	−0·17075	+0·04170	− 2·8400	−0·00176	−0·00353
	Y_2	+0·04410	+0·01052	− 5·7271	+0·00476	−0·00958
N $n=2\gamma-3\sigma+\varpi$	X_1	−0·05884	+0·03680	+0·02938	−0·01760	−0·01760
	X_2	−0·77758	−0·22337	−0·19384	+0·00254	+0·00254
	Y_1	−0·02059	−0·15245	−0·12210	+0·00020	+0·00041
	Y_2	+0·11381	−0·08544	−0·08081	+0·00007	+0·00015
O $n=\gamma-2\sigma$	X_1	−0·06485	+0·01673	+0·01582	−0·19240	−0·19340
	X_2	−0·34765	−0·07788	−0·08158	−0·18260	−0·18311
	Y_1	−0·34523	+0·08418	+0·08748	−0·00460	−0·00926
	Y_2	+0·04052	+0·03379	+0·03295	+0·00897	+0·01802

Method of equivalent multipliers.

60. As the determination of each of the ten quantities $\Sigma\ \delta h \cos(\sigma-\varpi)\ t$, $\Sigma\ \delta h \sin(\sigma-\varpi)\ t$, &c., by multiplying each of the 365 δh's by its proper cosine or sine and adding the results together, would be extremely laborious, the method of equivalent multipliers has been devised by Professor Adams.

The values of the respective cosines and sines are divided into eleven groups, according as they fall nearest to 1·0, ·9, ·8, ·7.........·2, ·1, 0. Then, as all the values of δh are to be multiplied by some value of the cosine or sine, and that value must fall into one of these groups, all the values of δh which belong to one of these groups are collected together, summed and the sum multiplied by the corresponding multiplier. Since there are as many positive as negative values of the cosine or sine, the signs of half of the δh's must be changed: this is effected mechanically as follows:—In the spaces in the forms for the entry of the δh's, those δh's whose signs are to be unchanged are to be entered on the left side of the space if positive, and to the right if negative. Thus, in the column corresponding to each multiplier, there are two sub-columns: these are separately summed and the difference of these sums gives the total of the column for the δh's whose signs are to be unchanged. This process is carried out in the upper half of the form and the result is called a. Exactly the same course is adopted in the lower half of the form with the δh's whose signs have to be changed, and the result is denoted by b. The complete sum of the δh's is thus a—b, and the value of a—b in each column is multiplied by the multiplier corresponding to that column, when the sum of the products will give the result required. A pair of forms, one for the cosine and the other for the sine series, is of course required for each long-period tide.

Solution of equations.

61. Having now obtained the 'cleared' values of the left-hand sides of the equations, the left-hand side of the first equation is divided by the coefficient of A in that equation, the left-hand side of the second equation by the coefficient of B in that equation and so on, the results being approximate values of A, B, C, &c. These are now substituted in the ten equations and the final values of A, B, C, &c. deduced. But the initial instant of time is the first $11^h\ 30^m$ in the year, instead of the first noon. Hence, if as before,

$$R^2 = A^2 + B^2 \text{ and } \tan \zeta_1 = \frac{B}{A},$$

then, in order to reduce the results to the normal form in which noon of the first day is the initial instant of time, the increment of the corresponding argument for $11^h\ 30^m$ must be added to ζ_1 to get ζ. Having thus determined R and ζ, H and κ will be found as before by multiplying R by its proper factor of reduction and by adding to ζ the initial argument.

62. From the analysis of the observations at any port, one value of H (the mean semi-range) and one value of κ (which may be called the mean epoch) are obtained for each constituent for each year of observation analysed. For the larger tides M_2, S_2, K, &c., the values obtained are very accordant, but in the smaller tides there are considerable discrepancies from year to year. The means of the values of H and κ for each tide are accepted as the best results.

Preparation of data for prediction of tides.

The computations of the R's and ξ's are carried out in the way described in the account of the reductions of the observations, the only difference being that to find R from H and ζ from κ, the formulæ $R = f H$ and $\zeta = \kappa - (V_0 + u)$ are used.

Suppose, now it is required to predict the tide for any open seaport, the values of the R and ζ of the 24 component tides which are to be set on the tide predicting machine, are computed for the 28th December of the year preceding the commencement of prediction. The values of R multiplied by a constant factor give the amplitudes and $(360° - \zeta)$ the phase angles to be set on the tide predicting machine. In the case of a Riverain port the procedure in computing is the same only the computation is limited to 8 component tides *viz*:—$2M_2K_1$, J, Q, P, M_2K_1, S_1, K_1 & O, as only these are set on the machine.

63. It may happen from time to time that the tide-gauge breaks down for a few days, from the stoppage of the clock, the choking of the tube, or some other accident and that other readings are not taken during the interruption. In this case there will be a hiatus in the values of δh. Now the whole process employed depends on the existence of 365 continuous values of δh. Unless, therefore, the year's observations are to be sacrificed, this hiatus must be filled. If not more than three or four days observations are wanting, it is best to plot out the values of δh graphically on each side of the hiatus and, filling in the gap with a curve drawn by hand, use the values of δh given by the conjectural curve. If the gap is somewhat longer, several plans might be adopted, for example, if there is another station in the neighbourhood the values of δh for that station might be inserted; or, the values of δh for another part of the year, in which the moon's and sun's declinations are as nearly as possible the same as they were during the gap might be used and, as a matter of fact, these methods have been used. When the hiatus is of considerable length, the preceding methods are inapplicable, and the plan employed is as follows:—The actual δh's are entered in their proper

Interruption in record.

places; then in the ten final equations all the terms with small coefficients are neglected, and in the terms whose coefficients are approximately 182·5, a coefficient equal to 182·5 diminished by half the number of days of hiatus, is substituted; the computations are then carried out, as if there was no gap, until the values of R and ξ are obtained for each long-period tide. From these approximate values of R and ζ, the height of each tide for each day of the gap is computed from the formula R cos $(nt—\zeta)$, where t is the number of days since the commencement of the year of observation and n the speed of the particular long-period tide per mean solar day. Thus five heights, above or below mean-water level, are obtained for each day of the gap. These five heights are added together and the sum is the missing δh for the particular day. The gap having been thus filled in with computed δh's, the whole computation is repeated with the completed series.

Where a break extends over two or three months in the first half of a working year, the observations antecedent to the break in that half-year are rejected, and the date of the working year put forward to the date following the end of the break: but if the break occurs in the second half of the working year, that year is considered to end at the commencement of the break, and to begin 365 days before it. And the year following the break will begin at the end of the break.

Forms.

64. In entering the heights of the water, read off the diagrams, in the computation forms, various precautions are taken to guard against error. The procedure adopted in regard to these forms will now be explained, in so far as is not self-evident from the forms themselves. The forms given below have for convenience been numbered consecutively from I to XI, but this is not the case in the actual forms.

Form I, S-Series.

65. After the diagrams have been prepared, the heights are successively measured and entered in the respective hour columns of this form, the first entry being that of 0^h of the 1st day. The first day is called 1 in the forms for short-period tides and 0 in those for long-period tides. The date for each day is generally written in pencil on the left-hand margin.

The measurements are made to hundredths of a foot, by means of a paper scale divided into tenths and hundredths in accordance with the working scale of the instrument which registered the diagram under treatment. When the reader has called out the entry for each 23rd hour, the recorder checks the record by calling out 'end of the day' and then gives the date of the next day.

The reading and entry of the heights in the S-Series is done in duplicate, the original set being generally measured by one, and the duplicate by another computer. While the measurement for the duplicate is going on, the first computer watches the entries in the original and if the reading differs by more than 0·01, 0·03 or 0·05 of a foot according as it is the natural, half or smaller scale, that height is at once remeasured and a correct value entered both in the original and the duplicate. The original and duplicate are now compared, and if there are any discrepancies larger than those above mentioned, they are noted, remeasured at the completion of the comparison, and correct values entered in both copies.

Form II, &c. M, &c., Series.

66. The heights are next copied from the original of the S-Series into the M-Series, from the M-Series into the O-Series, from that into the K-Series and so on, the last being now the 2MK-Series.

Where a double dot occurs in the forms, it indicates that two successive hourly values of the S-Series are to be entered, the first above the second : when a horizontal line takes the place of one of the usual dots, it means that no entry is to be made there, but that the next entry is to be in the next column to the right.

A black vertical line means that the solar day divides at the line and that the height immediately preceding corresponds to a 23rd solar hour. A double dot with a short black vertical line opposite the upper one, means that the entry made at the upper dot is the height corresponding to the 23rd hour of one solar day, and that at the lower dot to the 0 hour of the next solar day. By these marks, the copyist knows that he is at the end of a mean solar day.

At the right-hand side of all the forms, except the S-form, is a column headed S-hour giving the day and hour of the S-Series corresponding to the 23rd hour of the particular day of the series in question. This is a further check but is rarely used.

Comparison of entries.

67. When the 2MK-Series has been copied from the MK-Series, it is compared simultaneously with the original and duplicate of the S-Series, and if it agrees with the original the copying is perfect. The comparison with the duplicate guards against gross errors which may have escaped notice in the comparison of the original and duplicate.

Errors found in the 2MK-Series are searched for in the other series in the reverse order until an entry is reached in which the error does not occur, and corrections are made accordingly.

68. The heights in each column are then added together, the units, tens and hundreds being separately summed, and the sums entered at the bottom of the page. Some weeks afterwards they are verified by fresh computers.

Addition of hourly heights.

Besides this, for the S-Series, the sum of the 24 hourly heights is taken for each day and entered in the column 'daily sum,' which, divided by 24, gives the quantity in the column 'daily mean.'

69. For the S-Series the total of the 'daily sums' should be equal to the total of the horizontal line at the bottom of the page in each of the five pages of the form.

Checks

For the other series, for example the M-Series, the total of the heights on page 1 of M should be the total of the heights on page 1 of S, less the sum of the last 12 hours on page 1 of S, since the last entry on page 1 of M corresponds to $74^d\ 11^h$ of S. In comparing the totals after the first page, account must be taken of the number of entries in excess or defect at the beginning as well as at the end of each page, as compared with the entries on the corresponding page of S.

70. This requires no explanation; but care must be taken that the number of observations is correct: it should be the sum of the five quantities, one on each page, at the bottom of the page under the corresponding hour.

Form V. Summations and means.

The remainder of the forms for short-period tides are self-explanatory.

71. The mean height of the water for each day is taken from the column 'daily means' in the S-Series, and the mean height of the water for the whole year, or A_0, is determined in the form for the S-Series which corresponds to Form VI. The latter mean is subtracted from each of the former quantities, giving a number of small positive and negative quantities δh, one for each day. These are entered in Form IX, of which there are two for each long-period tide, in the manner described in the foot-note.

Form IX. Long-period tides.

72. In solving the first equation, the second line is obtained by introducing for B, C, D, etc., the first approximate values obtained in the preceding form. In solving the second equation, the second approximate value of A is introduced and the first of the other quantities, C, D, etc., and so on.

Form XI.

The manner of carrying out the remaining computations is sufficiently evident from an inspection of the forms given below.

SHORT-PERIOD TIDES.

Commencing 0 hours.

Astronomical time, 1st May, 1883.

Argument $(\gamma - \eta)$.

[KARACHI, 1883-84.

FORM I.—SERIES S.

Motion per mean Solar hour = 15°.

	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	Daily Sum.	Daily Mean.
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76	9·00	8·16	7·30	6·30	5·67	5·44	5·76	6·46	7·18	7·92	8·56	8·80	8·36	7·41	6·52	5·62	4·91	176·20	7·342
2	4·81	5·51	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90	5·77	4·75	4·36	4·97	5·86	6·95	8·15	9·07	9·42	9·09	8·14	6·90	5·76	177·62	7·401
3	5·00	4·83	5·50	6·72	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05	5·50	6·91	8·50	9·64	10·14	9·76	8·48	7·08	175·43	7·310
4	5·79	4·96	4·71	5·37	6·77	8·24	9·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	7·06	8·84	10·24	10·84	10·20	8·72	170·02	7·084
5	7·24	5·96	4·90	4·48	5·23	6·84	8·51	9·84	10·74	11·04	10·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	9·40	10·82	11·40	10·59	167·25	6·969
70	10·02	10·67	10·74	9·90	8·52	7·06	5·83	5·03	5·05	5·72	6·77	7·87	8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	6·77	8·07	175·60	7·317
71	9·23	10·04	10·50	10·14	9·00	7·60	6·43	5·30	4·72	5·00	5·84	6·87	7·68	8·18	8·50	8·27	7·32	6·35	5·83	5·64	5·48	5·40	6·24	6·48	172·04	7·168
72	6·83	7·27	7·66	8·04	8·24	8·31	6·90	5·86	5·06	4·77	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47	6·20	5·87	6·10	6·66	159·48	6·645
73	7·27	7·60	7·87	8·13	8·30	8·41	7·50	6·41	5·53	4·94	4·81	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6·38	6·34	6·54	163·56	6·815
74	6·96	7·68	8·45	8·92	8·96	8·46	7·88	6·87	5·92	5·00	4·44	4·32	4·64	5·11	5·69	6·52	7·19	7·74	7·87	7·76	7·38	7·02	6·87	6·77	164·42	6·851
Sum	646·01	647·27	641·04	624·89	604·65	591·02	584·87	587·09	591·63	592·54	587·31	570·00	535·33	489·49	442·73	408·68	393·70	398·91	430·37	472·53	519·28	564·88	604·18	634·41	13162·80	

Measured from Diagram by ______________________ *and* ______________________

Summed by ______________________ *Compared by* ______________________

Checked by ______________________

SHORT-PERIOD TIDES.

[KARACHI, 1883-84.

Commencing 0 hours.

Astronomical time, 1st May, 1883.

Argument $(\gamma - \sigma)$.

FORM II.—Series M.

Motion per mean Solar hour $= 14°·4920521$.

	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	S. hour	
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76	9·00	8·16	7·30	6·30	5·67	5·44	5·76	6·46 7·18	7·92	8·56	8·80	8·36	7·41	6·52	5·62	4·91	4·81	2^d	0^h
2	5·51	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90	5·77	4·75	4·36	4·97	5·86	6·95	8·15	9·07	9·42 9·09	8·14	6·90	5·76	5·00	4·83	3	1
3	5·50	6·72	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05	5·50	6·91	8·50	9·64	10·14	9·76	8·48	7·08	5·79	4·96 4·71	4	2
4	5·37	6·77	8·24	9·51 8·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	7·06	8·84	10·24	10·84	10·20	8·72	7·24	5·96	4·90	5	2
5	4·48	5·23	6·84	9·84	10·74	11·04	11·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	9·40	10·82	11·40	10·59	8·86	7·36	5·99	4·90	6	3
67	5·22	6·16	7·14	7·97	8·68	9·10	8·50	7·32	5·84	4·60	3·96	3·96	4·64	5·70	7·33	8·88	10·02	10·67	10·74	9·90	8·52	7·06	5·83	5·03	70	7
68	5·05	5·72	6·77	7·87 8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	6·77	8·07	9·23	10·04	10·50	10·14	9·00	7·60	6·43	5·30	4·72	71	8
69	5·00	5·84	6·87	7·68	8·18	8·50	8·27	7·32	6·35 5·83	5·64	5·48	5·40	6·24	6·48	6·83	7·27	7·66	8·04	8·24	8·31	6·90	5·00	5·06	4·77	72	9
70	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47	6·20	5·87	6·10	6·66 7·27	7·60	7·87	8·13	8·30	8·41	7·50	6·41	5·53	4·94	4·81	73	10
71	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6·38	6·34	6·54	6·96	7·68	8·45	8·92	8·96 8·46	7·88	6·87	5·92	5·00	4·44	4·32	74	11
Sum	381·34	447·11	555·96	641·43	700·00	735·57	722·91	647·44	565·07	456·26	375·65	330·69	347·89	425·92	512·91	596·94	673·81	739·51	728·38	662·97	575·35	489·01	403·65	366·47	No. of days	
o.	73	73	74	74	73	73	73	73	75	73	73	73	74	74	74	73	73	75	74	73	73	74	73	74	73^d	12^h

13802·24

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SHORT-PERIOD TIDES.

Commencing 0 hours

Astronomical time, 1st May, 1883.

Argument ($\gamma = 2\sigma$)

FORM III.—SERIES O.

KARACHI, 1883-84.

Motion per mean Solar hour = 13°·9430356

	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	S. hour	
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76 9·00	8·16	7·30	6·30	5·67	5·44	5·76	6·46	7·18	7·92	8·56	8·80	8·36	7·41 6·52	5·62	4·91	4·81	5·51	2d	1h
2	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90 5·77	4·75	4·36	4·97	5·86	6·95	8·15	9·07	9·42	9·09	8·14	6·90	5·76	5·00	4·83 5·50	6·72	3	3
3	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05 5·50	6·91	8·50	9·64	10·14	9·76	8·48	7·08	5·79	4·96	4·71	5·37	6·77	4	4
4	8·24 9·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	7·06	8·84 10·24	10·84	10·20	8·72	7·24	5·96	4·90	4·48	5·23	6·84	8·51	5	6
5	9·84	10·74	11·04 10·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	9·40	10·82	11·40	10·59	8·86 7·36	5·99	4·90	4·56	5·48	7·26	8·96	10·28	6	8
65	4·64 5·70	7·33	8·88	10·02	10·67	10·74	9·90	8·52	7·06	5·83	5·03	5·05	5·72	6·77	7·87 8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	70	21
66	6·77	8·07	9·23	10·04 10·50	10·14	9·00	7·60	6·43	5·30	4·72	5·00	5·84	6·87	7·68	8·18	8·50	8·27 7·32	6·35	5·83	5·64	5·48	5·40	6·24	6·48	71	23
67	6·83	7·27	7·66	8·04	8·24	8·31 6·90	5·86	5·06	4·77	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47 6·20	5·87	6·10	6·66	7·27	7·60	73	1
68	7·87	8·13	8·30	8·41	7·50	6·41	5·53	4·94 4·81	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6·38	6·34	6·54	6·96 7·68	8·45	8·92	74	3
69	8·96	8·46	7·88	6·87	5·92	5·00	4·44	4·32	4·64	5·11	5·69 6·52	7·19	7·74	7·87	7·76	7·38	7·02	6·87	6·77	6·73	7·20	7·88	8·37	8·73 8·74	75	5
Sum	536·93	520·49	523·78	509·52	509·01	493·46	508·06	509·57	535·11	532·66	566·22	569·10	587·27	588·44	606·51	594·24	583·08	572·95	571·37	565·98	562·28	560·03	551·53	552·86	No. of days.	
N.	75	73	75	74	74	74	74	75	75	74	74	75	73	74	75	74	74	73	75	74	75	74	75	74	74d	6h

13210·45

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SHORT-PERIOD TIDES.

[KARACHI, 1883-84

Commencing 0 hours.

Astronomical time, 1st May, 1883.

Argument $(\gamma + \delta - \varpi)$).

FORM IV.—SERIES J.

Motion per mean Solar hour = 15°·5854433.

	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	S. hour.
1	5·22	6·35	7·60	8·62	9·42	9·86	9·76	9·00	8·16	7·30	6·30	5·67	5·44	—	5·76	6·46	7·18	7·92	8·56	8·80	8·36	7·41	6·52	5·62	1d 22h
2	4·91	4·81	5·51	6·56	7·81	8·98	9·82	10·36	10·24	9·34	8·10	6·90	5·77	4·75	4·36	4·97	—	5·86	6·95	8·15	9·07	9·42	9·09	8·14	2 21
3	6·90	5·76	5·00	4·83	5·50	6·72	8·10	9·28	10·18	10·74	10·48	9·26	7·81	6·20	4·59	3·36	3·32	4·05	5·50	—	6·91	8·50	9·64	10·14	3 20
4	9·76	8·48	7·08	5·79	4·96	4·71	5·37	6·77	8·24	9·51	10·44	10·92	10·27	8·76	6·96	5·05	3·23	2·07	2·34	3·45	5·28	—	7·06	8·84	4 19
5	10·24	10·84	10·20	8·72	7·24	5·96	4·90	4·48	5·23	6·84	8·51	9·84	10·74	11·04	10·20	8·36	6·13	4·04	2·21	1·32	1·75	3·29	5·48	7·48	5 19
72	5·22	6·16	7·14	7·97	8·68	9·10	8·50	7·32	5·84	4·60	3·96	3·96	4·64	—	5·70	7·33	8·88	10·02	10·67	10·74	9·90	8·52	7·06	5·83	70 6
73	5·03	5·05	5·72	6·77	7·87	8·80	9·20	8·78	7·80	6·77	5·74	5·02	4·80	5·01	5·66	6·77	—	8·07	9·23	10·04	10·50	10·14	9·00	7·60	71 5
74	6·43	5·30	4·72	5·00	5·84	6·87	7·68	8·18	8·50	8·27	7·32	6·35	5·83	5·64	5·48	5·40	6·24	6·48	—	6·83	7·27	7·66	8·04	8·24	72 4
75	8·31	6·90	5·86	5·06	4·77	5·08	5·77	6·28	6·63	6·96	7·26	7·41	7·14	6·71	6·47	6·20	5·87	6·10	6·66	7·27	7·60	—	7·87	8·13	73 3
76	8·30	8·41	7·50	6·41	5·53	4·94	4·81	4·98	5·52	6·33	6·93	7·33	7·61	7·58	7·44	7·10	6·71	6·38	6·34	6·54	6·96	7·68	8·45	8·92	74 3
Sum	530·35	540·31	525·07	529·69	543·21	541·22	538·65	552·76	540·83	548·94	551·26	556·58	551·56	542·43	559·33	557·96	535·36	553·85	542·24	544·27	537·84	529·88	541·24	536·56	No. of days
No.	73	74	73	73	74	73	73	74	73	73	73	74	73	72	74	74	71	74	73	73	73	72	74	73	73d 4h

13030·39

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SHORT-PERIOD TIDES.

[KARACHI, 1883-84.

FORM V. SUMMATIONS AND MEANS OF SERIES M.

	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
Sum (p. 1)	381·34	447·11	555·96	641·43	700·00	735·57	722·91	647·44	565·07	456·26	375·65	330·69
,, (p. 2)	343·74	428·05	526·68	615·67	695·14	721·19	699·40	641·14	544·08	442·65	368·45	332·16
,, (p. 3)	345·81	419·59	505·63	596·08	681·28	718·74	709·65	634·70	545·90	459·52	385·92	345·39
,, (p. 4)	364·14	435·23	536·65	632·80	691·73	730·42	711·59	651·35	553·59	451·15	366·11	324·73
,, (p. 5)	353·49	423·23	527·79	625·39	690·07	724·24	714·22	631·25	539·98	433·03	358·82	318·28
Sum (pp. 1–5)	1788·52	2153·21	2652·71	3111·37	3458·22	3630·16	3557·77	3205·88	2748·62	2242·61	1854·95	1651·25
No. of Obs.	369	369	371	370	369	368	369	369	372	368	369	369
Means ...	4·847	5·835	7·150	8·409	9·372	9·865	9·642	8·688	7·389	6·094	5·027	4·475

12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	
347·89	435·92	512·91	596·94	673·81	739·51	728·38	662·97	575·35	489·01	403·65	366·47	13082·24
345·45	412·26	519·02	599·31	680·90	707·83	697·21	630·58	532·87	434·23	363·64	330·45	12612·10
360·08	422·98	511·07	602·91	657·22	693·69	676·94	624·31	528·24	424·97	354·08	320·60	12525·30
351·46	417·95	512·74	605·23	695·54	736·54	733·18	660·95	570·73	482·95	394·27	345·06	12956·09
330·04	400·14	503·91	591·91	660·43	706·30	702·20	627·34	541·38	443·44	362·66	328·93	12538·47
1734·92	2079·25	2559·65	2996·30	3367·90	3583·87	3537·91	3206·15	2748·57	2274·60	1878·30	1691·51	63714·20
370	369	371	368	368	369	369	368	369	369	368	369	
4·689	5·635	6·899	8·142	9·152	9·712	9·588	8·712	7·449	6·164	5·104	4·584	

SHORT-PERIOD TIDES.

FORM VI. ANALYSIS OF SERIES M.

	I		II	III I+II	IV I−II	V Lower half of IV reversed	VI IV+V	M	VII M×VI	M	VIII M×VI	IX IV−V	M	X M×IX	M	XI M×IX	XII First half of III	XIII Second half of III	XIV XII+XIII	XV XII−XIII
0h	4·847	12h	4·689	9·536	+·158	....	+·158	0	+·000	0	·000	+·158	1	+·158	1	+·158	9·536	19·230	28·766	−9·694
1	5·835	13	5·635	11·470	+·200	−·109	+·091	S_1	+·024	S_3	+·064	+·309	S_5	+·298	S_3	+·218	11·470	17·400	28·870	−5·930
2	7·150	14	6·899	14·049	+·251	−·077	+·174	S_2	+·087	1	+·174	+·328	S_4	+·284	0	·000	14·049	14·838	28·887	−0·789
3	8·409	15	8·142	16·551	+·267	−·070	+·197	S_3	+·139	S_3	+·139	+·337	S_3	+·238	$-S_3$	−·238	16·551	12·258	28·809	+4·293
4	9·372	16	9·152	18·524	+·220	−·060	+·160	S_4	+·139	0	·000	+·280	S_2	+·140	−1	−·280	18·524	10·131	28·655	+8·393
5	9·865	17	9·712	19·577	+·153	−·024	+·129	S_5	+·125	$-S_3$	−·091	+·177	S_1	+·046	$-S_3$	−·125	19·577	9·059	28·636	+10·518
6	9·642	18	9·588	19·230	+·054	... .	+·054	1	+·054	−1	−·054	+·054	0	·000	0	·000				
7	8·688	19	8·712	17·400	−·024															
8	7·389	20	7·449	14·838	−·060															
9	6·094	21	6·164	12·258	−·070			12	+·568	12	+·232		12	+1·164	12	−·267				
10	5·027	22	5·104	10·131	−·077															
11	4·475	23	4·584	9·059	−·109			$B_1=$	+·0473	$B_3=$	+·0193		$A_1=$	+·0970	$A_3=$	−·0223				

M	XVI M×XV	M	XVI M×XV	M	XVIII M×XV	M	XIX M×XV	XX First half of XIV	XXI Second half of XIV	XXII XX−XXI	M	XXIII M×XXII	M	XXIV M×XXII	XXV XX+XXI	M	XXVI M×XXV	M	XXVII M×XXV
0	·000	1	−9·694	0	·000	1	−9·694	28·766	28·809	−0·043	0	·000	1	−0·043	57·575	0	·000	1	+57·575
S_3	−2·965	S_4	−5·136	1	−5·930	0	·000	28·870	28·655	+0·215	S_4	+0·186	S_2	+0·108	57·525	S_4	+49·818	$-S_2$	−28·763
S_4	−0·683	S_3	−0·395	0	·000	−1	+·789	28·887	28·636	+0·251	S_4	+0·217	$-S_2$	−0·126	57·523	$-S_4$	−49·817	$-S_2$	−28·761
1	+4·253	0	·000	−1	−4·293	0	·000												
S_4	+7·269	$-S_3$	−4·196	0	·000	1	+8·393												
S_3	+5·259	$-S_4$	−9·109	1	+10·518	0	·000												
12	+13·173	12	−28·530	12	+0·295	12	−0·512				12	+0·403	12	−0·061		12	+0·001	12	+0·0
$B_2=$	+1·0978	$A_2=$	−2·3775	$B_6=$	+0·0246	$A_6=$	−0·0427				$B_4=$	+0·0336	$A_4=$	−0·0051		$B_8=$	+0·0001	$A_8=$	+0·0043

0 = nat. sin 0° = ·00000
S_1 = nat. sin 15° = ·25882
S_2 = nat. sin 30° = ·50000
S_3 = nat. sin 45° = ·70711
S_4 = nat. siu 60° = ·86603
S_5 = nat. sin 75° = ·96593
1 = nat. sin 90° = 1·00000

Computed by ______ *Checked by* ______ *Compared by* ______

SHORT-PERIOD TIDES. FORM VII.—VALUES OF V_0+u FOR KARACHI, 1883-84.

1883, May 1st, 0^h. N. Lat. 24° 47′, Long. 66° 58′ *E.* ($=4^h \cdot 4645$).

Average Long. Moon's Node for year beginning with 1st May 1883, Midyear, 0^h 31st October, 1883.

Auxiliary Tables N = 211°·884.

Extract from Auxiliary Tables.

$I = 19°·267$, $\nu = -8°·256$, $\xi = -7°·695$.

For K Tides $\nu' = -5·366$, $2\nu'' = -9·906$.

		h_0			s_0
Sid. Time at M.N. (*Naut. Alm.*, p. II.)	=	h m s 2 36 17·48	Moon's mean long. (*Naut. Alm.*)	=	332°·324
	=	2 36·2913		=	332·540
	=	2·604855	E. Long. Cor.	=	−2·448
		1·302428			
	=	39·073	s_0	=	330·092
Long. Cor.	=	−0·183	ξ	=	−7·695
h_0	=	38·890	$s_0-\xi$	=	337·787
ν	=	−8·256	$2(s_0-\xi)$	=	315·574
$h_0-\nu$	=	47·146			
$2h_0$	=	77·780	$2s_0$	=	300·184

Average Long. Moon's Perigee for L Tide (round numbers).

(*from below*) p_0 =	16°·079
motion in $184^d\ 13\frac{1}{2}^h$ =	20·560
$P+\xi$ =	36·039
$-\xi$ =	7·695
P =	44·334
$\cot\frac{1}{2}I$ = +	5·891
$\cot^2\frac{1}{2}I$ = +	34·704
$\frac{1}{6}\cot^2\frac{1}{2}I$ = +	5·784
$-\cos 2P$ = −	·023
$\frac{1}{6}\cot^2\frac{1}{2}I-\cos 2P$ = +	5·761

		p_0
Hansen's Tables de la Lune, p. 300. π for 1883, Jan. 0.	=	2°·484
Motion in interval from Jan. 0, or 121 days.	=	13·480
Constant	=	0·136
Sum	=	16·100
E. Long. Cor.	=	−0·021
p_0	=	16·079
s_0	=	330·092
s_0-p_0	=	314·013

Computation of R and Q (round numbers).

colog. $[\frac{1}{6}\cot^2\frac{1}{2}I-\cos 2P]$ = +	9·23950
log sin 2 P = +	9·99988
log tan R = +	9·23938
for L tide, R =	9°·845
log tan P = +	9·98990
log$\frac{1}{2}$ =	9·68997
log. tan Q = +	9·68887
For M_1 Tide, Q =	26·036

Extract from Auxiliary Table pl } $p_1 = 280°·940$

$h_0 = 38·890$

$h_0 - p_1 = 117·950$

Motion per mean Solar hour.	East Long. Correction.
$\eta = 0°·0410686$	$(-0°·183)$
$\sigma = 0°·5490165$	$(-2°·448)$
$\omega = 0°·0046419$	$(-0°·021)$

M SERIES.

$h_0-\nu$ =	47°·146
$-(s_0-\xi)$ =	22·213
§ M_1 =	69·359
(×2) for M_2 =	138·718
(×3) for M_3 =	208·077
(×4) for M_4 =	277·436
(×6) for M_6 =	56·154
(×8) for M_8 =	194·872
§ as above M_1 =	69·359
(*see above*) + Q =	26·036
$-\frac{1}{4}\pi$ =	270·000
for M_1 =	5·395

K_2 SERIES.

$2h_0$ =	77°·780
$-2\nu''$ =	9·906
For K_2 =	87·686

K_1 SERIES

h_0 =	38°·890
$-\nu'$ =	5·366
$-\frac{1}{4}\pi$ =	270·000
for K_1 =	314·256

λ SERIES.

For N =	184°·705
$-2h_0$ =	282·220
$+2s_0$ =	300·184
$+\pi$ =	180·000
For λ =	227·109

ν SERIES.

for M_2 =	138°·718
+ (s_0-p_0) =	314·013
$+2h_0$ =	77·780
$-2s_0$ =	59·816
for ν =	230·327

M_2 N SERIES.

for M_2 =	138°·718
+ for N =	184·705
for M_2 N =	323·423

$2M_2 K_1$ SERIES.

for M_4 =	277°·436
−for K_1 =	45·744
for $2M_2 K_1$ =	323·180

Mm SERIES.

$s_0 - p_0 = 314°·013$

Mf SERIES.

$2(s_0-\xi) = 315°·574$

SHORT-PERIOD TIDES.

FORM VIII.—EVALUATION OF SHORT-PERIOD TIDES. SERIES M.

Augmenting Factors.—For A_1, B_1 ..1·00286, A_2, B_2...1·01152, A_3, B_3 ..1·02617, A_4, B_4...1·04720, A_6, B_6 ..1·11072 A_8, B_8...1·20920

Log B_1 = +8·67486	Log B_2 = +0·04052	Log B_3 = +8·28556	Log B_4 = +8·52634	Log B_6 = +8·39094	Log B_8 = +6·00000
Log A_1 = +8·98677	Log A_2 = −0·37612	Log A_3 = −8·34830	Log A_4 = −7·70758	Log A_6 = −8·63043	Log A_8 = +7·63347
L. tan ζ_1 = +9·68809	L. tan ζ_2 = −9·66440	L. tan ζ_3 = −9·93726	L. tan ζ_4 = −0·81877	L. tan ζ_6 = −9·76051	L. tan ζ_8 = +8·36653
ζ_1 = 25°·995	ζ_2 = 155°·215	ζ_3 = 139°·124	ζ_4 = 98°·631	ζ_6 = 150°·053	ζ_8 = 1°·332
V_0+u = 5·395	V_0+u = 138·718	V_0+u = 208·077	V_0+u = 277·436	V_0+u = 56·154	V_0+u = 194·872
κ_1 = 31·390	κ_2 = 293·933	κ_3 = 347·201	κ_4 = 16·067	κ_6 = 206·207	κ_8 = 196·204
$(B_1)^2$ = ·002237	$(B_2)^2$ = 1·205165	$(B_3)^2$ = ·000372	(B_4) = ·001129	$(B_6)^2$ = ·400605	$(B_8)^2$ = ·000000
$(A_1)^2$ = · 9409	$(A_2)^2$ = 5·652506	$(A_3)^2$ = · 497	(A_4) = · 26	$(A_6)^2$ = · 1823	$(A_8)^2$ = · 18
$(R_1)^2$ = ·011646	$(R_2)^2$ = 6·857671	$(R_3)^2$ = ·000869	$(R_4)^2$ = ·001155	$(R_6)^2$ = ·002428	$(R_8)^2$ = ·000018
R_1 = ·1079	R_2 = 2·6187	R_3 = ·0295	R_4 = ·0340	R_6 = ·0493	R_8 = ·0043
Aug[tn] = · 3	Aug[tn] = · 301	Aug[tn] = · 8	Aug[tn] = · 16	Aug[tn] = · 55	Aug[tn] = · 9
Aug[td] R_1 = ·1082	Aug[td] R_2 = 2·6488	Aug[td] R_3 = ·0303	Aug[td] R_4 = ·0356	Aug[td] R_6 = ·0548	Aug[td] R_8 = ·0052
$1/f$ = ·7443	$1/f$ = ·9689	$1/f$ = ·9537	$1/f$ = ·9388	$1/f$ = ·9096	$1/f$ = ·8813
H_1 = ·0805	H_2 = 2·5664	H_3 = ·0289	H_4 = ·0334	H_6 = ·0498	H_8 = ·0046

Computed by ——————— ***Checked by*** ——————— ***Compared by*** ———————

$\Sigma dh \sin (\sigma - \varpi)$ FORM IX.—LONG-PERIOD TIDES. TIDE Mm. [KARACHI, 1883-84.

Multiplier.	1·0		·9		·8		·7		·6		·5		·4		·3		·2		·1		0
No. of the day.	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	
7 to 0	•	·27	•	·25	•	·22			•	·11			·12	•			·21	•			·15
	•	·20																			
8 − 13	•	·09	·08	•	·19	•			·18	•			·18	•			·33	•			
34 − 28	•	·01	·18	•	·23	•	·17	•			·16	•			·22	•			·25	•	
35 − 41	•	·02	·13	•	·27	•	·44	•			·52	•			·54	•			·67	•	
			&c.												&c.						
311 − 316	·16	•	•	·06	•	·04			•	·07			•	·09			•	·22			
337 − 331	•	·20	•	·32	•	·34	•	·38			•	·21			•	·29			•	·32	
338 − 344	•	·16	•	·07	•	·05	•	·08			•	·18			•	·19			•	·15	
364 − 359	·25	•	·02	•	•	·07			•	·21			•	·03			•	·01			
	3·10	4·32	3·19	2·70	2·46	1·49	0·87	1·36	0·94	1·55	1·38	1·15	1·25	1·30	1·47	1·48	1·88	0·88	1·44	1·15	
Total *a*	−1·22		+·049		+0·97		−0·49		−0·61		+0·13		−0·05		−0·01		+1·00		+0·29		
20 to 14	·10	•	•	·13	•	·09	·03	•			·11	•			·27	•					·39
21 − 27	·36	•	·40	•			·38	•			·43	•			·39	•			·28	•	
			·35	•																	
48 − 42	·43	•	·54	•			·58	•	·81	•			·98	•					·92	•	
	·29	•																			
			&c.												&c.						
325 − 330	·19	•	·27	•			·02	•	•	·10			•	·25					•	31	
351 − 345	•	·05	•	·33			•	·39			•	·38			•	·31			•	19	
			•	·19																	
352 − 358	·02	•	·06	•	·03	•	·02	•			·09	•			·17	•					·11
	2·37	4·13	2·16	3·84	0·98	2·02	1·51	2·15	1·83	0·99	0·92	2·60	2·40	0·71	0·99	1·78	1·53	0·56	1·56	1·32	
Total *b*	−1·76		−1·68		−1·04		−0·64		+0·84		−1·68		+1·69		−0·79		+0·97		+0·24		
a−*b*	+0·54		+2·17		+2·01		+0·15		−1·45		+1·81		−1·74		+0·78		+0·03		+0·05		
(*a*−*b*) × Mult	+0·54		+1·95		+1·61		+0·11		+ •		+0·91		+ •		+0·23		+0·01		+0·01		
	− •		− •		− •		− •		−0·87		− •		−0·70		− •		− •		− •		

$\Sigma(a-b) \times$ Mult. $= +5·37$. $\Sigma(a-b) \times$ Mult. $= -1·57$. $\Sigma dh \sin (\sigma - \varpi)t = \Sigma_{364}^{0} (a-b) \times$ Mult. $= +3·80$.

NOTE.—The arrows show the direction of the sequence of the entries of dh in the columns in which points are inserted, the values being entered under their proper signs—*e.g.*, in the first row (marked 7 to 0) with arrow from right to left, the entry (irrespective of sign) for day 0 is to be entered in column 0, for day 1 in column ·2, for day 2 in column ·4, and so on. After filling in the first two rows of the upper half, the first two rows of the lower half are to be filled, and so on alternately, the alternation of entry being indicated by the curved arrows. In the spaces containing double points two successive entries are to be filled in—the first entry above, the second below: *e.g.*, in row 7 to 0, column 1·0, there are double points, and the entry of dh for day 6 is to be made above the line and to left or right according as it is + or −, and for day 7 it is to be made below the line, and to left or right according as it is + or −.

*Computed by*__________ *Compared by*__________

LONG-PERIOD TIDES. [KARACHI, 1883-84.

FORM X.—SUMMATIONS AND EVALUATION OF LONG-PERIOD TIDES.—CLEARANCE FROM EFFECTS OF TIDES OF SHORT-PERIOD.

NOTE.—A *and* B *to be extracted from harmonic analysis for Tides of Short-Period.*

Tide Mm or $(\sigma-\varpi)$.

Cosine Series.

		Cosine Series.	Products.
M_2	A_2 = −2·378		
	multiplier = −0·0556		= +0·13
	B_2 = +1·098		
	multiplier = −0·1704		= −0·19
N	A = −0·035		
	multiplier = −0·0588		= +0·00
	B = +0·599		
	multiplier = −0·0776		= −0·05
O	A = −0·382		
	multiplier = −0·0649		= +0·02
	B = −0·405		
	multiplier = −0·3477		= +0·14

Total + = +0·29
Total − = −0·24

Total clearance = ±0·05
Uncleared $\Sigma dh \cos (\sigma-\varpi)t$ = +1·29

$\Sigma dh \cos (\sigma-\varpi)t$ = +1·34

Divisor 183·05. 1st approx. A = +0·007

Sine Series.

		Products.
M_2	A_2 = −2·378	
	multiplier = −3·1708	= +0·41
	B_2 = +1·098	
	multiplier = +0·0441	= +0·05
N_2	A = −0·035	
	multiplier = −0·0206	= +0·00
	B = +0·599	
	multiplier = +0·1138	= +0·07
O_1	A = −0·382	
	multiplier = −0·3452	= +0·13
	B = −0·405	
	multiplier = +0·0405	= −0·02

Total + = +0·66
Total − = −0·02

Total clearance = +0·64
Uncleared $\Sigma dh \sin (\sigma-\varpi)t$ = +3·80

$\Sigma dh \sin (\sigma-\varpi)t$ = +4·44

Divisor 181·95. 1st approx. B. = +0·024

Tide Mf or 2σ.

Cosine Series.

	Products.
A_2 = −2·378	
multiplier = +0·0030	= −0·01
B_2 = +1·098	
multiplier = −0·0377	= −0·04
A = −0·035	
multiplier = +0·0368	= −0·00
B = +0·599	
multiplier = −0·2234	= −0·13
A = −0·382	
multiplier = +0·0167	= −0·01
B = −0·405	
multiplier = −0·0779	= +0·03

Total + = +0·03
Total − = −0·19

Total clearance = −0·16
Uncleared $\Sigma dh \cos 2\sigma t$ = +7·10

$\Sigma dh \cos 2\sigma t$ = +6·94

Divisor 183·18. 1st approx C = +0·38

Sine Series.

	Products.
A_2 = −2·378	
multiplier = +0·0417	= − 0·10
B_2 = +1·098	
multiplier = +0·0105	= +0·01
A = −0·035	
multiplier = −0·1525	= +0·01
B = +0·598	
multiplier = −0·0854	= −0·05
A = −0·382	
multiplier = +0·0842	= −0·03
B = −0·405	
multiplier = +0·0338	= −0·01

Total + = +0·02
Total − = −0·19

Total clearance = −0·17
Uncleared $\Sigma dh \sin 2\sigma t$ = +2·14

$\Sigma dh \sin 2\sigma t$ = +1·97

Divisor 181·82. 1st approx. D = +0·011

The 1st approximations of the constants for other three tides are deduced in the same way, and are for MSf. C′ = −0·001, D′ = −0·009; for Sa, E = +0·089, F = −0·001; and for Ssa, G = −0·003, H = +0·189.

Computed by ______ *Checked by* ______ *Compared by* ______

LONG-PERIOD TIDES.

[KARACHI, 1883-84.

FORM XI—EVALUATION OF LONG-PERIOD TIDES.

$\Sigma_0^{364} dh.\cos\ (\sigma-\varpi)t=$	+1·34 =	+183·05A+	2·14B+	0·73C+	4·29D+	0·77C′+	5·04D′+	4·88E−	0·43F+	4·96G−	0·69H.
	+1·34 =	183·05A+	·05 +	·03 +	·05−	·00 −	·05 +	·43 +	·00 −	·01 −	·13
	+0·97 =	183·05A									
	+0·005=	A									
$\Sigma_0^{364} dh.\sin\ (\sigma-\varpi)t=$	+4·44 =	+ 2·14A+	181·95B−	4·15C+	1·02D−	4·90C′+	1·07D′+	3·80E+	0·34F+	3·88G+	0·69H·
	+4·44 =	+ ·01 +	181·95B−	·16 +	·01 +	·00 −	·01 +	·34 −	·00 −	·01 +	·13
	+4·13 =	181·95B									
	+0·023=	B									
$\Sigma_0^{364} dh.\cos\ 2\sigma t$ =	+6·94 =	+ 0·73A−	4·15B+	183·18C+	0·88D+	0·61C′+	0·92D′−	1·50E=	0·10F−	1·51G−	0·19H.
	+6·94 =	·00 −	·10 +	183·18C+	·01 −	·00 −	·01 −	·13 +	·00 +	·00 −	·04
	+7·21 =	183·18C									
	+0·039=	C									
	&c.				&c.			&c.			

LUNAR MONTHLY Mm.

Log B	=	+8·36173
Log A	=	+7·69897
L. tan ζ_1	=	+0·66276
ζ_1	=	77·735
Motion for $11\frac{1}{4}^h$ …	=	6·260
ζ	=	83·995
V_0+u	=	314·013
κ	=	38·008
B^2	=	0·009529
A^2	=	0· 25
Sum = R^2	=	0·000554
R	=	0·024
1/f	=	0·900
H	=	0·022

LUNAR FORTNIGHTLY Mf.

Log D	=	+7·95424
Log C	=	+8·59106
L. tan ζ_1	−	+9·36318
ζ_1	=	12·995
Motion for $11\frac{1}{4}^h$ …	=	12·627
ζ	=	25·622
V_0+u	=	315·574
κ	=	351·196
D^2	=	0·000081
C^2	=	0· 1521
Sum = R^2	=	0·001602
R	=	0·040
1/f	=	1·449
H	=	0·058

Luni-solar Fortnightly MSf.

Log D′	=	−8·04139
Log C′	=	+7·00000
L. tan ζ'	=	−1·04139
ζ_1	=	275·194
Motion for $11\frac{1}{4}^h$ …	=	11·683
ξ	=	286·877
V_0+u	=	221·282
κ	=	148·159
$(D')^2$	=	0·000121
(C'^2)	=	0· 1
Sum = R^2	=	0·000122
R	=	0·011
1/f	=	0·969
H	=	0·011

Solar Annual Sa.

Log F	=	−7·00000
Log E	=	+8·94448
L. tan ζ_1	=	−8·05552
ζ_1	=	359·349
Motion for $11\frac{1}{4}^h$ …	=	0·472
ζ	=	359·821
V_0+u	=	38·890
κ	=	38·711
F^2	=	0·000001
E^2	=	· 1444
Sum = R^2	=	0·001445
R = H	=	0·088

Solar Semi-Annual Ssa.

Log H	=	+9·27646
Log G	=	−7·60206
L. tan ζ_1	=	−1·67440
ζ_1	=	91·212
Motion for $11\frac{1}{4}^h$ …	=	0·945
ζ	=	92·157
V_0+u	=	77·780
κ	=	169·937
H^2	=	0·035721
G^2	=	· 16
Sum = R^2	=	0·035737
R = H	=	0·189

Computed by ______________ *Checked by* ______________ *Compared by* ______________

Correction to the tidal pamphlet, 'The Tides.' (1926).

Chapter I, Page 61, para 84 —

Substitute the words "Chapter I" for the words "the book" in line 1.

Geod. Br. P.O.—1926—310.

86. This table is the converse of Table 1, but had to be made out somewhat differently. The correct value to three places of decimals of a degree is all that is generally required in the computations. This is given in the 4th column, and it will be observed that the figures in the 4th column are arranged midway between those in the 3rd and also midway between those in the 5th column, where the actual values corresponding to the angles in the 3rd column are given.

Table II.

It will also be observed that the table is divided into six groups. The reason of this is as follows :—6 minutes = ·1 of a degree, 12 minutes = ·2, 18 minutes = ·3, and so on. Therefore, any number of seconds added to 6 minutes will give the same figures in the second and third place of decimals, (in the equivalent value of a degree expressed in decimals), as the similar number of seconds added to 12 minutes or 18 minutes would give. Similarly, 1 minute, or 7 minutes, or 13 minutes, and so many seconds would each have for the second and third place of decimals, the same figures in expressing their corresponding values in decimals of a degree ; (the first figure of the decimals of course alters).

To use the table, first look for the minutes in one of the groups of column 1 ; opposite it in the 2nd column will be found the first figure of the equivalent value in decimals of a degree. *Keeping to the same group*, look in the 3rd column for the seconds, (most probably the exact number will not be found), entering this column at the *space* between the number of seconds next less, and the number of seconds next greater than that looked for, in the *4th column opposite this space* the second and third figures of the corresponding value of the decimals of a degree will be found. If the actual number of seconds looked for is found in the third column, then the corresponding value is obtained in column 4 *opposite the space in column 3 below* the number of seconds.

84. The tables at the end of the book have been constructed to facilitate the computation required for the harmonic analysis of the tidal observations and for the preparation of data for tidal prediction. The following explanations and examples are given to illustrate the use of the tables.

Auxiliary Tables.

85. This table, for converting decimals of a degree into their corresponding values of minutes and seconds of arc, enters frequently into the computations, more especially in taking out the values of the trigonometrical functions from Shortrede's Logarithm tables. Its use hardly requires explanation.

Table I.

86. This table is the converse of Table 1, but had to be made out somewhat differently. The correct value to three places of decimals of a degree is all that is generally required in the computations. This is given in the 4th column, and it will be observed that the figures in the 4th column are arranged midway between those in the 3rd and also midway between those in the 5th column, where the actual values corresponding to the angles in the 3rd column are given.

Table II.

It will also be observed that the table is divided into six groups. The reason of this is as follows :—6 minutes = ·1 of a degree, 12 minutes = ·2, 18 minutes = ·3, and so on. Therefore, any number of seconds added to 6 minutes will give the same figures in the second and third place of decimals, (in the equivalent value of a degree expressed in decimals), as the similar number of seconds added to 12 minutes or 18 minutes would give. Similarly, 1 minute, or 7 minutes, or 13 minutes, and so many seconds would each have for the second and third place of decimals, the same figures in expressing their corresponding values in decimals of a degree; (the first figure of the decimals of course alters).

To use the table, first look for the minutes in one of the groups of column 1; opposite it in the 2nd column will be found the first figure of the equivalent value in decimals of a degree. *Keeping to the same group*, look in the 3rd column for the seconds, (most probably the exact number will not be found), entering this column at the *space* between the number of seconds next less, and the number of seconds next greater than that looked for, in the *4th column opposite this space* the second and third figures of the corresponding value of the decimals of a degree will be found. If the actual number of seconds looked for is found in the third column, then the corresponding value is obtained in column 4 *opposite the space in column 3 below* the number of seconds.

Example.—Required the decimals of a degree corresponding to 18′ 25″:—

Column 1, group 1, 18′	... ...	=	·3	in col. 2
„ 3, „ 1, 25″ (between 23″·4 and 27″·0)	...	=	·007	in col. 4
	Value required	=	·307	

Again :—If the decimals of a degree corresponding to 40′ 0″ are required: then as in the above example, 40′ 3″ would just equal ·668, and anything less than 40′ 3″ must be less than .668. Again, 39′ 59″·4 would just equal ·667, and anything greater than 39′ 59″·4 must be equal to ·667 or more. Therefore 40′ 0″ would equal ·667.

Table III.

87. This is extracted and deduced from Hansen's *Tables de la Lune*, pages 299 and 300 up to the year 1923. The values of p_0 the mean longitude of the moon's perigee, (or π as it is written in the tidal computation forms), is given for what is called January 0 of each year; but in the preface to Hansen's Tables it will be found that January 0 and December the 31st mean the same date; therefore the values given in the tables are for 0 hours December 31st of the preceding year. But this is not the case for leap-year; the values given in the tables as regards those years are for 0 hours January 1st.

If the value of π for 0 hours January 1st is required for any year which is not a leap-year, take the value of π given opposite in the next column, and *add* one day's motion (Table V); if it is a leap-year, the one day's motion has not to be added.

Example.—Required p_0 or π for 0 hours January 1st, 1891.

In Table III opposite 1891 · · .	328°00632
In Table V one day's motion for π ·	0·11140
p_0 or π for 0 hours January 1st, 1891 =	328°·11772

Again :—Required p_0 or π for 0 hours January 1st, 1892.

Opposite 1892 in Table III is 8°·78020, which is the value required, for 1892 being leap-year, the one day's motion is not added.

Since the year 1923 the value of p_0 or π, the mean longitude of the moon's perigee, given in the Nautical Almanac is derived from the formula in Brown's tables instead of from the older formula given in Hansen's tables which was in use prior to that date and from which the values of p given in G. T. Survey Vol. 16 were obtained :—vide Vol. 16 pp 61 & 317 part I.

The formula in use since 1923 is :—

$334^\circ\ 19'\ 46''\cdot40 + 4069^\circ\ 2'\ 2''\cdot52\ T - 37''\cdot17\ T^2 - 0''\cdot045\ T^3$,
where T is the time expressed in terms of a Julian century of 36525 mean solar days elapsed since midnight at Greenwich on Jan. 0—1 1900, which is taken as the origin of time.

The speed of p_0 is $0^\circ\cdot1114040803$1 per mean solar day, so that for simplicity the first 2 terms of the formula above may be written :—

$334^\circ.329556 - 0^\circ\cdot11140408031 \times$ No. of days.

An example of the computation is given below—

Required p_0 or π the mean longitude of the moon's perigee for 0 hours on Jan. 1, 1923.

		°
Absolute term		334·329556
Motion for 8401 days at $0^\circ\cdot11140408031$ $=935^\circ\cdot905679$	or	215·905679
		550.235235
	or	190·235235
$-37\cdot17\ T^2 - 0\cdot045\ T^3$ for T = ·23		— ·000546
∴ p_0 or π	=	190°·234689

or 190°·2347 as given in the Nautical Almanac.

The values from 1924 onwards depend on the new formula above. *They are shown in italics in table III.*

The values for Jan. 1 computed by the new formula were found in defect of those published in the old edition of the table after addition of ·136 and one day's motion at ·11404 to convert the latter to Jan. 1 instead of Jan. 0 by :—

... ·00159 in 1923
... ·00324 in 1936
... ·00608 in 1949

The tabular values for Jan. 0 from 1924 onwards were accordingly corrected by interpolation from the above, and still require the constant 0·136 added.

These values may be also obtained from the N. Almanac, which is usually available in time for any particular year's computations.

88. These tables are to be employed together ; in using Table V great care must be taken that the proper number of days from January 0 is taken by means of Table IV. The motion of p_0 or π is given for every day from 1 to 366.

Tables IV and V.

89. This table gives the correction to the value of p_0 or π on account of difference of longitude from Greenwich. The correction is required only to three places of decimals of a degree, as far as the tidal computations are concerned, and the table is constructed on the principle explained in Table II.

Table VI.

Column 3 shows the exact correction for the difference of longitude given immediately opposite in column 1; and column 2 gives the correction which has to be used, for all values occurring *between the longitudes* given in column 1.

Example.—Required correction for p_0 or π for 30° 30′ E. longitude. By table II 30′ = ·500 of a degree.

From Table VI.—30°·500 lies between 27°·468 and 30°·700 in column 1, and in column 2, opposite the space between those two longitudes is ·009, which is the correction required, and the sign is—.

90. This table will be found much more convenient than Crelle's for the particular multiplications required, and admits of much more rapid computation. The three augmenting factors R_3, R_6, and R_8 occur so seldom, that it has not been thought necessary to extend the tables on their account.

Table VII.

The usual multiples of the factors are one integer and three places of decimals, or two figures only preceded by 0; for instance, 1·412, or 0·026. Sometimes, however, two integers and three places of decimals have to be multiplied by the augmenting factor. The use of the table hardly requires explanation, care being taken to put down the decimal point correctly. The values in the tables are the products of the factors by whole numbers.

91. These tables require no explanation. They give the value of S_1, S_3, S_4, and S_5 multiplied by every number between ·001 and ·999: from which the products of these factors with other numbers can be rapidly obtained, care being taken about the decimal point.

Tables VIII, IX, X, and XI.

92. This table has been made up in order to permit of the natural numbers corresponding to logarithms, with indices 6, 7, and 8, being taken out much more rapidly than could be done with Hutton's Logarithm Tables. The natural number corresponding to the given logarithm is only required correct to three places of decimals. The table has been made up on the same principle as explained in the case of Table II.

Table XII.

93. This table gives the values of N, (longitude of moon's ascending node), for 0 hour January 1, Greenwich mean time, for each year from 1850 up to 1949.

Table XIII.

Since the year 1924 the value of N, as given in the Nautical Almanac, is derived from the formula in Brown's tables, instead of from the older formula given in Hansen's tables, which was in use prior to that date, and from which the values of N given in G. T. Survey

Vol. 16 were obtained :—vide Vol. 16 pp. 61 and 324 part 1.

The formula in use since 1924 is :—

$259^\circ 10' 59''\cdot79 - 1934^\circ 8' 31''\cdot23\,T + 7''\cdot48\,T^2 + 0''\cdot008\,T^3$, where T is the time expressed in terms of a Julian century of 36525 mean solar days elapsed since midnight at Greenwich on Jan. 0-1, 1900, which is taken as the origin of time.

The speed of N per mean solar day is $0^\circ.05295392220$ so that for simplicity the first two terms of formula (1) may be written $259^\circ\cdot183275 - 0^\circ\cdot05295392220 \times$ No. of days.

An example of the computation is given below:—

Required the mean longitude of the Moon's ascending node for 0 hours on Jan. 1 1923.

Absolute term	=	$259^\circ\cdot183275$
Motion in 8401 days at $0^\circ\cdot05295392220$		
$= -444^\circ\cdot865899$ (or deducting 360°)	=	$-84\cdot865899$
$7''\cdot48T^2 + 0''\cdot008T^3$ for T at .23	=	$0\cdot000110$
$\therefore N$	=	$174^\circ\cdot317486$

or $174^\circ\cdot3175$ as given in the Nautical Almanac.

The tabular values from 1924 onwards depend on the new formula above. *They are shown in italics in Table XIII.* The values for January 1 computed by the new formula were in excess of those published in the old edition of the table by :—

.................·0034 in 1923

.................·0042 in 1949

The old values were corrected accordingly by interpolation from 1924 onwards.

The values may also be obtained from the N. Almanac, which will usually be available in time for any particular year's computations.

94. This table shows the amount to be subtracted from the values

Table XIV. given in Table XIII, to obtain N at any particular date. The mean value of N to be used in the tidal reductions is the value at mid-year of the observations : and as half a year after 0 hour of the first day under analysis falls at midnight, the values in Table XIV are computed for each midnight.

95. This table gives the value of p_1, the solar perigee, for 0 hour

Table XV. January 1, from 1850 to 1949.

The formula for p_1, the mean longitude of the solar perigee, given in the Nautical Almanac for 1917 p. 590, is derived from the formula given in Newcomb's tables.

The formula is $p_1 = 281^\circ{\cdot}13\ 15''{\cdot}0 + 6189''{\cdot}03\ T + 1''{\cdot}63\ T^2 + 0{\cdot}012\ T^3$ where T has the same significance as above.

The first two terms may be written for simplicity $281^\circ{\cdot}220833 + 0^\circ{\cdot}00004706845 \times$ No. of days.

An example of computation is given below :—

Required p_1 the mean longitude of the solar perigee for 0 hrs. Jan. 1. 1923.

Absolute term		$281^\circ{\cdot}220833$
Motion for 8401 days at $0^\circ{\cdot}00004706845$...	=	·395422
$1''{\cdot}63\ T^2 + 0''{\cdot}012\ T^3$ for T at ·23 ...	=	·000024
$\therefore p_1$	=	$281^\circ{\cdot}616279$

The tabular values from 1924 onwards depend on the new formula above. *They are shown in italics in Table XV.*

The values for January 1st computed by the new formula were in excess of those published in the old Edition of the table by:—

... ·0055 in 1923
... ·0064 in 1936
... ·0075 in 1949

The old values were corrected accordingly by interpolation from 1924 onwards.

These values may also be obtained from the N. Almanac which will usually be available in time for any particular year's computations.

96. This table shows the increment to be added to the quantities given in Table XV to obtain the value on certain days of the year, as the value of the solar perigee (p_1) is required for mid-year of the observations.

Table XVI

97. This table gives the values of I, (inclination of the lunar orbit to the equator), ν, (the right ascension of the intersection of the lunar orbit and the equator), and ξ, (the longitude 'in the moon's orbit' of the intersection), corresponding to each half degree of N, (the longitude of the moon's ascending node), from 0° up to 180°.

Table XVII.

When N is negative, I has the same value as when N is positive; but ν and ξ change sign with N.

The values of I, ν and ξ, corresponding to N at mid-year, will be easily found by interpolation between the two nearest half degrees.

98. This table is subdivided into seven parts (1), (2), &c., and is used for the determination of the factors $1/f$ and f required in calculating H from R and *vice versâ*.

Table XVIII.

The values are given corresponding to each 0°.1 of I, the inclination of the lunar orbit to the equator. The values required in the computation are those corresponding to I for mid-year; so that I is first obtained from Table XVII to correspond to N at mid-year, and then the $1/f$ or f will be easily calculated by interpolation from the particular part of this table.

Table XIX.

99. This table gives the values of ν' corresponding to each 0°·1 of I. The ν' is required in computing the values of $h_0-\nu'-\frac{1}{2}\pi$, the initial argument for the tide K_1.

Table XX.

100. This table is similar to Table XIX, and gives the value of $2\nu''$ employed in determining the initial argument, $2h_0-2\nu''$, for the tide K_2.

Attention should be paid to the notes at the foot of these tables.

Harmonic analysis discontinued

101. Harmonic analysis has been discontinued at all ports except Basrah on the advice of Dr. A. T. Doodson and Prof H. Lamb. Observations were continued at the minor ports for only 5 years or less; at the larger ports observations extended over longer periods. At the following working ports observations still continue merely for purposes of local investigation and check on predictions:—Aden, Karachi, Bombay (Apollo Bandar and Prince's Dock), Madras, Calcutta (Kidderpore), Rangoon and Moulmein. In addition tidal observations are being carried out at Basrah with a view to harmonic analysis.

THE PREPARATION OF DATA FOR TIDAL PREDICTION.

102. The result of the harmonic analysis has been the dissection of the aggregate tidal wave,and it gives the tidal constants for each separate constituent tide for each port.

To carry out prediction, it is necessary to carry out the synthesis of these constituent tides and recompound them in their proper relative positions in relation to the position of the moon and sun at the moment chosen, for the commencement of prediction at the port in question, and to obtain the amplitudes and phase angles to be set on the tide-predicting machine at the commencement of the predictions.

From the harmonic analyses, the values of κ and H have been obtained year by year for each tide for each port, and the mean values extracted after a long period of observations, varying from about 5 to 45 years in the case of some of the ports.

As these average values are now adopted, these values of κ and H are entered in form 17 Tid for each of the separate tides to be set on the tidal machine for each port. The values of (V_0+u) and f however have now to be determined for the actual year of prediction.

103. The method of procedure is best illustrated by a sample of the computations for the data for tidal prediction for the tide-tables for 1923.

COMPUTATIONS FOR DATA FOR 1923 TIDE-TABLES.

Year begins 0 hour (noon) 28th December 1922, and is equal to 365 days.

Midyear = midnight 28th-29th June 1923. (In case of a leap year mid-year would fall at noon 28th June.)

N. (Longitude of moon's ascending node.)

Table 13 for 0 hour (noon) 1st January 1923 = 174°·3141

Table 14 December up to midnight 28th-29th June 1923 = −9·4523

164·8618

or 164°·862 for Greenwich

I. (Inclination of the lunar orbit to the equator.)

Table 17 for 164°·5 = 18°·544 and 165°·0 = 18°·529 Diff. for 0°·5 = −0°·015

for Diff. ·0367 to ·0377 (see various values of N on page 211)

=18°·544 − 0°·011

=18°·533 for all ports

ν. (Right Ascension of the intersection of the lunar orbit and the equator.)

Table 17 for 164°·5 = 4°·323

and for 165·0 = 4·190

Diff. for 0·5 = −0·133

Corrections corresponding to differences varying

from 0·367 to 0·370	= −0·098	ν corresponding	= 4°·225
„ 0·371 to 0·374	= −0·099	„	= 4·224
„ 0·375 to 0·377	= −0·100	„	= 4·223

ζ. (Longitude in the moon's orbit of the intersection.)

Table 17 for 164°·5 = 4°·040

„ 165·0 = 3·916

Diff „ 0·5 = −0·124

Corrections corresponding to differences varying

from 0·367 to 0·368	= −0·091	ζ corresponding	= 3°·949
„ 0·370 to 0·372	= −0·092	„	= 3·948
0·373 to 0.377	= −0·093	„	= 3·947

ν for determining $(h_0 - \nu' - \frac{1}{2}\pi)$, the initial argument for tide K_1

Table 19 for 18°·5 = 2°·515

„ 18·6 = 3·049

Diff. .. ·1 = 0·534 and for 0·033 = 0·176

Hence for 18·533 = 2·691 for all ports.

$2\nu''$ for determining $(2h_0 - 2\nu'')$, the initial argument for tide K_2.

Table 20 for 18°·5 = 4°·548

„ 18·6 = 5·531

Diff. „ 0·1 = 0·98

0·033 = 0·324

Hence for 18°·533 = 4°·872 for all ports.

COMPUTATIONS FOR DATA FOR 1923 TIDE-TABLES.

No.	Ports	N For Greenwich	N E. Long. Corr.	N For Port	I	ν	ζ	ν′	2 ν″
			+		+	+	+	+	+
1	Suez ...	164°·862	0°·005	164°·867	+ 18°·533 for all ports	4°·225	3°·949	2°·691 for all ports	4°·872 for all ports
2	Perim ...	,,	0·006	·868		,,	,,		
3	Aden ...	,,	0·007	·869		,,	,,		
4	Maskat ...	,,	0·009	·871		,,	3·948		
5	Bushire ...	,,	0·007	·869		,,	3·949		
6	Karachi ..	,,	0·010	·872		4·224	3·948		
7	Okha ...	,,	,,	,,		,,	,,		
8	Porbandar ...	,,	,,	,,		,,	,,		
9	Port Albert Victor	,,	0·011	·873		,,	,,		
10	Bhavnagar ...	,,	,,	,,		,,	,,		
11	Bombay (A.B) ...	,,	,,	,,		,,	,,		
12	Do. (Prince's Dock)	,,	,,	,,		,,	,,		
13	Marmagao ...	,,	,,	,,		,,	,,		
14	Karwar ...	,,	,,	,,		,,	,,		
15	Beypore ...	,,	,,	,,		,,	,,		
16	Cochin ...	,,	,,	,,		,,	,,		
17	Tuticorin ...	,,	,,	,,		,,	,,		
18	Minicoy ...	,,	,,	,,		,,	,,		
19	Pamban ...	,,	0·012	·874		,,	3·947		
20	Galle ...	,,	,,	,,		,,	,,		
21	Colombo ..	,,	,,	,,		,,	,,		
22	Trincomalee ...	,,	,,	,,		,,	,,		
23	Negapatam ...	,,	,,	,,		,,	,,		
24	Madras ...	,,	,,	,,		,,	,,		
25	Cocanada ...	,,	,,	,,		,,	,,		
26	Vizagapatam ...	,,	,,	,,		,,	,,		
27	False Point ...	,,	0·013	·875		,,	,,		
28	Akyab ...	,,	0·014	·876		4·223	,,		
29	Diamond Island ...	,,	,,	,,		,,	,,		
30	Mergui ...	,,	0·015	·877		,,	,,		
31	Port Blair ...	,,	0·014	·876		,,	,,		
32	Dublat ...	,,	0·013	·875		4·224	,,		
33	Diamond Harbour	,,	,,	,,		,,	,,		
34	Kidderpore ...	,,	,,	,,		,,	,,		
35	Chittagong ...	,,	0·014	·876		4·223	,,		
36	Bassein ...	,,	,,	,,		,,	,,		
37	Elephant Point ...	,,	,,	,,		,,	,,		
38	Rangoon ...	,,	,,	,,		,,	,,		
39	Amherst ...	,,	,,	,,		,,	,,		
40	Moulmein ...	,,	,,	,,		,,	,,		

COMPUTATIONS FOR DATA FOR 1923 TIDE-TABLES.

p_1

Table 15.	For 0 hours 1st January 1923 ...	=	281°·6108
Table 16.	Increment to 0 hours 29th June 1923	=	+ 0·00838
			281·61918
Deduct for 12 hours to bring it to midnight 28th-29th June 1923		−	0·00002
For all ports	p_1	=	281·61916 or 281°·619

p_0 or π

From Table III for Jany. 0 1923 *i.e.* noon Dec. 31,1922	=	189°·98899
Deduct for 3 days *i.e.* from 0 hrs. 31st Dec. to 0 hrs. 28th Dec. 1922, *vide* Table V ...	=	−0·33421
Hence for noon 28th December 1922 ...	=	189·65467
Add constant *vide* footnote Table III ...		+0·136

p_0 or π = 189°·79067 or 189°·791 for Greenwich,

Constant for Computing P from p_0

P = p_0 + constant − ζ or values of ζ for various ports

P = p_0 + 20·560 −3·949 = p_0 + 16·611 for ports from Nos. 1 to 3 and 5.

P = p_0 + 20·560 −3·948 = p_0 + 16·612 for port No. 4 and for ports from Nos. 6 to 18.

P = p_0 + 20·560 −3·947 = p_0 + 16·613 for ports from No. 19 to end.

I = 18°·533 for all ports.

$\frac{1}{2}I$ = 9°·2665 = 9° 15′ 59″·4

Log. cot. $\frac{1}{2}I$ = 10·7873970 Log. tan $\frac{1}{2}I$ = 9·21260

cot. $\frac{1}{2}I$ or NN = 6·129 $\tan^2 \frac{1}{2}I$ = 8·42520

$\cot^2. \frac{1}{2}I$ = 37·564641

$\frac{1}{6}\cot^2. \frac{1}{2}I$ = 6·2608

h_0 (*sidereal time at mean noon*)

	hrs. mts. secs.
Nautical Alamanac for 28th Dec. 1922	= 18 24 41·12
	hrs. mts.
	= 18 24·6853
	hrs.
	= 18·4114217
	or 276°·171326
	= 276°·171
	for Greenwich

s_0 (*moon's mean longitude*)

Nautical Almanac for 27th Dec. 1922	=	19°·4754
Daily Motion for 1 day	=	+ 13·1764
∴ for noon 28th Dec. 1922	=	32·6518
Correction p. 598 NA −40″·4	=	−0°·011
	=	32°·6406
		for Greenwich

COMPUTATION FOR DATA FOR 1923 TIDE-TABLES.

$I = 18^\circ{\cdot}533$ for all ports

From Table 18 (1) f for 18·5 = 1·03672
and „ 0·033 = − 0·0001947

Augmenting factor f for 18·533 = 1·0365253 = 1·037

for M_2, N, 2N, ν, MS, 2SM and MSf tides
$(1{\cdot}037)^2$ = 1·075 for M_4, 2MS, and MN tides
$(1{\cdot}037)^3$ = 1·115 „ M_6 tide
$(1{\cdot}075)^2$ = 1·156 for M_8 tide

From Table 18 (3) f for 18·5 = 0·81333

and „ 0·033 = + 0·00132

Augmenting factor f for 18·533 = 0·81465 = 0·815 for O and Q tides
From Table 18 (5) f for 18·5 = 0·83416

and „ 0·033 = + 0·00128

Augmenting factor f for 18·533 = 0·83544 or 0·835 for J. tide
From Table 18 (8) f for 18·5 = 0·88625

and „ 0·033 = + 0·00079

Augmenting factor f for 18·533 = 0·88704 or 0·887 for K_1 tide
From Table 18 (9) f for 18·5 = 0·75462

and „ 0·033 = + 0·00123

Augmenting factor f for 18·533 = 0·75585 or 0·756 for K_2 tide
$M_2 K_1$ = 1·037 × 0·887 = 0·919819 or 0·920 for $M_2 K_1$ tide

$2 M_2 K_1$ = $M_4 K_1$ = 1·075 × 0·887 = 0·953525 or 0·954 for $2 M_2 K_1$ tide

Augmenting factor
$\frac{1}{f}$ for M_2 from Table 18 (1) for 18·5 = 0·96458
and „ 0·033 = + 0·00018
0·96476 or 0·965

for M_2, N, 2N, ν, MS, 2SM, and MSf. tides.

COMPUTATIONS FOR DATA

No.	Name of Port	h_0		s_0		p_0		P		2 P
		E.L. Corr.	For Port	E.L. Corr.	For Port	E.L. Corr.	For Port	(Constant $-\xi$)	P = (p_0 + const $-\zeta$)	
	Greenwich *	*	276·171	*	32·641	*	189·791			
		−		−		−		+		
1	Suez ...	0°·089	276°·082	1°·191	31°·450	0°·010	189°·781	16°·611	206°·392	52°·784
2	Perim ...	0·119	276·052	1·589	31·052	0·013	·778	,,	·389	·778
3	Aden ...	0·123	276·048	1·647	30·994	0·014	·777	,,	·388	·776
4	Muskat ...	0·160	276·011	2·145	30·496	0·018	·773	·612	·385	·770
5	Bushire ...	0·139	276·032	1·857	30·784	0·016	·775	·611	·386	·772
6	Karachi ...	0·183	275·988	2·451	30·190	0·021	·770	·612	·382	·764
7	Okha ...	0·189	275·982	2·529	30·112	,,	,,	,,	,,	,,
8	Porbandar ...	0·191	275·980	2·548	30·093	0·022	·769	,,	·381	·762
9	Port Albert Victor	0·196	275·975	2·618	30·023	,,	,,	,,	,,	,,
10	Bhavnagar ...	0·198	275·973	2·641	30·000	,,	,,	,,	,,	,,
11	Bombay (A.B) ...	0·199	275·972	2·666	29·975	0·023	·768	,,	·380	·760
12	Do. (Prince's Dock)	0·199	275·972	2·667	29·974	,,	,,	,,	,,	,,
13	Marmagao ...	0·202	275·969	2·701	29·940	,,	,,	,,	,,	,,
14	Karwar ...	0·203	275·968	2·712	29·929	,,	,,	,,	,,	,,
15	Beypore ...	0·208	275·963	2·774	29·867	,,	,,	,,	,,	·
16	Cochin ...	0·209	275·962	2·791	29·850	0·024	·767	,,	·379	·758
17	Tuticorin ...	0·214	275·957	2·860	29·781	,,	,,	,,	,,	,,
18	Minicoy ...	0·200	275·971	2·674	29·967	0·023	·768	,,	·380	·760
19	Pamban ...	0·217	275·954	2·899	29·742	0·025	·766	·613	·379	·758
20	Galle ...	0·220	275·951	2·936	29·705	,,	,,	,,	·	,,
21	Colombo ...	0·219	275·952	2·922	29·719	,,	,,	,,	,,	,,
22	Trincomalee ...	0·222	215·949	2·972	29 669	,,	,,	,,	,,	,,
23	Negapatam ...	0·219	275·952	2·922	29·719	,,	,,	,,	,,	,,
24	Madras ...	0·220	275 951	2·939	29·702	,,	,,	,,	,,	,,
25	Coconada ...	0·225	275 946	3 010	29·631	,,	,,	,,	,,	,,
26	Vizagapatam ...	0·228	275·943	3·048	29·593	0·026	·765	,,	·378	·756
27	False Point ...	0·238	275·933	3·177	29 464	0·027	·764	,,	·377	·754
28	Akyab ...	0·254	275·917	3·400	29·241	0·029	·762	,,	·375	·750
29	Diamond Island...	0·258	275·913	3·451	29·190	,,	,,	,,	,,	,,
30	Mergui ...	0·270	275·901	3·609	29·032	0·031	·760	,,	·373	·746
31	Port Blair ...	0·254	275·917	3·395	29·246	0·029	·762	,,	·375	·750
32	Dublat ...	0·241	275 930	3·226	29·415	0·027	·764	,,	·377	·754
33	Diamond Harbour	0·241	275 930	3·228	29·413	,,	,,	,,	,,	,,
34	Kidderpore ...	0·242	275.929	3·233	29·408	,,	,,	,,	,,	,,
35	Chittagong ...	0·251	275·920	3·361	29·280	0·028	·763	,,	·376	·752
36	Bassein ...	0·260	275·911	3·469	29·172	0·029	·762	,,	·375	·750
37	Elephant Point ...	0·264	275·907	3·525	29·116	0·030	·761	..	·374	·748
38	Rangoon ...	0·263	275·908	3·520	29·121	,,	,,	,,	,,	,,
39	Amherst ...	0·267	275·904	3·571	29·070	,,	,,	,,	,,	,,
40	Moulmein ...	0·267	275·904	3·573	29·068	,,	,,	,,	,,	,,

FOR 1923 TIDE TABLES.

2 P (1)	Log sin 2 P (2)	Log cos 2 P (3)	Cos 2P or N.N. (4)	$\frac{1}{6}\cot^2 \frac{1}{2}I$ (5)	(5) − (4) = (6)	Colog of (6) = (7)	Log Tan R. = (2) + (7)	R	
	+	+	+	+	+	+	+		
52° 47′ 2″·4	9·90111	9·78163					9·14860	8° 0′ 52″·24	8·015
46 40·8	9·90108	9·78169					9·14857	8 0 50·28	8·014
46 33·6	9·90106	9·78171					9·14855	8 0 48·97	,,
46 12·0	9·90103	9·78177					9·14852	8 0 47·00	8·013
46 19·2	9·90104	9·78175					9·14853	8 0 47·65	,,
45 50·4	9·90099	9·78183					9·14848	8 0 44·38	8·012
,,	,,	,,					,,	,,	,,
45 43·2	9·90098	9·78185					9·14847	8 0 43·72	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
45 36·0	9·90097	9·78187					9·14846	8 0 43·07	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
45 28·8	9·90096	9·78189					9·14845	8 0 42·41	,,
,,	,,	,,					,,	,,	,,
45 36·0	9·90097	9·78187	+ 0·605 for all ports	+ 6·261 for all ports	+ 5·656 for all ports	+ 9·24749 for all ports	9·14846	8 0 43·07	,,
45 28·8	9·90096	9·78189					9·14845	8 0 42·41	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
45 21·6	9·90095	9·78191					9·14844	8 0 41·76	,,
45 14·4	9·90094	9·78193					9·14843	8 0 41·10	8·011
45 0·0	9·90091	9·78197					9·14840	8 0 39·13	,,
,,	,,	,,					,,	,,	,,
44 45·6	9·90089	9·78201					9·14838	8 0 37·82	,,
45 0·0	9·90091	9·78197					9·14840	8 0 39·13	,,
45 14·4	9·90094	9·78193					9·14843	8 0 41·10	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
45 7·2	9·90093	9·78195					9·14842	8 0 40·44	,,
45 0·0	9·90091	9·78197					9·14840	8 0 39·13	,,
44 52·8	9·90090	9·78199					9·14839	8 0 38·48	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,
,,	,,	,,					,,	,,	,,

COMPUTATION OF DATA FOR 1923 TIDE-TABLES.

No. of Port.	Log. Tan. ½ I =(1)	2 log. Tan ½ I or (1)² =(2)	Log. of Nat number 12=(3)	Log. cos 2 P =(4)	(2)+(3)+(4) =(5)	N.N of (5) =(6)	Nat number 1 −(6)=(7)	Log. of (7) =(8)	½ log. of (7) =(9)	Log. of $\frac{1}{f}$ for M_2 =(10)	(10)−(9) =(11)	Co-log. of (11) =(12)	NN. of (12) or f for L_2
1	9·21280 for all ports	8·42520 for all ports	1·07918 for all ports	9·78163	9·28601	0·1932	0·8068	9·90677	9·95339	9·99453 for all ports	0·03114	9·96886	0·931
2				169	607	"	"	"	"		"	"	"
3				171	609	"	"	"	"		"	"	"
4				177	615	0·1933	0·8067	671	336		117	883	"
5				175	613	"	"	"	"		"	"	"
6				183	621	"	"	"	"		"	"	"
7				"	"	"	"	"	"		"	"	"
8				185	623	"	"	"	"		"	"	"
9				"	"	"	"	"	"		"	"	"
10				"	"	"	"	"	"		"	"	"
11				187	625	"	"	"	"		"	"	"
12				"	"	"	"	"	"		"	"	"
13				"	"	"	"	"	"		"	"	"
14				"	"	"	"	"	"		"	"	"
15				"	"	"	"	"	"		"	"	"
16				189	627	"	"	"	"		"	"	"
17				"	"	"	"	"	"		"	"	"
18				187	625	"	"	"	"		"	"	"
19				189	627	"	"	"	"		"	"	"
20				"	"	"	"	"	"		"	"	"
21				"	"	"	"	"	"		"	"	"
22				"	"	"	"	"	"		"	"	"
23				"	"	"	"	"	"		"	"	"
24				"	"	"	"	"	"		"	"	"
25				"	"	"	"	"	"		"	"	"
26				191	629	"	"	"	"		"	"	"
27				193	631	"	"	"	"		"	"	"
28				197	635	0·1934	0·8066	666	333		120	880	"
29				"	"	"	"	"	"		"	"	"
30				201	639	"	"	"	"		"	"	"
31				197	635	"	"	"	"		"	"	"
32				193	631	0·1933	0·8067	671	336		117	883	"
33				"	"	"	"	"	"		"	"	"
34				"	"	"	"	"	"		"	"	"
35				195	633	"	"	"	"		"	"	"
36				197	635	0·1934	0·8066	666	333		120	880	"
37				199	637	"	"	"	"		"	"	"
38				"	"	"	"	"	"		"	"	"
39				"	"	"	"	"	"		"	"	"
40				"	"	"	"	"	"		"	"	"

COMPUTATION OF DATA FOR 1923 TIDE TABLES.

h_0 or S_s +	ν +	$h_0-\nu$ +	s_0 +	ζ +	$s_0-\zeta$	V_0+u for M_1 = $(h_0-\nu)-(s_0-\zeta)$	V_0+u for M_2, MS, −2 SM & −MSf	V_0+u for M_4 & 2MS	V_0+u for M_6	No. of Port.
°	°	°	°	°	°	°	°	°	°	
276·082	4·225	271·857	31·450	3·949	27·501	244·356	128·712	257·424	26·136	1
·052	,,	·827	·052	,,	·103	·724	129·448	258·896	28·344	2
·048	,,	·823	30·994	,,	·045	·778	·556	259·112	·668	3
·011	,,	·786	·496	3·948	26·548	245·238	130·476	260·952	31·428	4
·032	,,	·807	·784	3·949	·835	244·972	129·944	259·888	29·832	5
275·988	4·224	·764	·190	3·948	·242	275·522	131·044	262·088	33·132	6
·982	,,	·758	·112	,,	·164	·594	·188	·376	·564	7
·980	,,	·756	·093	,,	·145	·611	·222	·444	·666	8
·975	,,	·751	·023	,,	·075	·676	·352	·704	34·056	9
·973	,,	·749	30·000	,,	·052	·697	·394	·788	·182	10
·972	,,	·748	29·975	,,	·027	·721	·442	·884	·326	11
·972	,,	·748	·974	,,	·026	·722	·444	·888	·332	12
·969	,,	·745	·940	,,	25·992	·753	·506	263·012	·518	13
·968	,,	·744	·929	,,	·981	·763	·526	·052	·578	14
·963	,,	·739	·867	,,	·919	·820	·640	·280	·920	15
·962	,,	·738	·850	,,	·902	·836	·672	·344	35·016	16
·957	,,	·733	·781	,,	·833	·900	·800	·600	·400	17
·971	,,	·747	·967	,,	26·019	·728	·456	262·912	34·368	18
·954	,,	·730	·742	3·947	25·795	·935	·870	263·740	35·610	19
·951	,,	·727	·705	,,	·758	·969	·938	·876	·814	20
·952	,,	·728	·719	,,	·772	·956	·912	·824	·736	21
·949	,,	·725	·669	,,	·722	246·003	132·006	264·012	36·018	22
·952	,,	·728	·719	,,	·772	245·956	131·912	263·824	35·736	23
·951	,,	·727	·702	,,	·755	·972	·944	·888	·832	24
·946	,,	·722	·631	,,	·684	246·038	132·076	264·152	36·228	25
·943	,,	·719	·593	,,	·646	·073	·146	·292	·438	26
·933	,,	·709	·464	,,	·517	·192	·384	·768	37·152	27
·917	4·223	·694	·241	,,	·294	·400	·800	265·600	38·400	28
·913	,,	·690	·190	,,	·243	·447	·894	·788	·682	29
·901	,,	·678	·032	,,	·085	·593	133·186	266·372	39·558	30
·917	,,	·694	·246	,,	·299	·395	132·790	265·580	38·370	31
·930	4·224	·706	·415	,,	·468	·238	·476	264·952	37·428	33
·930	,,	·706	·413	,,	·466	·240	·480	·960	·440	33
·929	,,	·705	·408	,,	·461	·244	·488	·976	·464	34
·920	4·223	·697	·280	,,	·333	·364	·728	265·456	38·184	35
·911	,,	·688	·172	,,	·225	·463	·926	·852	·778	36
·907	,,	·684	·116	,,	·169	·515	133·030	266·060	89·090	37
·908	,,	·685	·121	,,	·174	·511	·022	·044	·066	38
·904	,,	·681	·070	,,	·123	·558	·116	·232	·348	39
·904	,,	·681	·068	,,	·121	·560	·120	·240	·360	40

COMPUTATION OF DATA

No. of Port.	h_0	$-\nu'$	$-\frac{1}{2}\pi$ or $+270°\cdot000$	$=(V_0+u)$ for K_1	$2h_0$	$-2\nu''$	$=(V_0+u)$ for K_2
1	276°·082	−2°·691	+270°·000	183°·391	192°·164	−4°·872	187°·292
2	·052	,,	,,	·361	·104	,,	·232
3	·048	,,	,,	·357	·096	,,	·224
4	·011	,,	,,	·320	·022	,,	·150
5	·032	,,	,,	·341	·064	,,	·192
6	275·988	,,	,,	·297	191·976	,,	·104
7	·982	,,	,,	·291	·964	,,	·092
8	·980	,,	,,	·289	·960	,,	·088
9	·975	,,	,,	·284	·950	,,	·078
10	·973	,,	,,	·282	·946	,,	·074
11	·972	,,	,,	·281	·944	,,	·072
12	·972	,,	,,	·281	·944	,,	·072
13	·969	,,	,,	·278	·938	,,	·066
14	·968	,,	,,	·277	·936	,,	·064
15	·968	,,	,,	·272	·926	,,	·054
16	·962	,,	,,	·271	·924	,,	·052
17	·957	,,	,,	·266	·914	,,	·042
18	·971	,,	,,	·280	·942	,,	·070
19	·954	,,	,,	·263	·908	,,	·036
20	·951	,,	,,	·260	·902	,,	·030
21	·952	,,	,,	·261	·904	,,	·032
22	·949	,,	,,	·258	·898	,,	·026
23	·952	,,	,,	·261	·904	,,	·032
24	·951	,,	,,	·260	·902	,,	·030
25	·946	,,	,,	·255	·892	,,	·020
26	·943	,,	,,	·252	·886	,,	·014
27	·933	,,	,,	·242	·866	,,	186·994
28	·917	,,	,,	·226	·834	,,	·962
29	·913	,,	,,	·222	·826	,,	·954
30	·901	,,	,,	·210	·802	,,	·930
31	·917	,,	,,	·226	·834	,,	·962
32	·930	,,	,,	·239	·860	,,	·988
33	·930	,,	,,	·239	·860	,,	·988
34	·929	,,	,,	·238	·858	,,	·986
35	·920	,,	,,	·229	·840	,,	·968
36	·911	,,	,,	·220	·822	,,	·950
37	·907	,,	,,	·216	·814	,,	·942
38	·908	,,	,,	·217	·816	,,	·944
39	·904	,,	,,	·213	·808	,,	·936
40	·904	,,	,,	·213	·808	,,	·936

FOR 1923 TIDE TABLES.

$(h_0-\nu)$	$-2(s_0-\xi)$ or Mf	+ ½π or 90°·000	$=(V_0+u)$ for O	½π or $-h_0$	$+90°·000$	$=(V_0+u)$ for P
271°·857	−55°·002	+90°·000	306°·855	83°·918	+90°·000	173°·918
·827	−54·206	,,	307·621	·948	,,	·948
·823	−54·090	,,	·733	·952	,,	·952
·786	−53·096	,,	308·690	·989	,,	·989
·807	−53·670	,,	·137	·968	,,	·968
·764	−52·484	,,	309·280	84·012	,,	174·012
·758	·328	,,	·430	·018	,,	·018
·756	·290	,,	·466	·020	,,	·020
·751	·150	,,	·601	·025	,,	·025
·749	·104	,,	·645	·027	,,	·027
·748	·054	,,	·694	·028	,,	·028
·748	·052	,,	·696	·028	,,	·028
·745	−51·984	,,	·761	·031	,,	.031
·744	·962	,,	·782	·032	,,	·032
·739	·838	,,	·901	·037	,,	·037
·738	·804	,,	·934	·038	,,	·038
·733	·666	,,	310·067	·043	,,	·043
·747	−52·038	,,	309·709	·029	,,	·029
·730	−51·590	,,	310·140	·046	,,	·046
·727	·516	,,	·211	·049	,,	·049
·728	·544	,,	·184	·048	,,	·048
·725	·444	,,	·281	·051	,,	·051
·728	·544	,,	·184	·048	,,	·048
·727	·510	,,	·217	·049	,,	·049
·722	·368	,,	·354	·054	,,	·054
·719	·292	,,	·427	·057	,,	·057
·709	·034	,,	·675	·067	,,	·067
·694	−50·588	,,	311·106	·083	,,	·083
·690	·486	,,	·204	·087	,,	·087
·678	·170	,,	·508	·099	,,	·099
·694	·598	,,	·096	·083	,,	·083
·706	·936	,,	310·770	·070	,,	·070
·706	·932	,,	·774	·070	,,	·070
·705	·922	,,	·783	·071	,,	·071
·697	·666	,,	311·031	·080	,,	·080
·688	·450	,,	·238	·089	,,	·089
·684	·338	,,	·346	·093	,,	·093
·685	·348	,,	·337	·092	,,	·092
·681	·246	,,	·435	·096	,,	·096
·681	·242	,,	·439	·096	,,	·096

COMPUTATION FOR DATA FOR 1923 TIDE TABLES.

No. of port.	½ π or $(h_0 - \nu) + (s_0 - p_0) - 270 \cdot 000)$ or Mm			$= (V_0 + u)$ for J	$(V_0 + u)$ for O $- (s_0 - p_0)$		$= (V_0 + u)$ for Q
1	271°·857	+ 201°·669	+ 270°·000	23°·526	306°·855	− 201°·669	105°·186
2	·827	·274	+ ,,	·101	307·621	·274	106·347
3	·823	·217	+ ,,	·040	·733	·217	·516
4	·786	+ 200·723	+ ,,	22·509	308·690	− 200·723	107·967
5	·807	+ 201·009	+ ,,	·816	·137	− 201·009	·128
6	·764	+ 200·420	+ ,,	·184	309·280	− 200·420	108·860
7	·758	·342	+ ,,	·100	·430	·342	109·088
8	·756	·324	+ ,,	·080	·466	·324	·142
9	·751	·254	+ ,,	·005	·601	·254	·347
10	·749	·231	+ ,,	21·980	·645	·231	·414
11	·748	·207	+ ,,	·955	·694	·207	·487
12	·748	·206	+ ,,	·954	·696	·206	·490
13	·745	·172	+ ,,	·917	·761	·172	·589
14	·744	·161	+ ,,	·905	·782	.161	·621
15	·739	·099	+ ,,	·838	·901	·099	·802
16	·738	·083	+ ,,	·821	·934	·083	·851
17	·733	·014	+ ,,	·747	310·067	·014	110·053
18	·747	·199	+ ,,	·946	309·709	·199	109·516
19	·730	+ 199·976	+ ,,	·706	310·140	− 199·976	119·164
20	·727	·939	+ ,,	·666	·211	·939	·272
21	·726	·953	+ ,,	·681	·184	·953	·231
22	·725	·903	+ ,,	·628	·281	·903	·378
23	·728	·953	+ ,,	·681	·184	·953	·231
24	·727	·936	+ ,,	·663	·217	·936	·281
25	·722	·865	+ ,,	·587	·354	·865	·489
26	·719	·828	+ ,,	·547	·427	·823	·599
27	·709	·700	+ ,,	·409	·675	·700	·975
28	·694	·479	+ ,,	·173	311·106	·479	111·627
29	·690	·428	+ ,,	·118	·204	·428	·776
30	·678	·272	+ ,,	20·950	·508	·272	112·236
31	·694	·484	+ ,,	21·178	·096	·484	111·612
32	·706	·651	+ ,,	·357	310·770	·651	·119
33	·706	·649	+ ,,	·355	·774	·649	·125
34	·705	·644	+ ,,	·349	·783	·644	·139
35	·697	·517	+ ,,	·214	311·031	·517	·514
36	·688	·410	+ ,,	·098	·238	·410	·823
37	·684	·355	+ ,,	·039	·346	·355	·991
38	·685	·360	+ ,,	·045	·337	·360	·977
39	·681	·309	+ ,,	20·990	·435	·309	112·126
40	·681	·307	+ ,,	·988	·439	·307	·232

COMPUTATION OF DATA FOR 1923 TIDE-TABLES.

No. of Port.	(V_0+u) for $M_2+(s_0-p_0)+\pi-R$				$=(V_0+u)$ for L	(V_0+u) for $M_2-(s_0-p_0)$		$=(V_0+u)$ for N
1	128°·712 +	201°·669 +	180°·000 +	351°·985	142°·366	128°·712 −	201°·669	287°·043
2	129·448	·274	,,	·986	142·708	129·448	·274	288·174
3	·556	·217	,,	·986	142·759	·556	·217	·339
4	130·476 +	200·723	,,	·987	143·186	130·476 −	200·723	289·753
5	129·944 +	201·009	,,	,,	142·940	129·944 −	201·009	288·935
6	131·044 +	200·420	,,	·988	143·452	131·044 −	200·420	290·624
7	·188	·342	,,	,,	·518	·188	·342	·846
8	·222	·324	,,	,,	·534	·222	·324	·898
9	·352	·254	,,	,,	·594	·352	·254	291·098
10	·394	·231	,,	,,	·613	·394	·231	·163
11	·442	·207	,,	,,	·637	·442	·207	·235
12	·444	·206	,,	,,	·638	·444	·206	·238
13	·506	·172	,,	,,	·666	·506	·172	·334
14	·526	·161	,,	,,	·675	·526	·161	·365
15	·640	·099	,,	,,	·727	·640	·099	·541
16	·672	·083	,,	,,	·743	·672	·083	·589
17	·800	·014	,,	,,	·802	·800	·014	·786
18	·456	·199	,,	,,	·643	·456	·199	·257
19	·870 +	199·976	,,	,,	·834	·870 −	199·976	·894
20	·938	·939	,,	,,	·865	·938	·939	·999
21	·912	·253	,,	,,	·853	·912	·953	·959
22	132·006	·903	,,	,,	·897	132·006	·903	292·103
23	131·912	·953	,,	,,	·853	131·912	·953	291·959
24	·944	·936	,,	,,	·868	·944	·936	292·008
25	132·076	·865	,,	,,	·929	132·076	·865	·211
26	·146	·828	,,	,,	·962	·146	·828	·318
27	·384	·700	,,	989	144·073	·384	·700	·684
28	·800	·479	,,	,,	·268	·800	·479	293·321
29	·894	·428	,,	,,	·311	·894	·428	·466
30	133·186	·272	,,	,,	·447	133·186	·272	·914
31	132·790	·484	,,	,,	·263	132·790	·484	·306
32	·476	·651	,,	,,	·116	·476	·651	292·825
33	·480	·649	,,	,,	·118	·480	·649	·831
34	·488	·644	,,	,,	·126	·488	·644	·844
35	·728	·517	,,	,,	·366	·728	·517	293·211
36	·926	·410	,,	,,	·325	·926	·410	·516
37	133·030	·355	,,	,,	·374	133·030	·355	·675
38	·022	·360	,,	,,	·371	·022	·360	·662
39	·116	·309	,,	,,	·414	·116	·309	·807
40	·120	·307	,,	,,	·416	·120	·307	·813

Not required for Riverain Ports except Bassein. (L and N, Ports 32–40)

COMPUTATION OF DATA

No. of Port.	(V_0+u) for $M_2+(s_0-p_0)+2h_0-2s_0$				$=(V_0+u)$ for V	(V_0+u) for 2 SM $=$ $-(V_0+u)$ for M_2	(V_0+u) for T $=$ $-(h_0-p_1)$	(V_0+u) for $N-(s_0-p_0)$	
1	128·712+	201·669+	192·164+	297·100	99·645	231·288	5·537	287·043−	201·669
2	129·448	·274	·104+	·896	100·722	230·552	·567	288·174	·274
3	·556	·217	·096+	298·012	·881	·444	·571	·339	·217
4	130·476+	200·723	·022+	299·008	102·229	229·524	·608	289·753−	200·723
5	129·944+	201·009	·064+	298·432	101·449	230·056	·587	298·935−	201·009
6	131·044+	200·420+	191·976+	299·620	103·060	228·956	·631	290·624−	200·420
7	·188	·342	·964	·776	·270	·812	·637	·846	·342
8	·232	·324	·960	·814	·320	·778	·639	·898	·324
9	·352	·254	·950	·954	·510	·648	·644	291·098	·254
10	·394	·231	·946+	300·000	·571	·606	·646	·163	·231
11	·442	·207	·944	·050	·643	·558	·647	·235	·207
12	·444	·206	·944	·052	·646	·556	·647	·238	·206
13	·506	·172	·938	·120	·736	·494	·650	·334	·172
14	·526	·161	·936	·142	·765	·474	·651	·365	·161
15	·640	·099	·926	·266	·931	·360	·656	·541	·099
16	·672	·083	·921	·300	·979	·328	·657	·589	·083
17	·800	·014	·914	·438	104·166	·200	·662	·786	·014
18	·456	·199	·942	·066	103·663	·544	·648	·257	·199
19	·870+	199·976	·908	·516	104·270	·130	·665	·894−	199·976
20	·938	·939	·902	·590	·369	·062	·668	·999	·939
21	·912	·953	·904	·562	·331	·088	·667	·959	·953
22	132·006	·903	·898	·662	·469	227·994	·670	292·103	·903
23	131·912	·953	·904	·562	·331	228·088	·667	291·959	·953
24	·944	·936	·902	·596	·378	·056	·668	292·008	·936
25	132·076	·865	·892	·738	·571	227·924	·673	·211	·865
26	·146	·828	·886	·814	·674	·854	·676	·318	·828
27	·384	·700	·866+	301·072	105·022	·616	·686	·684	·700
28	·800	·479	·834	·519	·631	·200	·702	293·321	·479
29	·894	·428	·826	·620	·768	·106	·706	·466	·428
30	133·186	·272	·802	·936	106·196	226·814	·718	·914	·272
31	132·790	·484	·834	·508	105·616	227·210	·702	·306	·484
32	·476	·651	·860	·170	·157	·524	·689	292·825	·651
33	·480	·649	·860	·174	·163	·520	·689	·831	·649
34	·488	·644	·858	·184	·174	·512	·690	·844	·644
35	·728	·517	·840	·440	·525	·272	·699	293·211	·517
36	·926	·410	·822	·656	·814	·074	·708	·516	·410
37	133·030	·355	·814	·768	·967	226·970	·712	·675	·355
38	·022	·360	·816	·758	·956	·978	·711	·662	·360
39	·116	·309	·808	·860	106·093	·884	·715	·807	·309
40	·120	·307	·808	·864	·099	·880	·715	·813	·307

FOR 1923 TIDE-TABLES.

$=(V_0+u)$ for 2 N	(V_0+u) for M_2+ (V_0+u) for N		$=(V_0+u)$ for MN_4 or M_2N	(V_0+u) for M_2+ (V_0+u) for K_1		$=(V_0+u)$ for MK_3 or M_2K_1
85·374	128·712	+287·043	55·755	128·712	+183·391	312·103
86·900	129·448	+288·174	57·622	129·448	·361	·809
87·122	·556	·339	·895	·556	·357	·913
89·030	130·476	+289·753	60·229	130·476	·320	313·796
87·926	129·944	+288·935	58·870	129·944	·341	·285
90·204	131·044	+290·624	61·668	131·044	·297	314·341
·504	·188	·846	62·034	·188	·291	·479
·574	·222	·898	·120	·222	·289	·511
·844	·352	+291·098	·450	·352	·284	·636
·932	·394	·163	·557	·394	·282	·676
91·028	·442	·235	·677	·442	·281	·723
·032	·444	·238	·682	·444	·281	·725
·162	·506	·334	·840	·506	·278	·784
·204	·526	·365	·891	·526	·277	·803
·442	·640	·541	63·181	·640	·272	·912
·506	·672	·589	·261	·672	·271	·943
·772	·800	·786	·586	·800	·266	315·066
·058	·456	·257	62·713	·456	·280	314·736
·918	·870	·894	63·764	·870	·263	315·133
92·060	·938	·999	·937	·938	·260	·198
·006	·912	·950	·871	·912	·261	·173
·200	132·006	+292·103	64·109	132·006	·258	·264
·006	131·912	+291·959	63·871	131·912	·261	·173
·072	·944	+292·008	·052	·944	·260	·204
·346	132·076	·211	64·287	132·076	·255	·331
·490	·146	·318	·464	·146	·252	·398
·984	·384	·684	65·068	·384	·242	·626
93·842	·800	+293·321	66·121	·800	·226	316·026
94·038	·894	·466	·360	·894	·222	·116
·642	133·186	·914	67·100	133·186	·210	·396
03·822	132·790	·306	66·096	132·790	·226	·016
·174	·476	+292·825	65·301	·476	·239	315·715
·182	·480	·831	·311	·480	·239	·719
·200	·488	·844	·332	·488	·238	·726
·694	·728	+293·211	·939	·728	·229	·957
94·106	·926	·516	96·442	·926	·220	316·146
·520	133·030	·675	·705	133·030	·216	·246
·302	·022	·662	·684	·022	·217	·239
·408	·116	·807	·923	·116	·213	·329
·506	·120	·813	·933	·120	·213	·333

COMPUTATION OF DATA FOR 1923 TIDE-TABLES.

No. of Port.	(V_0+u) for M_4 – (V_0+u) for K_1		= (V_0+u) for $2MK_3$ or $2M_2K_1$
1	257·424 –	183·391	74·033
2	258·896	·361	75·535
3	259·112	·357	·755
4	260·952	·320	77·632
5	259·888	·341	76·547
6	262·088	·297	78·791
7	·376	·291	79·085
8	·444	·289	·155
9	·704	·284	·420
10	·788	·282	·506
11	·884	·281	·603
12	·888	·281	·607
13	263·012	·278	·734
14	·052	·277	·775
15	·280	·272	80·008
16	·344	·271	·073
17	·600	·266	·334
18	262·912	·280	79·632
19	263·740	·263	80·477
20	·876	·260	·616
21	·824	·261	·563
22	264·012	·258	·754
23	263·824	·261	·563
24	·888	·260	·628
25	264·152	·255	·897
26	·292	·252	81·040
27	·768	·242	·526
28	265·600	·226	82·374
29	·788	·222	·566
30	266·372	·210	83·162
31	265·580	·226	82·354
32	264·952	·239	81·713
33	·960	·239	·721
34	·976	·238	·738
35	265·456	·229	82·227
36	·852	·220	·632
37	266·060	·216	·844
38	·044	·217	·827
39	·232	·213	83·019
40	·240	·213	·027

COMPUTATION OF DATUM BELOW MEAN SEA LEVEL FOR 1923 TIDE TABLES.

Mean of A_0 up to 1st January 1918 above zero of gauge	From abstracts of results of Harmonic Analysis of Tidal observations	Datum above or below zero of gauge.	Datum below M. S. L.
1. Aden ...	5·846	1·600 above	4·246
2. Karachi ...	7·209	2·000 „	4·209
3. Bombay (Apollo Bandar) ...	10·230	2·000 „	8·230
4. „ (Prince's Dock) ...	8·244	14·000 below	22·244
5. Madras ...	2·283	0·350 above	1·933
6. Kidderpore ...	10·717	0·000 identical	10·717
7. Rangoon ...	10·248	0·000 identical	10·248
8. Moulmein ...	8·492	2·660 above	5·832
9. Port Blair ...	4·754	1·160 above	5·584

104. From these computations the values of (V_0+u) and f obtained are entered in form 17 Tid, and hence, by subtracting (V_0+u) from κ already entered, ζ is obtained, and by multiplying H by f, the value R is obtained.

The entries for Karachi for data for the 1923 predictions are shown below, as extracted from 24 forms (17 Tid) for the 24 tides whose components are set on the machine.

17. Tid.

VALUES OF R AND Z FOR 0 HOURS 28th DECEMBER 1922. DATA FOR 1923 TIDE-TABLES FOR KARACHI.

Tides	κ	V_0+u	Z = $\kappa-(V_0+u)$	H	f	R = H × f
	°	°	°	feet	feet	feet
M_2	293·76	131·04	162·72	2·564	1·037	2·659
M_4	93·55	262·09	197·46	0·026	1·075	0·028
M_6	204·97	33·13	171·84	0·047	1·115	0·052
S_1	175·50	0·00	175·50	0·088	1·000	0·088
S_2	323·13	0·00	323·13	0·959	1·000	0·959
K_1	46·19	183·30	222·89	1·308	0·837	1·160
K_2	318·75	187·10	131·65	0·268	0·756	0·203
O	46·77	309·28	97·49	0·660	0·815	0·538
N	277·87	290·62	347·25	0·614	1·037	0·637
ν	279·54	103·06	176·48	0·143	1·037	0·148
2η(SSA)	152·41	191·98	320·43	0·150	1·000	0·150
T	284·96	5·63	279·33	0·080	1·000	0·080
P	44·57	174·01	230·56	0·395	1·000	0·395
J	62·97	22·18	40·79	0·080	0·835	0·073
Q	48·79	108·86	299·93	0·141	0·815	0·115
L	296·56	143·45	153·11	0·080	0·931	0·074
μ	267·89	262·09	5·80	0·065	1·075	0·070
MS	318·66	131·04	187·62	0·034	1·037	0·035
2SM	112·33	228·96	243·37	0·019	1·037	0·020
η (SA)	71·60	275·99	155·61	0·144	1·000	0·144
2N	245·05	90·20	154·85	0·092	1·037	0·095
M_2N	232·16	61·67	170·49	0·049	1·075	0·053
M_2K_1	76·50	314·34	122·16	0·043	0·920	0·040
$2M_2K_1$	79·41	78·79	0·62	0·020	0·954	0·019

Computation of Form 1 Tid Pred.

105. From the above data the form 1 Tid Pred for the settings required on the tide predicting machine for height only, or for height and time combined, is computed as shown in the sample of the form given on p 86.

COMPUTATION OF FORM 1 TID PRED.

1 Tid. Pred. (Height Sheet) for year 1923

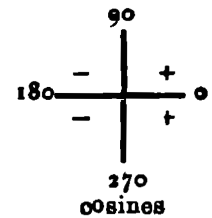
Port Karāchi

Scale = $1\frac{1}{2}''$ to 1 foot. Scale factor M = 19·05 $C = \frac{M}{12\cdot7} = 1\cdot5$

$A_0 = 5\cdot21' = 7\cdot8''$

S_2 (Standard time) = $S_2 - 31°\cdot07 = 36°\cdot87 - 31°\cdot07 = 5°\cdot80$

Δ Z = change in (360° − ζ) after 369 days



(360° − ζ) = Z	359°·38 $2M_2K_1$	2SM 116°·63	172°·38 MS	J 319°·21	60°·07 Q	T 80°·67	206°·89 L	ν 183°·52	354°·20 μ	K_2 228°·35	129°·44 P	N 12°·75	189°·51 M_2N	
Δ Z	+2·7	−3·3	+3·2	+144·7	+218·5	−3·7	+144·1	+147·4	+6·4	+7·3	−3·7	+222·2	+225·4	Sum upper Series
Z + Δ Z = ζ′	2·08	113·33	175·58	103·91	278·57	76·97	350·99	330·92	0·60	235·65	125·79	234·95	54·91	
R	* * *	0·020	* * *	0·073	* · *	0·080	* * * * *	0·148	* * *	0·203	* * *	0·637	* * *	1·161
	0·019	* * *	0·035	* * *	0·115	* * *	0·074	* * *	0·070	* * *	0·395	* * *	0·053	0·761
log R / log M	$\bar{2}$·2788 / 1·2799	$\bar{2}$·3010	$\bar{2}$·5441	$\bar{2}$ 8633	$\bar{1}$·0607	$\bar{2}$·9031	$\bar{2}$·8692	$\bar{1}$·1703	$\bar{2}$·8451	$\bar{1}$·3075	$\bar{1}$·5966	$\bar{1}$·8041	$\bar{2}$·7243	
log RM	$\bar{1}$·5587	$\bar{1}$·5809	$\bar{1}$·8240	0·1432	0·3406	0 1830	0·1491	0·4502	0·1250	0·5874	0 8765	1·0840	0·0042	
R M	* * *	0·38	* * *	1·39	* * *	1·52	* * *	2·82	* * *	3·87	* * *	12·13	* * *	22·11
	0·36	* * *	0·67	* * *	2·19	* * *	1·41	* * *	1·33	* * *	7·53	* * *	1·01	14·50
log cos Z	0·0000	$\bar{1}$·6515	$\bar{1}$·9962	$\bar{1}$·8792	$\bar{1}$·6981	$\bar{1}$·2099	$\bar{1}$·9504	$\bar{1}$·9992	$\bar{1}$·9978	$\bar{1}$·8225	$\bar{1}$·8029	$\bar{1}$·9892	$\bar{1}$·9940	
log (RM cos Z)	$\bar{1}$·5587	$\bar{1}$·2324	$\bar{1}$·8202	0·0224	0·0387	$\bar{1}$·3929	0·0995	0·4494	0·1228	0·4099	0·6794	1·0732	$\bar{1}$·9982	
RM cos Z +	0·36			1·05	1 09	0·25			1·33			11·84		15·92
RM cos Z −		0·17	0·66				1·26	2·82		2·57	4·78		1·00	13·26
log cos ζ′	$\bar{1}$·9997	$\bar{1}$·5977	$\bar{1}$·9987	$\bar{1}$·3809	$\bar{1}$·1732	$\bar{1}$·3531	$\bar{1}$·9946	$\bar{1}$·9414	0·0000	$\bar{1}$·7515	$\bar{1}$·7665	$\bar{1}$·7591	$\bar{1}$·7596	
log (RM cos ζ′)	$\bar{1}$·5584	$\bar{1}$·1786	$\bar{1}$·8227	$\bar{1}$·5241	$\bar{1}$·5138	$\bar{1}$·5361	0·1437	0·3916	0·1250	0·3389	0·6430	0·8431	$\bar{1}$·7638	
RM cos ζ′ +	0·36				0·33	0·34	1·39	2·46	1·33				0·58	6·79
RM cos ζ′ −		0·15	0·66	0·33						2·18	4·40	6·97		14·69

(360° − ζ) = Z	M_2K_1 237·84	204·39 (η SA)	2η(SSA) 39·57	168·54 M_4	M_6 188·16	184·50 S_1	M_2 197·28	36·87 S_2	K_1 137·11	262·51 O	2N 205·15		
Δ Z	+6·9	+3·6	+7·3	+6·4	+9·6	0·0	+3·2	0·0	+3·6	−0·5	+81·2	Sum upper Series	Sum upper & lower
Z + Δ Z = ζ′	244·74	207·99	46·87	174·94	197·76	184·50	200·48	36·87	140·71	262·01	286·35		
R	0·040	* * *	0·150	* * *	0·052	* * *	2·659	* * *	1·160	* * *	0·095	1·161	5·317
	* * *	0·144	* * *	0·028	* * *	0·088	* * *	0·959	* * *	0·538	* * *	0·761	2·518
log R log M	$\bar{2}$·6021 1·2799	$\bar{1}$·1584	$\bar{1}$·1761	$\bar{2}$·4472	$\bar{2}$·7160	$\bar{2}$·9445	0·4247	$\bar{1}$·9818	0·0645	$\bar{1}$·7308	$\bar{2}$·9777	Diff. = a_1 C × a_1 = R_1	+2·799 " +4·199
log RM	$\bar{1}$·8820	0·4383	0·4560	$\bar{1}$·7271	$\bar{1}$·9959	0·2244	1·7046	1·2617	1·3444	1·0107	0·256		
R M	0·76	* * *	2·86	* * *	0·99	* * *	50·65	* * *	22·10	* * *	1·81	22·11	101·28
	* * *	2·74	* * *	0·53	* * *	1·68	* * *	18·27	* * *	10·25	* * *	14·50	47·97
log cos Z	$\bar{1}$·7261	$\bar{1}$·9595	$\bar{1}$·8870	$\bar{1}$·9912	$\bar{1}$·9955	$\bar{1}$·9987	$\bar{1}$·9799	$\bar{1}$·9031	$\bar{1}$·8649	$\bar{1}$·1151	$\bar{1}$·9568	Diff. = a_2	+53·31
log (RM cos Z)	$\bar{1}$·6081	0·3978	0·3430	$\bar{1}$·7183	$\bar{1}$·9914	0·2231	1·6845	1·1648	1·2093	0·1258	0·2144	$\frac{a_2}{12·7}$ = Σ	" +4·198
RM cos Z +			2·20					14·62				15·92	32·74
RM cos Z −	0·41	2·50		0·52	0·98	1·67	48·37		16·19	1·34	1·64	13·26	86·88
log cos ζ′	$\bar{1}$·6302	$\bar{1}$·9460	$\bar{1}$·8349	$\bar{1}$·9983	$\bar{1}$·9788	$\bar{1}$·9987	$\bar{1}$·9716	$\bar{1}$·9031	$\bar{1}$·8888	$\bar{1}$·1433	$\bar{1}$·4495	Diff. = a_3	−54·14
log (RM cos ζ′)	$\bar{1}$·5122	0·3843	0·2909	$\bar{1}$·7254	$\bar{1}$·9747	0·2231	1·6762	1·1648	1·2332	0·1540	$\bar{1}$·7071	$\frac{a_3}{12·7}$ = R_2	" −4·26
RM cos ζ′ +			1·95					14·62			0·51	6·79	23·87
RM cos ζ′ −	0·33	2·42		0·53	0·94	1·67	47·44		17·11	1·43		14·69	86·56
												Diff. = a_4	−62·69
												$\frac{a_4}{12·7}$ = R_3	" −4·94

Computed by *Checked by*

106. The scale for the port is entered from table XXI at the end of Chapter I. The scale factor M is also entered from the same table.

$$C = \frac{\text{M, the scale factor}}{12 \cdot 7}$$

As the machine is set in millimetres, and the amplitude of the motion given by the machine is double of that required, owing to the rope passing round the pulleys, 12·7 has to be taken as the dividing factor, being one half of 25.4 the number of millimetres per inch. A_0 is the height of mean sea level above the datum of soundings and is entered from table on page 83. S_2 is the correction applied to the S_2 dial pointer on the machine, to change its phase angle from local mean to standard time. These changes in phase have been tabulated in table XXI.

S_2 is the dial pointer, by means of which the tide-predicting machine is set in phase for the commencement of prediction. It has been selected for setting as well as stopping the machine for time, as it completes two revolutions in one day's movement, and it is thus sufficiently rapid to obtain a good time check, while admitting of the machine being under proper control, so as to stop it at any desired moment.

The entries for $(360 - \zeta)$ or Z are obtained from the data in form 17 Tid. for each tide.

ΔZ are the changes in Z after 369 days, which are based on the speed of each tide. These movements are entered in the form for 369 days motion, and they are also tabulated in table XXII, which shows their motion up to the first of every month, in case an intermediate setting is required for any month in the year, if the machine breaks down from any cause. The summation of $(Z + \Delta Z)$ gives ζ'. The values of R are entered from form 17 Tid, for each tide.

The remaining entries on the form require no explanation except the quantities R_1, R_2, R_3.

The sums of the upper and lower series for R having been obtained; their difference a_1, multiplied by C, gives R_1, the first check for the height of the pen, above or below mean sea-level, to be verified on the machine after the amplitudes have been set.

The sums of the upper and lower series for RM having been obtained, their difference a_2, divided by 12.7, again gives the value R_1 as a check on the previous working. The sums of the upper and lower series for RM cos Z having been obtained, their difference a_3, divided by 12·7, gives R_3, the check for the height of the pen, above or below mean sea level; to be verified on the machine after the phase angles are set by the dial pointers.

The sums of the upper and lower series for RM cos ζ having been obtained; their difference a_4, divided by 12·7, gives R_3, the check for height of pen, to be verified on the machine after the curve has been run for 369 days.

Computation of form 2 Tid. Pred.

107. The form 2 Tid. Pred. is intended for use when the settings on the tide-predicting machine are required, for purposes of prediction of times of high and low water only, by the Chronograph method.

The form will always be computed after the form 1 Tid. Pred., for height only, has been completed, and advantage has been taken in the arrangement of the computation to utilise the results already obtained on form 1 Tid. Pred., whenever the entries required are the same.

2 Tid. Pred. (Time Sheet) for year 1923.

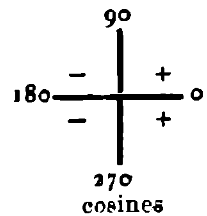
Port Karachi

Correction to Standard Time + $62^m \cdot 13$

After setting, run machine back until $S_2 = \zeta_\tau - \frac{62° \cdot 13}{2} = 126° \cdot 87 - 31° \cdot 07 = 95° \cdot 80$

n = Speed component in degrees per mean solar hour.

$T = \frac{n}{40}$



	$2M_2K_1$	2SM	MS	J	Q	T	L	ν	μ	K_2	P	N	M_2N	
$(360° - \Omega) + 90° - \zeta_\tau$	69·38	206·63	262·38	49·21	150·07	170·67	296·89	278·52	84·20	318·35	219·44	102·75	279·51	Sum upper Series
$\zeta' + 90° - \zeta'_\tau$	92·08	203·38	265·58	193·91	8·57	166·97	80·99	60·92	90·60	325·65	215·74	324·95	144·91	
RMT	* * *	0·30	* * *	0·54	* * *	1·14	* * * * *	2·01	* * *	2·91	* * *	8·63	* * *	15·53
	0·39	* * *	0·98	* * *	0·73	* * *	1·04	* * *	0·93	* * *	2·81	* * *	1·45	8·33
log T	0·0307	$\bar{1}$·8895	0·1687	$\bar{1}$·5907	$\bar{1}$·5250	$\bar{1}$·8745	$\bar{1}$·8682	$\bar{1}$·8530	$\bar{1}$·8446	$\bar{1}$·8763	$\bar{1}$·5728	$\bar{1}$·8519	0·1570	
log RM	$\bar{1}$·5587	$\bar{1}$·5809	$\bar{1}$·8240	0·1132	0·3406	0·1830	0·1491	0·4502	0·1250	0·5874	0·8765	1·0840	0·0042	
log RMT	$\bar{1}$·5894	$\bar{1}$·4704	$\bar{1}$·9927	$\bar{1}$·7339	$\bar{1}$·8656	0·0675	0·0173	0·3032	$\bar{1}$·9696	0·4637	0·4493	0·9359	0·1612	
log cos ζ_τ	$\bar{2}$·0334	$\bar{1}$·9513	$\bar{1}$·1225	$\bar{1}$·8152	$\bar{1}$·9378	$\bar{1}$·9942	$\bar{1}$·6555	$\bar{2}$·7882	$\bar{1}$·0046	$\bar{1}$·8734	$\bar{1}$·8878	$\bar{1}$·3438	$\bar{1}$·2181	
log (RMT cos ζ_τ)	$\bar{3}$·6228	$\bar{1}$·4217	$\bar{1}$·1152	$\bar{1}$·5491	$\bar{1}$·8034	0·0517	$\bar{1}$·6728	$\bar{1}$·0914	$\bar{2}$·9742	0·3371	0·3371	0·2797	$\bar{1}$·3793	
RMT cos ζ_τ +	0·00			0·35			0·47	0·12	0·09	2·17			0·24	3·44
RMT cos ζ_τ −		0·26	0·13		0·64	1·13					2·17	1·90		6·23
log cos ζ'_τ	$\bar{2}$·5568	$\bar{1}$·9630	$\bar{2}$·8869	$\bar{1}$·9871	$\bar{1}$·9951	$\bar{1}$·9886	$\bar{1}$·1950	$\bar{1}$·6867	$\bar{2}$·0200	$\bar{1}$·9168	$\bar{1}$·9094	$\bar{1}$·9131	$\bar{1}$·9128	
log (RMT cos ζ'_τ)	$\bar{2}$·1492	$\bar{1}$·4334	$\bar{2}$·8796	$\bar{1}$·7210	$\bar{1}$·8607	0·0461	$\bar{1}$·2123	$\bar{1}$·9899	$\bar{3}$·9896	0·3805	0·3587	0·8490	0·0740	
RMT cos ζ'_τ +					0·73		0·16	0·98		2·40		7·06		11·33
RMT cos ζ'_τ −	0·01	0·27	0·08	0·53		1·11			0·01		2·28		1·19	5·48

	M_2K_1	η (SA)	2η (SSA)	M_4	M_6	S_1	M_2	S_2	K_1	O	2N		
$(360° - \zeta) + 90° = \zeta_\tau$	327·84	294·39	129·57	258·54	278·16	274·50	287·28	126·87	227·11	352·51	295·15	Sum upper Series	Sum upper & lower
$\zeta' + 90° = \zeta_\tau'$	334·74	297·99	136·87	264·94	287·76	274·50	290·48	126·87	230.71	352·01	16·35		
RMT	0·84	* * *	0·01	* * *	2·15	* * *	36·70	* * *	8·31	* * *	1·26	15·53	64·80
	* * *	0·00	* * *	0·77	* * *	0·63	* * * * *	13·71	* * *	3·57	* * *	8·33	27·01
log T log RM	0·0416 $\bar{1}$·8820	$\bar{3}$·0107 0·4383	$\bar{3}$·3118 0·4560	0·1611 $\bar{1}$·7271	0·3372 $\bar{1}$·9959	$\bar{1}$·5740 0·2244	$\bar{1}$·8601 1·7046	$\bar{1}$·8751 1·2617	$\bar{1}$·5752 1·3444	$\bar{1}$·5423 1·0107	$\bar{1}$·8435 0·2576	Diff. =	+37·79
log RMT log cos ζ_τ	$\bar{1}$·9236 $\bar{1}$·9277	$\bar{3}$·4490 $\bar{1}$·6159	$\bar{3}$·7678 $\bar{1}$·8041	$\bar{1}$·8882 $\bar{1}$·2982	0·8331 $\bar{1}$·1519	$\bar{1}$·7984 $\bar{2}$·8946	1·5647 $\bar{1}$·4728	1·1368 $\bar{1}$·7781	0·9196 $\bar{1}$·8330	0·5530 $\bar{1}$·9963	0·1011 $\bar{1}$·6284	$\frac{\text{Diff.}}{12·7} = B_1$	″ +2·08
log (RMT cos ζ_τ)	$\bar{1}$·8513	$\bar{3}$·0649	$\bar{3}$·5719	$\bar{1}$·1864	$\bar{1}$·4850	$\bar{2}$·6930	1·0375	0·9149	0·7326	0·5493	$\bar{1}$·7295		
RMT cos ζ_τ +	0·71	0·00			0·31	0·05	10·90			3·54	0·54	3·44	19·49
−			0·00	0·15				8·22	5·66			6·23	20·26
log cos ζ_τ'	$\bar{1}$·9563	$\bar{1}$·6715	$\bar{1}$·8632	$\bar{2}$·9458	$\bar{1}$·4843	$\bar{2}$·8946	$\bar{1}$·5439	$\bar{1}$·7781	$\bar{1}$·8016	$\bar{1}$·9958	$\bar{1}$·9821	Diff. =	−0·77
log (RMT cos ζ_τ')	$\bar{1}$·8799	$\bar{3}$·1205	$\bar{3}$·6310	$\bar{2}$·8340	$\bar{1}$·8174	$\bar{2}$·6930	1·1086	0·9149	0·7212	0·5488	0·0832	$\frac{\text{Diff.}}{12·7} = B_2$	″ −0·06
RMT cos ζ_τ' +	0·76	0·00			0·66	0·05	12·84			3·54	1·21	11·33	30·39
−			0·00	0·07				8·22	5·26			5·48	19·03
												Diff. =	+11·36
												$\frac{\text{Diff.}}{12·7} = B_2$	″ +0·89

Computed by *Checked by*

108. The correction to standard time is entered in minutes, and the correction to the phase of S_2 is obtained by dividing this by 2, or from table XXI. T is an arbitrary factor which is obtained by dividing n the speed component in degrees per m.s. hour by 40, and was selected with a view to keep the motion of the pen-box on the machine within the limits of the pen-guide.

The entries for $(360° - \zeta) + 90°$ or ζ_T are obtained from form 1 Tid. Pred. by adding 90° to the values of Z, or from the data given in 17 Tid.

The reason for increasing the phase angles by 90° is as follows.

The height of the tide at any moment is the sum of a number of simple tides such as RM cos $(nt - \zeta)$ so that

$$h = \Sigma \text{ RM} \cos (nt - \zeta).$$

High or low water occurs when this is a maximum or minimum, *i.e.* when $\frac{dn}{dt} = o$ or $\Sigma \text{ RM}n \cos (nt - \zeta + 90°) = o$.

Hence if the machine is set with amplitude RMn and phase angle $\zeta + 90°$ it will draw a curve which will cut the mean level line (corresponding to all components set to zero) at the instants of high and low water. (Actually these amplitudes are multiplied by $\frac{1}{40}$ to bring them within convenient ranges for the machine).

The form 2 Tid. Pred. closely resembles 1 Tid. Pred.

$$(\zeta' + 90°) \text{ or } \zeta'_T$$

is the phase angle as altered after having completed 369 days movement and is obtained from the form 1 Tid. Pred. by adding 90° to the value of ζ' in form 1 Tid Pred. It is also directly obtainable from form 17 Tid. and table XXII being the value of $[360° - \zeta + 90° +$ change of movement in 369 days].

RMT is obtained from the RM of form 1 Tid. Pred. by multiplying RM by the factor T.

The remaining entries on the form are quite clear from the form itself, several of the entries being the same as in form 1 Tid. Pred.

The checks to the pen height R_1, R_2, R_3 are obtained in a similar manner to those in form 1 Tid. Pred., except that there is no double check on the value of R_1.

PREDICTION FOR RIVERAIN PORTS.

109. The tide-predicting machine, combining some 24 tidal components, gives accurate results for open coast stations. But for tidal stations situated in rivers at some distance from their mouths or in estuaries having a considerable shallow foreshore, the number of overtides and shallow water components on the machine do not suffice to represent, with the accuracy desired, the actual tidal curve. In addition to the above, the tides are greatly affected by freshets in the rivers, which alter the normal times of high and low water very considerably during several months of the year.

In order therefore to predict the times and heights of tides at Riverain Ports, recourse is had to the method of referring the observed times and heights of tides to the apparent time of moon's transits preceding them by about 1¼ to 1½ days, for reasons explained in para 112.

The computation is carried out on form 3 Tid. Pred, the computed times and heights of the semi-diurnal tide being corrected for time, date, declination and parallax, as shown in the form, as well as for corrections taken from the diurnal chart run on the machine for the 8 components $2M_2K_1$, J, Q, P, M_2K_1, S_1, K_1, and O only*.

110. An explanation of the principles on which corrections to the times of high and low-water, found for semi-diurnal tides, are applied from the chart of the diurnal tide is given in the following note by Dr. J. de Graaff Hunter, M. A., Sc. D., F. Inst. P.

Although the semi-diurnal tide is compounded of a number of simple tides of periods slightly differing, yet the semi-diurnal tide of one day may be represented tolerably well by

$$A_2 \cos (30^\circ f_2 h)$$

where A_2 varies slowly from day to day, but may be considered constant for one day.

h is the number of solar hours measured from last maximum of semi-diurnal tide.

f_2 is a factor, nearly unity, which is the ratio of the average speed of semi-diurnal tides to 30° p. h.

In a similar way

$$A_1 \cos (15^\circ f_1 h + a_1)$$

may be supposed to represent the diurnal tide for the day under consideration.

* Mr. Brookes of the National Physical Laboratory, Teddington, England used only 6 components *viz* :—S_1, K_1, O, J, P, Q, as these gave a less complex curve.

The problem is to find how far the maximum of the combined tide $A_2 \cos(30^\circ f_2\, h) + A_1 \cos(15^\circ f_1 h + a_1)$ is delayed after that of the tide $A_2 \cos(30^\circ f_2\, h)$.

Now the fundamental idea underlying the principle of the tidal prediction under consideration, is that A_1 is small compared with A_2.

Now the maximum required occurs when :—

$$A_2 \sin(30^\circ f_2\, h)\, 30 f_2 + A_1 \sin(15^\circ f_1\, h + a_1)\, 15 f_1 = 0$$

or, dividing by 15, when :—

$$2f_2\, A_2 \sin(30^\circ f_2 h) + f_1 A_1 \sin(15^\circ f_1 h + a_1) = 0.$$

If we consider the above equation the solution sought is a small value of h since A_1 is small compared with A_2 and the maximum occurs when $h=0$ if A_1 vanishes.

Now as h is small the above equation may be written approximately

$$2f_2 A_2\,(2Rf_2 h) + f_1 A_1 \sin a_1 + f_1 A_1 \cos a_1\,(Rf_1 h) = 0$$

where R is the radian measure of 15°.

The equation reduces to

$$R\,(4f_2^2 A_2 + f_1^2 A_1 \cos a_1)\, h + f_1 A_1 \sin a_1 = 0$$

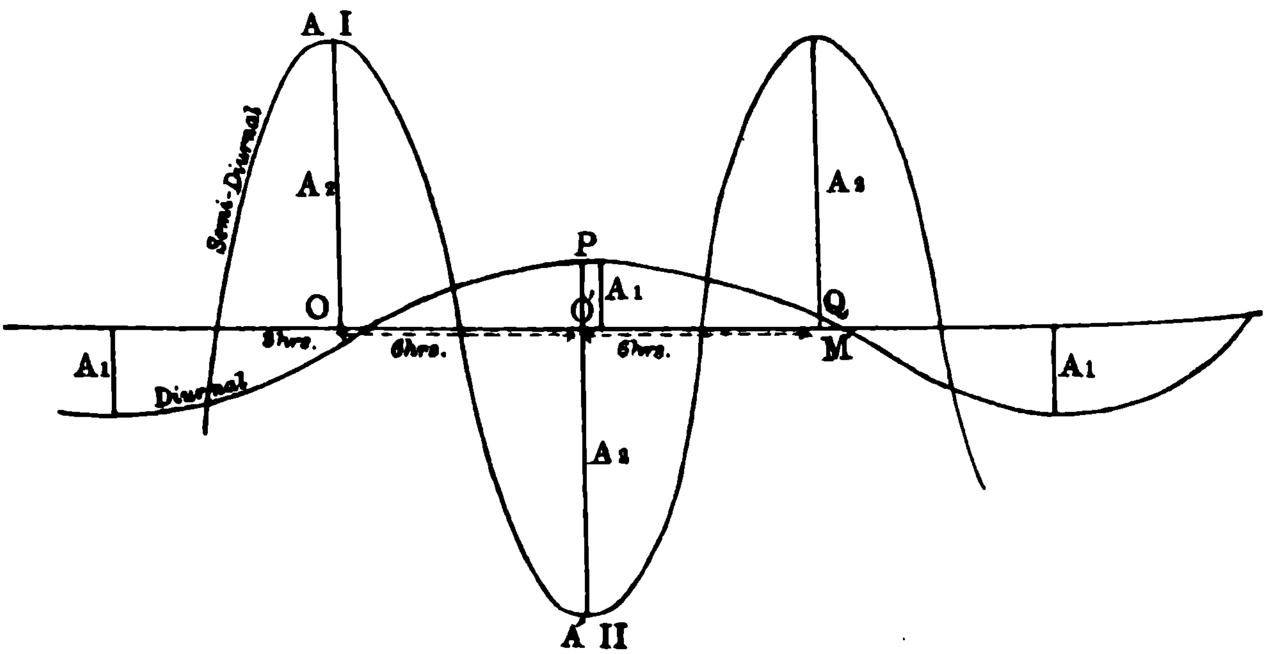
Now R, the radian measure of $15^\circ = \cdot 262$ approximately, and so

$$h = -\frac{f_1 A_1 \sin a_1}{\cdot 262\,(4f_2^2 A_2 + f_1^2 A_1 \cos a_1)}$$

Now f_1, f_2 are both slightly less than unity in proportion something like $\frac{1}{29}$, and further $f_1^2 A_1 \cos a_1$ is small compared with $4 f_2^2 A_2$, so that we can write :—

$h = -\dfrac{A_1 \sin a_1}{A_2}\left(\text{putting } \dfrac{f_1}{\cdot 262 \times 4f_2^2} = 1\right)$, expressed in hours of time.

Now $A_1 \sin a_1$ is the value of the height of the diurnal tide for $15^\circ f_1 h = 90^\circ$, or $h=6$ hours, measured from the time maximum of the semi-diurnal tide, and A_2 is $\frac{1}{2}$ the range of the semi-diurnal tide.



Correction to the tidal pamphlet, 'The Tides.' (1926).

Chapter I, Page 95, lines 2, 3 & 5—

Read PO′ *for* PN.

Geod. Br. P.O.—20-7-27—300.

111. The above method of deducing the time corrections has been applied in the method of computation, explained in paras 129 to 132, instead of the old method of dividing the differences between alternate values of the diurnal correction in *heights*, expressed in hundredths of a foot, by 5, when the resulting quotient represented minutes of *time*, as this older method had no theoretical justification. (Vide G. T. S. Vol. XVI p. 341 Part 1.)

112. It is necessary now to explain how the computations are carried out in detail.

The method of reduction now employed is, as already stated, to refer the observed times and heights of low-waters, extending over a considerable period, to the apparent times of transits of the moon, preceding the time of high or low-water. The best transit to which to refer the observations can be determined from the values of κ, deduced from the harmonic analysis of the two chief tides of the port in question or of those of an adjacent port. The difference between the values of κ of the mean lunar semi-diurnal tide and that of the mean solar semi-diurnal tide, divided by twice the moon's mean daily synodic motion, will give the mean value of the retardation of the times of high or low-water after the moon's transit.

Thus if we take the value of $\dfrac{\kappa \text{ for } S_2 - \kappa \text{ for } M_2}{24 \cdot 38}$

for the port required we obtain values varying from 21^h to 37^h for L waters and 27^h to 40^h for H waters and on the average about $1\frac{1}{4}$ days for the approximate interval after moon's transit to which the predictions have to be referred.

NOTE:—The method of prediction for Riverain Ports is thus based on the assumption that the diurnal inequality is small. In the case of Basrah it is at times quite large. This may account partially for the errors which occur in the predicted times at this port. The period of tidal observations at Basrah however has been too short for satisfactory results to be obtained, on which a definite opinion can be pronounced.

In the figure

$$-A_1 \sin a_1 = PN,$$

(1) H water I is delayed by $\frac{PN}{AO}$ hours $= \frac{PN}{\frac{1}{2} \text{ range of semi diurnal tide}}$

(2) L water II is advanced by $\frac{QM}{A'O'}$ hours $= \frac{QM}{\frac{1}{2} \text{ range of semi diurnal tide}}$

In both these cases PN and QM are positive, being above the mean sea level line. If they fall below, it indicates that the corrections are of opposite sign to the above.

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113. The following table exhibits the approximate interval after the moon's transit to which the predictions for the Indian Riverain Ports, by this method, are referred.

	Interval after moon transit to time of	
	L. Water	H. Water
Basrah ...	21^h	27^h
Dublat, (Saugor Island), Hooghly R	29^h	35^h
Diamond Harbour ...	32^h	36^h
Kidderpore, (Calcutta) ...	35^h	39^h
Chittagong ...	38^h	38^h
Elephant Point, (Rangoon R) ...	35^h	40^h
Amherst, (Moulmein R) ...	34^h	39^h
Moulmein ...	37^h	40^h

114. Having determined the best transit to which the times and heights are to be referred, as above, it is possible, by comparisons of previous moon's transits and times and heights corresponding, to obtain monthly means of the approximate intervals between moon's transits and the times of H & L water, as well as for the values of heights of H and L water corresponding to moon's transits, by grouping the results into 12 groups, *i.e.*:—those occurring between $0^h\ 0^m$ and $1^h\ 0^m$, or $12^h\ 0^m$ and $13^h\ 0^m$, into one group; those between $1^h\ 0^m$ and $2^h\ 0^m$, or $13^h\ 0^m$ and $14^h\ 0^m$, into a second group; and similarly for each of the twelve hours, and then taking out the required means. These means give the average times and heights of H and L water for values of the moon's transit about $0^h\ 30^m$, $1^h\ 30^m$, $2^h\ 30^m$, etc. If the means of the moon's transits do not equal 30m exactly, they are made to do so, the times and heights dependent on them being corrected accordingly. Having now the values for each hour of moon's transit, viz. at $0^h\ 30^m$, $1^h\ 30^m$, etc.; the intermediate values of times and heights are obtained by interpolation for each 10^m of the moon's transit; *i.e.*:—five values being inserted between each of the 12 original quantities.

115. As the effect of freshets on the tides of Riverain Ports in India varies considerably from month to month, but the same effects occur with fair regularity from year to year in the same months, the observations for a considerable period of years are taken separately month by month; *i.e.*:—all January results together, etc.; and by grouping the hourly values for each separate month for $0^h\ 0^m$ and $1^h\ 0^m$, or 12^h and 13^h, etc.; as above, monthly lists of mean values are obtained.

116. In order to obtain good values, it was necessary to treat the observations extending over a considerable period of years in this manner, as the number of observations in any one month, dependent on a certain hour of the moon's transit, was insufficient to give reliable mean values. It may be here remarked that it is not necessary to find the time of moon's transit for the particular place in question, and, in practice, it has been the rule to refer the times and heights to the transits at Greenwich, which are given in the Nautical Almanac. In the predictions, the Greenwich transits are again used, whereby a saving of time is effected both in the reductions, and the actual predictions.

Plotting of monthly mean values on charts.

117. The monthly mean values of heights and times, corresponding to the apparent times of moon's transit, so obtained, were then brought in terms of mean times of transit with a view to plotting them on charts. The values for *height* were plotted without alteration. In the case of the *times* however, the curves were drawn with the monthly mean values somewhat modified, so as to give the curves an easier gradient and enable the values to be read with more precision. For this purpose the monthly mean values relating to the mean times of transit were subtracted from the times of moon's transit in the case of low-waters and *vice versa* in the case of high-waters. With these residual values, after applying the correction to standard time, when necessary, the plotting of the curves was carried out. These residuals were taken to represent the value for the 15th day of each particular month, so that no smoothing was required between the values obtained for the end of one month and the begining of the next. Two sets of charts were prepared, one for high-water, comprising 8 separate charts, for heights and times of high-waters, and another similar 8 charts, for heights and times of low-waters. Each chart exhibits 4 curves, covering a period of three months, and one set covers a whole year of heights or times of high or low-waters. From these monthly charts, entries are made in form 3 Tid. Pred. according to dates as explained in para 126 *et seq.*

Corrections for lunar parallax and declination.

118. The next step in the work is to ascertain the corrections due to the lunar parallax and the lunar declination.

The tables, given on pages 337 to 339, and, rearranged and enlarged, on pages 342 to 352 of Chapter 8, Part I. G. T. S. Volume 16, are strictly speaking for the London Docks, but can be used, for the semi-diurnal tides of any port, for the corrections for *times* of high-water without alteration.

For *heights* however, corrections are applied in proportion to the ranges of the semi-diurnal tides at London and those of the port in question. The corrections for the times of low-water at the London Docks have not been determined, but have been assumed the same as for the high-water and have been so used in the prediction of the times of low-waters for the Indian Riverain ports. The corrections for the heights of low-waters will be of opposite sign to those for high-waters. The times of low-water in the reductions are referred to the same moon's transit as the *succeeding* high-water, as the low-waters precede the high-waters by a shorter interval than that between the low-water and the *previous* high-water. At Riverain ports, which are at a considerable distance from a river's mouth, the high-water succeeds the low-water by a few hours only, the time being coincident at the place where the tidal flow ceases.

Charts for parallax and declination corrections.

119. Charts have also been prepared for corrections to time and height for moon's horizontal parallax and declination, corresponding to tables 1,2 and 5,6 given in G. T. Survey Vol. 16 pages 342-345 and 350-351. From these latter charts, which are in apparent time, the corrections to mean time for each day in the year can be taken out and entered as described in the footnote to form A_2 in para 122. The corrections from these charts would have been more easily obtainable, had these charts been made to correspond with mean time. It may be found simpler in future to construct fresh charts corresponding to mean time, or to work in apparent time throughout, either from charts, or by means of tables, (as explained in para 124).

Multiplying factors for height corrections.

120. The corrections to *time* taken from these charts are the actual corrections required, but in the case of the corrections to *height*, the corrections have to be multiplied by an appropriate factor before entry.

A note to this effect is given in the heading of each chart. As the figures for time corrections on the charts are in black, and those for height are in red, there is no likelihood of confusion.

The appropriate factors by which the height corrections have to be multiplied are :—

Name of port.	Factor for H. Water.	Factor for L. Water.
Dublat, (Saugor Island), Hooghly R ...	1·2	−1·2
Diamond Harbour	1·4	−1·0
Kidderpore, (Calcutta)	1·2	−0·3
Chittagong	1·0	−0·5
Elephant Point, (Rangoon R) ...	1·6	−1·4
Rangoon	1·4	−0·9
Amherst, (Moulmein R)	1·8	−1·8
Moulmein	1·6	−0·2*
Basrah	0·3	−0·2

121. It is now necessary to draw up certain lists to facilitate the entries in the computation form 3 Tid Pred. The form A_1 is first prepared, as per the sample given below, with data from the Nautical Almanac, month by month throughout the year for prediction, and starting at least 2 days before the year commences, as the predictions are referred to transits 1¼ or more days previous.

Form A_1.

Form A_1 is used for reference in the subsequent computations for all the Riverain ports.

To simplify the work, the declinations are calculated for the nearest hour of moon's transit, and the values entered to the nearest tenth of a degree. Also the values of horizontal parallax for noon or midnight from the Nautical Almanac are entered, if the time of transit is within 3 hours of noon or midnight, as the case may be. The means of consecutive values for noon or midnight are entered, if the time of transit lies between 3 and 9 hours of noon or midnight, the values being kept to the nearest second. The values of declination and parallax are computed for alternate times of moon's transit only.

* The factor for Moulmein −0·2 is correct, as now given. The mistake in the value −1·2 given in G.T. Volume XVI p. 340 part I, was pointed out by Mr. Brookes of the National Physical laboratory.

Form A_1, 1925.

1 Year.	2 Month.	3 Date.	4 Moon's transit in mean time from N.A.	5 Moon's declination corresponding to column 4 from N.A.	6 Moon's horizontal parallax corresponding to column 4 from N.A.
			h. m.	°	m. s.
1924	Dec. ...	30	3 50	−11·8	58−57
"	" ...	...	*16 17	...	...
"	" ..	31	4 42	− 7·5	57−59
"	" ...	...	17 06	...	...
"	" ...	..	†AT 5 26	...	.
1925	Jany. ..	1	5 30	− 3·0	57−03
"	" ...	...	17 53	..	...
"	" ...	2	6 16	+ 1·6	56−13
"	" ...	..	18 38	..	...
"	" ...	3	7 00	+ 6·0	55−29
"	" ..	...	19 23	...	...
"	" ...	4	7 45	+10·0	54−55
"	" ...	...	20 07	...	...
"	" ...	5	8 29	+13·5	54−29
"	" ...	...	20 52	...	...
"	" ...	6	9 15	+16·4	54−08
"	" ...	...	21 38	..	...
"	" ...	7	10 01	+18·5	54−00
"	" ...	...	22 25	...	...
"	" ...	8	10 49	+19·8	53−58
"	" ...	...	23 13	...	...
"	" ...	9	11 37	+20·1	54−01
"	" ...	...	... ...	...	...
"	" ...	10	0 01	...	...
"	" ...	...	†AT 12 17	...	...
"	" ...	..	12 25	+19·5	54−10
"	" ...	11	0 49	...	...
"	" ...	...	13 13	+18·0	54−23
			etc., to the end of the year.		

122. From form A_1 it is then necessary to enter the corresponding corrections for declination and parallax from the charts prepared for this purpose (vide para 119), in the form A_2, of which a sample is given below. The entries are made as shown in the columns for time and height corrections.

Form A_2.

* The 24 hour system has been adopted expressing these times, instead of the 12 hour system, a. m. or p. m.

† Apparent time of moon's transit is entered in this column every 10th day commencing with 1st January.

Form A_2, 1925.

Year	Month	Date	Corrections to time				Corrections to height		
			(1) for moon's declination	(2) for moon's horztl parallax	(3) E. Errors in corrections (1) & (2), due to arguments being in mean instead of apparent time	(4) Sum of (1), (2)&E for time	(5) for moon's declination	(6) for moon's horizontal parallax	(7) Sum of (5)& (6) for height
1924	December	30	+1 0	+4 4	+ ·3	+5·7	+·10	+·47	+·57
,,	,,	,,							
,,	,,	31	+2·2	+2 3	+·2	+4·7	+·22	+·24	+·46
,,	,,	,,							
	Corresponding to A. T.		+2·5	+0·1		+2·6*			
1925	January	1	+2 4†	+0·1†	+0·1	+2·6	+·28	+·01	+·29
,,	,,	,,							
,,	,,	2	+1·0	−0·7	0·1	+0·4	+·32	−·20	+·12
,,	,,	,,							
,,	,,	3	−1·4	+2 6	0·1	+1·3	+·26	−·35	−·09
,,	,,	,,							
,,	,,	4	−1 5	+5·7	0·2	+4·4	+·16	−·45	−·29
,,	,,	,,							
,,	,,	5	−0 6	+7·1	0·2	+6·7	+·05	−·52	−·47
,,	,,	,,							
,,	,,	6	+0·7	+7·0	0·2	+7·9	−·06	−·58	−·64
,,	,,	,,							
,,	,,	7	+1 4	+5·4	0·3	+7·1	−·15	−·63	−·78
,,	,,	,,							
,,	,,	8	+1·4	+3·6	0·3	+5·3	−·23	−·64	−·87
,,	,,	,,							
,,	,,	9	+0·8	+1·7	0·4	+2·9	−·24	−·63	−·87
,,	,,	,,							
,,	,,	10	+0 2†	+0·2†	+0·4	+0·8	−·21	−·62	−·83
,,	,,	,,							
				etc., to	the end of the year.				
	Corresponding to A. T.		+0·3	+0·5		+0·8*			
1925		11	−0·4	−1·3	† +0·4	−1·3	−·13	−·57	−·70

*Column (3). E is a correction which is necessary, as the argument adopted in reading the charts is mean instead of apparent time. It is obtained by entering the values corresponding to the apparent time, given in form A_1, for every 10th day. By comparing these with the mean time corrections, it is possible to grade the differences, and enter column E:—for instance, taking the sum of +2·4 +0·1 marked † above, we obtain +2·5, which compared with +2·6 marked *, gives the entry in Column E as +0·1 for January 1st. Similarly, at the end of 10 days, by comparing sum of quantities +0·2, +0·2 marked †, we obtain +0·4, which compared with +0·8, marked *, gives the entry in Column E for 10th January as +0·4. The differences are then graded back smoothly between +0·4 and +0·1, as shown in Column E.

123. For convenience in making the entries in form 3 Tid Pred. a resumé of forms A_1 & A_2 is made out on form A_3.

Form A_3.

Columns 1 to 4 are copied direct from form A_1.

Columns 5 & 6 are sums of the corrections for time and height respectively from columns 4 & 7 of form A_2. The time corrections are entered to the nearest minute only. This table can be utilised for all Riverain ports.

FORM A_3, 1925.

(1)	(2)	(3)	(4)	(5)	(6)
				Corrections for	
Year	Month	Date (civil)	Mean Time of moon's transit hrs. – mts.	Time mts.	Height feet
1924	December	30	3 – 50	+6	+ ·57
,,	,,	,,	16 – 17	...	...
,,	,,	31	4 – 42	+5	+ ·46
,,	,,	,,	17 – 06	...	...
1925	January	1	5 – 30	+3	+ ·29
,,	,,	1	17 – 53	...	...
,,	,,	2	6 – 16	0	+ ·12
,,	,,	2	18 – 38	...	...
,,	,,	3	7 – 00	+1	– ·09
,,	,,	3	19 – 23	...	...
,,	,,	4	7 – 45	+4	–29
,,	,,	4	20 – 07	...	...
			etc., to the end of the year.		

124. An alternative method of entering the parallax and declination corrections similar to that adopted by Mr. Brookes of the National Physical Laboratory, Teddington, England is here given. The working is carried out in apparent time direct, the entries being made from Tables 1 to 7, given in G. T. Survey Volume XVI, p. 342 *et seq*. These tables are in apparent time, and the working is less cumbrous than that from the charts now in use, which are based on mean, instead of apparent time. If these charts are reconstructed to correspond with mean time, the method of entry of the form below would be suitable, and its use would obviate the preparation of forms A_1, A_2 & A_3 as at present. The form would be entered as shown :—

Date 1913	Transits of moon in mean time in order from N.A.	Eqn of time.	Appt. time of moon's transit	Decln. of moon at mean time of transit	Horl. parallax at apparent time of transit	Corrections to time Minutes			Sum of tables 1·2·3. Corrn. to time of H&L water.	Corrections to height Feet.			Sum of tables 5·6·7	Sum of tables 5·6·7 × factor given on p. 340 Vol. XVI. Part I. Corrn. to height of H.&L.W.
						Corrn. Table 1	Corrn. Table 2	Corrn. Table 3		Corrn. Table 5	Corrn. Table 6	Corrn. Table 7		
	h. m.	m.	h. m.	° ′	′ ″									
Dec. 30 1912 Dec. 31 1912	Entries for these days of previous year have to be made so as to refer the tides to transits about 1¼ days previously.											Usually neglected		
Jany. 1 1913 ...	7 25·5 19 49·6	−3·7 −3·9	7 22 19 46	15·5 ...	57·17 ...	−0 ...	−1 ...	−7 ...	−8 −6*	−02 ...	+·06 ...		+·04 −·15*	
Jany. 2 ...	8 14·2 20 39·4	−4·1 −4·4	8 10 20 35	20·6 ...	56·40 ...	+3 ...	+1 ...	−7 ...	−3 0*	−·26 ...	−·08 ...		−·34 −·51*	
Jany. 3 ...	9 5·2 21 31·6	−4·6 −4·9	9 1 21 27	24·6 ...	56·70 ...	+6 ...	+2 ...	−6 ...	+2 +3*	−·48 ...	−·20 ...		−·68 −·79*	etc.

The values marked* are not actually worked out but interpolated, which saves the labour of entering the corrections for every day, alternate entries being sufficient.

125. Method of entering form 3 Tid Pred.

Form 3 Tid Pred. The prediction of high and low-waters are both carried out on this form, high being treated separately and low separately.

In cols. 1 & 2 of the form are entered the civil dates and mean time of moon's transit at Greenwich from form A_3.

126. From the monthly time curve, prepared as described in para 117, are entered the values in col. 3 of the form. Also, the date of the tide being calculated by adding the approximate time interval in hours, (*vide para* 113), to the time of moon's transit in col. 2, the entry is made in col. 9. Again, from the monthly height curve, the values are entered in col. 10 of the form.

127. In reading the monthly time and height charts, the values corresponding to each date are interpolated between the two consecutive monthly curves described in para 117, which give the values for the 15th days of these months. The interpolations are made by taking proportional parts as follows :—

Date of month	Proportional parts
15	—
18	0·1
21	0·2
24	0·3
27	0·4
30	0·5
3	0·6
6	0·7
9	0·8
12	0·9
15	1·0

The values read from the time charts are added to the time of moon's transit in the case of high-waters, and subtracted therefrom in the case of low-waters, in order to obtain the approximate predicted times of high and low-waters. In order to minimise the work, it has been decided to read values for time and height corresponding to alternate times of transit only, and to interpolate for the intermediate values by taking the mean. In the case of heights, no further corrections are made to the interpolated mean values, but in the case of times, the mean of alternate quantities is not sufficiently correct, and a correction for the 2nd. differences of monthly mean values is accordingly applied and entered in col. 6, from charts prepared for these second differences, (Δ_2). The charts for 2nd. differences are prepared by taking the monthly values corresponding to hours of moon's transit modified for 49 minutes, the mean interval between alternate times of transit, ($-\frac{1}{8}\left(\frac{49}{60}\right)^2 \Delta_2$). The correction factors for the actual intervals between alternate values have been calculated for 42 to 64 minutes, and are shown on these charts. The values measured from the charts are multiplied by the factors, and entered in column 6.

128. The declination and parallax corrections, worked out on form A_3, are now entered in cols. 4 & 11 of form 3 Tid Pred., those for height being multiplied by the factor for the port before entry.

These can also be worked out by the alternative method given in para 124.

The summation of cols. 2, 3 and 4, gives the entry to be made in col. 5, giving the approximate mean time of the tide, to which cols. 6 and 7 are afterwards added, in order to get the predicted time in col. 8.

Reading the Diurnal Chart.

129. A celluloid scale, specially prepared for the purpose on the 24 hours system on the scale of either 3 or 4 inches to 1 foot, is then taken for reading the Diurnal Chart. Each of these celluloid scales consists of a horizontal and a vertical scale joined at right angles at the centre, or mean sea level.

The horizontal scale is prepared on the 24 hours system, and each length of 3 inches on it represents a full day. It is subdivided to show single hours.

The vertical scale, which is at right angles to the horizontal scale described above, actually has 2 scales on it. The one on the left gives the height correction to be entered in col. 12. The other scale, on the right, is in a position to read 6 hours to the right of the left-hand scale, and therefore, without moving the scale, enables the curve to be read at a different point from that at which the heights were read, and approximately 6 hours away from the time of maximum of the semi-diurnal tide, as required by the theory already explained in para 110.

130. The horizontal scale is set in position to correspond with the arguments of approximate mean time and date given in cols. 5 and 9 of the form respectively. In setting the scale for reading, as

above, it is best to place the horizontal scale to correspond with the mean sea level line first, and afterwards to shift it to the approximate time at which the reading has to be taken. The corrections for height and time are then read.

The height correction is entered directly in col. 12, being positive, if measured above mean sea level, and negative, if below.

The time correction, as already explained, is read without moving the scale at a point nearly 6 hours later on the curve, and allowance has been made for this in the width of the scale. The reading has to be divided by half the semi-diurnal range of the port, and the result is then entered as the time correction in col. 7.

Both height and time corrections are read from the Diurnal Chart so as to correspond to alternate approximate times of tide in col. 5, and the intermediate values are interpolated between them.

131. Half the semi-diurnal range, the divisor required, is now obtained practically, by taking half the difference in height between the high and low-waters entered in col. 10.

132. The time corrections, when entered in col. 7, are allotted correct signs in accordance with the following rules:—

Time corrections, if measured above mean sea level, are positive for high-water and negative for low-water.

Time corrections, if measured below mean sea level, are negative for high-water and positive for low-water.

This follows from the theory already explained in para 110.

133. The columns of the form, now having all been entered, the summation of cols. 10, 11 and 12 gives the entry to be made in col. 13.

This completes the computation on form 3 Tid Pred., of which a sample showing the entries made in the various columns is given overleaf.

The predictions for time and height are now copied from cols. 8, 9 and 13 of 3 Tid Pred. into one of the forms 5,6,7 or 8 Tid Pred., according to the port in question, and, when completed, the predictions are ready for the press.

* Formerly the value A_0, the difference in height between datum of sounding and mean sea level, was taken, and the difference between this and the height read from the monthly curve accepted as the divisor required.

3 Tid. Pred.

Port *For*...19

Approx. Interval..............................*hrs.* *Factor* = − ....

Water

Scale = ″to 1 foot — Time* — Height†

1 Date of moon's transit (Civil). from N.A	2 Mean time of moon's transit from N.A. h m	3 § Time from monthly curve h m	4 Corrns. for declin. and parallax m	5 Sum = Approx. mean time $=t_a$ h m	6 Corrn. for 2nd diff. to interpolated values m	7 Corrn. ‡ for diurnal tide for t_a m	8 Sum = **Predicted time** h m	9 **Date of tide**	10 Height from monthly curve ft.	11 Corrns. for declin. and parallax ×factor for port ft.	12 Corrn. for diurnal tide for t_a ft.	13 Sum = **Predicted height** ft.	14 Time readings from diurnal curve
Civil date of moon's transit, time of which is given in column 2.	Mean time of moon's transit at Greenwich from N.A. Part IV.	Time from monthly curve corresponding to date and mean time of transit in columns 1 and 2.	Corrections to time for declination and parallax taken from form A_3 for year of prediction for the date in column 1.	Sum of columns 2, 3, and 4.	Corrections of the 2nd order differences in the monthly values corresponding to hours of moon's transit. This is to be applied *only* to the interpolated values.	Taken from the Diurnal Chart run off on the machine. The correction is the reading, as entered in column 14, divided by half the semidiurnal range of the port.	Sum of columns 5, 6, and 7.	This is the date in column 1 with approximate interval added.	Height from monthly height curve corresponding to date and mean time of transit in columns 1 and 2.	Corrections to heights for declination and parallax taken from form A_3 for year of prediction for the date in col. 1, multiplied by factor for the port.	Taken from the Diurnal Chart run off on the machine. The correction is the reading taken on the left hand vertical scale on celluloid at the time in column 5.	Sum of columns 10, 11, and 12.	Readings taken from the Diurnal Chart on the right hand vertical scale on celluloid, 6 hours after the time in col. 5.

* Time is Indian standard mean time
Burma standard Do.
Local Do.

† Height above

‡ Corrn. = $\dfrac{\text{Time readings}}{\frac{1}{2}\text{ range}}$

§ $\dfrac{+\text{ for High}}{-\text{ for Low}}$ Water

The port of Bassein, though actually a Riverain port, is not treated as such for purposes of prediction. Predictions for Bassein are made by means of the tidal curve run on the machine, in the same manner as for an open sea port, the following corrections being applied to the times of high and low-water.

Date.		Correction to high-water in minutes.	Correction to low-water in minutes.	Date.		Correction to high-water in minutes.	Correction to low-water in minutes.
Jan.	1	+18	− 1	July	3	− 4	+ 7
	3	+19	0		10	− 6	+10
	10	+22	+ 2		17	− 8	+12
	17	+23	+ 4		24	−10	+13
	24	+23	+ 6		31	−12	+12
	31	+23	+ 7				
				Aug.	7	−14	+11
Feb.	7	+22	+ 8		14	−16	+10
	14	+22	+ 8		21	−17	+ 8
	21	+21	+ 9		28	−19	+ 7
	28	+20	+ 9				
				Sep.	4	−20	+ 5
Mar.	6	+19	+ 9		11	−21	+ 3
	13	+18	+ 8		18	−22	+ 2
	20	+16	+ 6		25	−23	+ 1
	27	+15	+ 3				
				Oct.	2	−23	− 1
Apr.	3	+14	0		9	−24	− 3
	10	+12	− 3		16	−23	− 6
	17	+11	− 5		23	−21	− 8
	24	+ 9	− 7		30	−19	−11
May	1	+ 8	− 8	Nov.	6	−16	−14
	8	+ 7	− 9		13	−13	−15
	15	+ 6	−10		20	− 9	−16
	22	+ 5	−10		27	− 5	−15
	29	+ 4	− 9				
				Dec.	4	0	−13
June	5	+ 2	− 7		11	+ 5	−10
	12	+ 1	− 4		18	+10	− 7
	19	− 1	− 1		25	+14	− 4
	26	− 2	+ 3		32	+18	− 1

For Bassein the following data are used.

Component	κ	H	(V_0+u)
M_2	48·3	2·294	As for
M_4	325·7	0·241	Diamond
M_6	237·0	0·097	Island
S_1	135·6	0·066	,,
S_2	90·9	0·754	,,
η	152·3	1·864	,,
2η	322·3	0·570	,,
O	39·2	0.186	,,
K_1	47·6	0·374	,,
K_2	107·6	0·168	,,
P	50·6	0·127	,,
N	44·9	0·359	,,
L	52·6	0·163	,,
ν	1·5	0·138	,,
T	set at zero		,,
μ	178.1	0·266	,,
J	239·2	0·015	,,
Q	48·5	0·011	,,
MS	10·5	0·178	,,
2SM	300·4	0·077	,,
2N	337·1	0·116	,,
M_2N	315·8	0·079	,,
M_2K_1	301·4	0·095	,,
$2M_2K_1$	258·5	0·073	,,

TABLE I.—For converting Decimals of a Degree into Minutes and Seconds.

1

Parts of a Degree	Minutes
·1	6
·2	12
·3	18
·4	24
·5	30
·6	36
·7	42
·8	48
·9	54

2

Parts of a Degree	Minutes and Seconds	Parts of a Degree	Minutes and Seconds	Parts of a Degree	Minutes and Seconds
·001	0 3·6	·034	2 2·4	·067	4 1·2
2	0 7·2	35	2 6·0	68	4 4·8
3	0 10·8	36	2 9·6	69	4 8·4
4	0 14·4	37	2 13·2	·070	4 12·0
5	0 18·0	38	2 16·8	71	4 15·6
6	0 21·6	39	2 20·4	72	4 19·2
7	0 25·2	·040	2 24·0	73	4 22·8
8	0 28·8	41	2 27·6	74	4 26·4
9	0 32·4	42	2 31·2	75	4 30·0
·010	0 36·0	43	2 34·8	76	4 33·6
11	0 39·6	44	2 38·4	77	4 37·2
12	0 43·2	45	2 42·0	78	4 40·8
13	0 46·8	46	2 45·6	79	4 44·4
14	0 50·4	47	2 49·2	·080	4 48·0
15	0 54·0	48	2 52·8	81	4 51·6
16	0 57·6	49	2 56·4	82	4 55·2
17	1 1·2	·050	3 0·0	83	4 58·8
18	1 4·8	51	3 3·6	84	5 2·4
19	1 8·4	52	3 7·2	85	5 6·0
·020	1 12·0	53	3 10·8	86	5 9·6
21	1 15·6	54	3 14·4	87	5 13·2
22	1 19·2	55	3 18·0	88	5 16·8
23	1 22·8	56	3 21·6	89	5 20·4
24	1 26·4	57	3 25·2	·090	5 24·0
25	1 30·0	58	3 28·8	91	5 27·6
26	1 33·6	59	3 32·4	92	5 31·2
27	1 37·2	·060	3 36·0	93	5 34·8
28	1 40·8	61	3 39·6	94	5 38·4
29	1 44·4	62	3 43·2	95	5 42·0
·030	1 48·0	63	3 46·8	96	5 45·6
31	1 51·6	64	3 50·4	97	5 49·2
32	1 55·2	65	3 54·0	98	5 52·8
33	1 58·8	66	3 57·6	99	5 56·4

Example.—Required the value of 0°·875 in minutes and seconds

From Part 1 ... ·8 = 48′

" 2 .. ·075 = 4′ 30″

·875 = 52′ 30″

TABLE II.—For converting Minutes and Seconds into Decimals of a Degree.

Minutes.	Decimals of a Degree.	Seconds and Decimals of Seconds.	Decimals of a Degree correct to three places.	Actual value of Decimals of a Degree.	Minutes.	Decimals of a Degree.	Seconds and Decimals of Seconds.	Decimals of a Degree correct to three places.	Actual value of Decimals of a Degree.
0	·0	0·0	·000	·0000	3	·0	1·8	·051	·0505
6	·1	1·8	·001	·0005	9	·1	5·4	·052	·0515
12	·2	5·4	·002	·0015	15	·2	9·0	·053	·0525
18	·3	9·0	·003	·0025	21	·3	12·6	·054	·0535
24	·4	12·6	·004	·0035	27	·4	16·2	·055	·0545
30	·5	16·2	·005	·0045	33	·5	19·8	·056	·0555
36	·6	19·8	·006	·0055	39	·6	23·4	·057	·0565
42	·7	23·4	·007	·0065	45	·7	27·0	·058	·0575
48	·8	27·0	·008	·0075	51	·8	30·6	·059	·0585
54	·9	30·6	·009	·0085	57	·9	34·2	·060	·0595
		34·2	·010	·0095			37·8	·061	·0605
		37·8	·011	·0105			41·4	·062	·0615
		41·4	·012	·0115			45·0	·063	·0625
		45·0	·013	·0125			48·6	·064	·0635
		48·6	·014	·0135			52·2	·065	·0645
		52·2	·015	·0145			55·8	·066	·0655
		55·8	·016	·0155			59·4		·0665
		59·4		·0165					
			·017					·067	
1	·0	3·0	·018	·0175	4	·0	3·0	·068	·0675
7	·1	6·6	·019	·0185	10	·1	6·6	·069	·0685
13	·2	10·2	·020	·0195	16	·2	10·2	·070	·0695
19	·3	13·8	·021	·0205	22	·3	13·8	·071	·0705
25	·4	17·4	·022	·0215	28	·4	17·4	·072	·0715
31	·5	21·0	·023	·0225	34	·5	21·0	·073	·0725
37	·6	24·6	·024	·0235	40	·6	24·6	·074	·0735
43	·7	28·2	·025	·0245	46	·7	28·2	·075	·0745
49	·8	31·8	·026	·0255	52	·8	31·8	·076	·0755
55	·9	35·4	·027	·0265	58	·9	35·4	·077	·0765
		39·0	·028	·0275			39·0	·078	·0775
		42·6	·029	·0285			42·6	·079	·0785
		46·2	·030	·0295			46·2	·080	·0795
		49·8	·031	·0305			49·8	·081	·0805
		53·4	·032	·0315			53·4	·082	·0815
		57·0		·0325			57·0		·0825
			·033					·083	
2	·0	0·6	·034	·0335	5	·0	0·6	·084	·0835
8	·1	4·2	·035	·0345	11	·1	4·2	·085	·0845
14	·2	7·8	·036	·0355	17	·2	7·8	·086	·0855
20	·3	11·4	·037	·0365	23	·3	11·4	·087	·0865
26	·4	15·0	·038	·0375	29	·4	15·0	·088	·0875
32	·5	18·6	·039	·0385	35	·5	18·6	·089	·0885
38	·6	22·2	·040	·0395	41	·6	22·2	·090	·0895
44	·7	25·8	·041	·0405	47	·7	25·8	·091	·0905
50	·8	29·4	·042	·0415	53	·8	29·4	·092	·0915
56	·9	33·0	·043	·0425	59	·9	33·0	·093	·0925
		36·6	·044	·0435			36·6	·094	·0935
		40·2	·045	·0445			40·2	·095	·0945
		43·8	·046	·0455			43·8	·096	·0955
		47·4	·047	·0465			47·4	·097	·0965
		51·0	·048	·0475			51·0	·098	·0975
		54·6	·049	·0485			54·6	·099	·0985
		58·2	·050	·0495			58·2	·100	·0995
3	·0	1·8		·0505		·1	1·8		·1005

TABLE III.—Values of p the Mean Longitude of the Moon's Perigee, (the π of the computation forms), for every year from 1850 to 1949.

Year	p or π	Year	p or π	Year	p or π	Year	p or π
	°		°		°		°
1850	99·7292	1875	36·9609	1900	334·1913	1925	*271·4236*
51	140·3918	76	77·7348	1	14·8540	26	*312·0860*
52	181·1657	77	118·3973	2	55·5166	27	*352·7484*
53	221·8283	78	159·0598	3	96·1793	28	*33·5223*
54	262·4908	79	199·7223	4	136·9533	29	*74·1848*
1855	303·1534	1880	240·4962	1905	177·6160	1930	*114·8473*
56	343·9273	81	281·1587	6	218·2786	31	*155·5098*
57	24·5899	82	321·8212	7	258·9413	32	*196·2837*
58	65·2524	83	2·4837	8	299·7153	33	*236·9462*
59	105·9149	84	43·2576	9	340·3779	34	*277·6087*
1860	146·6889	1885	83·9200	1910	21·0406	1935	*318·2712*
61	187·3514	86	124·5825	11	61·7032	36	*359·0450*
62	228·0139	87	165·2450	12	102·4772	37	*39·7074*
63	268·6765	88	206·0189	13	143·1399	38	*80·3698*
64	309·4504	89	246·6814	14	183·8025	39	*121·0321*
1865	350·1129	1890	287·3439	1915	224·4651	1940	*161·8059*
66	30·7755	91	328·0063	16	265·2392	41	*202·4682*
67	71·4380	92	8·7802	17	305·9018	42	*243·1306*
68	112·2119	93	49·4427	18	346·5644	43	*283·7929*
69	152·8744	94	90·1051	19	27·2270	44	*324·5667*
1870	193·5369	1895	130·7676	1920	68·0010	1945	*5·2290*
71	234·1994	96	171·5415	21	108·6637	46	*45·8914*
72	274·9734	97	212·2039	22	149·3263	47	*86·5537*
73	315·6359	98	252·8664	23	189·9889	48	*127·3274*
74	356·2984	99	293·5289	24	*230·7611*	49	*167·9898*

1. These values are for January 0 (*i.e.*, noon December 31st of preceding year), except in the case of leap-years, when the values are for 0 hour January 1st.

2. The values given in the above table require 0° 136 to be added to give the true values of p or π, (see page 36, Preface to Hansen's Tables), but as the form for the computation of tidal observations has been constructed, showing the constant 0°·136 to be added, it has been thought advisable not to make this correction in above table.

3. The values from 1924 onwards depend on the new formula given in para 87.

These new values are shown in italics in the above table. The values for Jan. 1 computed by the new formula were found in defect of those published in the old edition of the table after addition of 0°·136 and one day's motion at 0°·111404 to convert the latter to Jan. 1 instead of Jan. 0, by 0°·00159 in 1923, 0°·00324 in 1936, and 0°·00608 in 1949.

The tabular values for Jan. 0 from 1924 onwards were accordingly corrected by interpolation from the above. and still require the constant 0°·136 added.

The values may also be obtained from the N. Almanac, which is usually available in time for any particular year's computations.

TABLE IV.—Number of Days from January 0.

Month.	Common year.	Leap-year	Month.	Common year.	Leap-year
January 0 ...	0	—1	July 0 ...	181	181
February 0 ...	31	30	August 0 ...	212	212
March 0 ...	59	59	September 0 ...	243	243
April 0 ...	90	90	October 0 ...	273	273
May 0 ...	120	120	November 0 ...	304	304
June 0 ...	151	151	December 0 ...	334	334

TABLE V.—Value of Movement of p or π for 1 to 366 Days, at 0°·1114040803l per mean solar day.

Days.	π	Days.	π	Days.	π	Days.	π	Days.	π	Days.	π
	°		°		°		°		°		°
1	0·11140	62	6·90705	123	13·70270	184	20·49835	245	27·29400	306	34·08965
2	0·22281	63	7·01846	124	13·81411	185	20·60975	246	27·40540	307	34·20105
3	0·33421	64	7·12986	125	13·92551	186	20·72116	247	27·51681	308	34·31246
4	0·44562	65	7·24127	126	14·03691	187	20·83256	248	27·62821	309	34·42386
5	0·55702	66	7·35267	127	14·14832	188	20·94397	249	27·73962	310	34·53526
6	0·66842	67	7·46407	128	14·25972	189	21·05537	250	27·85102	311	34·64667
7	0·77983	68	7·57548	129	14·37113	190	21·16678	251	27·96242	312	34·75807
8	0·89123	69	7·68688	130	14·48253	191	21·27818	252	28·07383	313	34·86948
9	1·00264	70	7·79829	131	14·59393	192	21·38958	253	28·18523	314	34·98088
10	1·11404	71	7·90969	132	14·70534	193	21·50099	254	28·29664	315	35·09229
11	1·22544	72	8·02109	133	14·81674	194	21·61239	255	28·40804	316	35·20369
12	1·33685	73	8·13250	134	14·92815	195	21·72380	256	28·51944	317	35·31509
13	1·44825	74	8·24390	135	15·03955	196	21·83520	257	28·63085	318	35·42650
14	1·55966	75	8·35531	136	15·15095	197	21·94660	258	28·74225	319	35·53790
15	1·67106	76	8·46671	137	15·26236	198	22·05801	259	28·85366	320	35·64931
16	1·78247	77	8·57811	138	15·37376	199	22·16941	260	28·96506	321	35·76071
17	1·89387	78	8·68952	139	15·48517	200	22·28082	261	29·07646	322	35·87211
18	2·00527	79	8·80092	140	15·59657	201	22·39222	262	29·18787	323	35·98352
19	2·11668	80	8·91233	141	15·70798	202	22·50362	263	29·29927	324	36·09492
20	2·22808	81	9·02373	142	15·81938	203	22·61503	264	29·41068	325	36·20633
21	2·33949	82	9·13513	143	15·93078	204	22·72643	265	29·52208	326	36·31773
22	2·45089	83	9·24654	144	16·04219	205	22·83784	266	29·63349	327	36·42913
23	2·56229	84	9·35794	145	16·15359	206	22·94924	267	29·74489	328	36·54054
24	2·67370	85	9·46935	146	16·26500	207	23·06064	268	29·85629	329	36·65194
25	2·78510	86	9·58075	147	16·37640	208	23·17205	269	29·96770	330	36·76335
26	2·89651	87	9·69215	148	16·48780	209	23·28345	270	30·07910	331	36·87475
27	3·00791	88	9·80356	149	16·59921	210	23·39486	271	30·19051	332	36·98615
28	3·11931	89	9·91496	150	16·71061	211	23·50626	272	30·30191	333	37·09756
29	3·23072	90	10·02637	151	16·82202	212	23·61766	273	30·41331	334	37·20896
30	3·34212	91	10·13777	152	16·93342	213	23·72907	274	30·52472	335	37·32037
31	3·45353	92	10·24918	153	17·04482	214	23·84047	275	30·63612	336	37·43177
32	3·56493	93	10·36058	154	17·15623	215	23·95188	276	30·74753	337	37·54317
33	3·67633	94	10·47198	155	17·26763	216	24·06328	277	30·85893	338	37·65458
34	3·78774	95	10·58339	156	17·37904	217	24·17469	278	30·97033	339	37·76598
35	3·89914	96	10·69479	157	17·49044	218	24·28609	279	31·08174	340	37·87739
36	4·01055	97	10·80620	158	17·60184	219	24·39749	280	31·19314	341	37·98879
37	4·12195	98	10·91760	159	17·71325	220	24·50890	281	31·30455	342	38·10020
38	4·23336	99	11·02900	160	17·82465	221	24·62030	282	31·41595	343	38·21160
39	4·34476	100	11·14041	161	17·93606	222	24·73171	283	31·52735	344	38·32300
40	4·45616	101	11·25181	162	18·04746	223	24·84311	284	31·63876	345	38·43441
41	4·56757	102	11·36322	163	18·15887	224	24·95451	285	31·75016	346	38·54581
42	4·67897	103	11·47462	164	18·27027	225	25·06592	286	31·86157	347	38·65722
43	4·79038	104	11·58602	165	18·38167	226	25·17732	287	31·97297	348	38·76862
44	4·90178	105	11·69743	166	18·49308	227	25·28873	288	32·08438	349	38·88002
45	5·01318	106	11·80883	167	18·60448	228	25·40013	289	32·19578	350	38·99143
46	5·12459	107	11·92024	168	18·71589	229	25·51153	290	32·30718	351	39·10283
47	5·23599	108	12·03164	169	18·82729	230	25·62294	291	32·41859	352	39·21424
48	5·34740	109	12·14304	170	18·93869	231	25·73434	292	32·52999	353	39·32564
49	5·45880	110	12·25445	171	19·05010	232	25·84575	293	32·64140	354	39·43704
50	5·57020	111	12·36585	172	19·16150	233	25·95715	294	32·75280	355	39·54845
51	5·68161	112	12·47726	173	19·27291	234	26·06855	295	32·86420	356	39·65985
52	5·79301	113	12·58866	174	19·38431	235	26·17996	296	32·97561	357	39·77126
53	5·90442	114	12·70007	175	19·49571	236	26·29136	297	33·08701	358	39·88266
54	6·01582	115	12·81147	176	19·60712	237	26·40277	298	33·19842	359	39·99406
55	6·12722	116	12·92287	177	19·71852	238	26·51417	299	33·30982	360	40·10547
56	6·23863	117	13·03428	178	19·82993	239	26·62558	300	33·42122	361	40·21687
57	6·35003	118	13·14568	179	19·94133	240	26·73698	301	33·53263	362	40·32828
58	6·46144	119	13·25709	180	20·05273	241	26·84838	302	33·64403	363	40·43968
59	6·57284	120	13·36849	181	20·16414	242	26·95979	303	33·75544	364	40·55109
60	6·68424	121	13·47989	182	20·27554	243	27·07119	304	33·86684	365	40·62489
61	6·79565	122	13·59130	183	20·38695	244	27·18260	305	33·97824	366	40·77389

TABLE VI.—Value of the Movement of p or π for differences of Longitude Greenwich.

Difference of Longitude	Value of π	Actual values of π corresponding to Degrees in Column I	Difference of Longitude	Value of π	Actual values of π corresponding to Degrees in Column I
°	°	°	°	°	°
0·000		·0000	88·865		·0275
	·000			·028	
1·616		·0005	92·097		·0285
	·001			·029	
4·847		·0015	95·328		·0295
	·002			·030	
8·079		·0025	98·560		·0305
	·003			·031	
11·310		·0035	101·791		·0315
	·004			·032	
14·542		·0045	105·023		·0325
	·005			·033	
17·773		·0055	108·254		·0335
	·006			·034	
21·005		·0065	111·486		·0345
	·007			·035	
24·236		·0075	114·717		·0355
	·008			·036	
27·467		·0085	117·949		·0365
	·009			·037	
30·699		·0095	121·180		·0375
	·010			·038	
33·930		·0105	124·411		·0385
	·011			·039	
37·162		·0115	127·643		·0395
	·012			·040	
40·393		·0125	130·874		·0405
	·013			·041	
43·625		·0135	134·106		·0415
	·014			·042	
46·856		·0145	137·337		·0425
	·015			·043	
50·088		·0155	140·569		·0435
	·016			·044	
53·319		·0165	143·800		·0445
	·017			·045	
56·551		·0175	147·032		·0455
	·018			·046	
59·782		·0185	150·263		·0465
	·019			·047	
63·014		·0195	153·495		·0475
	·020			·048	
66·245		·0205	156·726		·0485
	·021			·049	
69·477		·0215	159·958		·0495
	·022			·050	
72·708		·0225	163·189		·0505
	·023			·051	
75·939		·0235	166·421		·0515
	·024			·052	
79·171		·0245	169·652		·0525
	·025			·053	
82·402		·0255	172·884		·0535
	·026			·054	
85·634		·0265	176·115		·0545
	·027			·055	
88·865		·0275	179·346		·0555
				·056	
			182·578		·0565

Correction for E. Longitude –, for W. Longitude +,

TABLE VII.—Products of Augmenting Factors R_1, R_2, and R_4 multiplied by 1 to 99.

$R_1 = \cdot 0028.$

	0	10	20	30	40	50	60	70	80	90	
0	·0000	·0280	·0560	·0840	·1120	·1400	·1680	·1960	·2240	·2520	0
1	·0028	·0308	·0588	·0868	·1148	·1428	·1708	·1988	·2268	·2548	1
2	·0056	·0336	·0616	·0896	·1176	·1456	·1736	·2016	·2296	·2576	2
3	·0084	·0364	·0644	·0924	·1204	·1484	·1764	·2044	·2324	·2604	3
4	·0112	·0392	·0672	·0952	·1232	·1512	·1792	·2072	·2352	·2632	4
5	·0140	·0420	·0700	·0980	·1260	·1540	·1820	·2100	·2380	·2660	5
6	·0168	·0448	·0728	·1008	·1288	·1568	·1848	·2128	·2408	·2688	6
7	·0196	·0476	·0756	·1036	·1316	·1596	·1876	·2156	·2436	·2716	7
8	·0224	·0504	·0784	·1064	·1344	·1624	·1904	·2184	·2464	·2744	8
9	·0252	·0532	·0812	·1092	·1372	·1652	·1932	·2212	·2492	·2772	9

$R_2 = 0115.$

	0	10	20	30	40	50	60	70	80	90	
0	·0000	·1150	·2300	·3450	·4600	·5750	·6900	·8050	·9200	1·0350	0
1	·0115	·1265	·2415	·3565	·4715	·5865	·7015	·8165	·9315	1·0465	1
2	·0230	·1380	·2530	·3680	·4830	·5980	·7130	·8280	·9430	1·0580	2
3	·0345	·1495	·2645	·3795	·4945	·6095	·7245	·8395	·9545	1·0695	3
4	·0460	·1610	·2760	·3910	·5060	·6210	·7360	·8510	·9660	1·0810	4
5	·0575	·1725	·2875	·4025	·5175	·6325	·7475	·8625	·9775	1·0925	5
6	·0690	·1840	·2990	·4140	·5290	·6440	·7590	·8740	·9890	1·1040	6
7	·0805	·1955	·3105	·4255	·5405	·6555	·7705	·8855	1·0005	1·1155	7
8	·0920	·2070	·3220	·4370	·5520	·6670	·7820	·8970	1·0120	1·1270	8
9	·1035	·2185	·3335	·4485	·5635	·6785	·7935	·9085	1·0235	1·1385	9

$R_4 = \cdot 0472.$

	0	10	20	30	40	50	60	70	80	90	
0	·0000	·4720	·9440	1·4160	1·8880	2·3600	2·8320	3·3040	3·7760	4·2480	0
1	·0472	·5192	·9912	1·4632	1·9352	2·4072	2·8792	3·3512	3·8232	4·2952	1
2	·0944	·5664	1·0384	1·5104	1·9824	2·4544	2·9264	3·3984	3·8704	4·3424	2
3	·1416	·6136	1·0856	1·5576	2·0296	2·5016	2·9736	3·4456	3·9176	4·3896	3
4	·1888	·6608	1·1328	1·6048	2·0768	2·5488	3·0208	3·4928	3·9648	4·4368	4
5	·2360	·7080	1·1800	1·6520	2·1240	2·5960	3·0680	3·5400	4·0120	4·4840	5
6	·2832	·7552	1·2272	1·6992	2·1712	2·6432	3·1152	3·5872	4·0592	4·5312	6
7	·3304	·8024	1·2744	1·7464	2·2184	2·6904	3·1624	3·6344	4·1064	4·5784	7
8	·3776	·8496	1·3216	1·7936	2·2656	2·7376	3·2096	3·6816	4·1536	4·6256	8
9	·4248	·8968	1·3688	1·8408	2·3128	2·7848	3·2568	3·7288	4·2008	4·6728	9

TABLE VIII.—Products of $S_1 \times \cdot 001$ up to $S_1 \times 1 \cdot 000$.
$S_1 = \sin 15° = \cdot 25882$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000259	·000518	·000776	·001035	·001294	·001553	·001812	·002071	·002329	·00
·01	·002588	·002847	·003106	·003365	·003623	·003882	·004141	·004400	·004659	·004918	·01
·02	·005176	·005435	·005694	·005953	·006212	·006471	·006729	·006988	·007247	·007506	·02
·03	·007765	·008023	·008282	·008541	·008800	·009059	·009318	·009576	·009835	·010094	·03
·04	·010353	·010612	·010870	·011129	·011388	·011647	·011906	·012165	·012423	·012682	·04
·05	·012941	·013200	·013459	·013717	·013976	·014235	·014494	·014753	·015012	·015270	·05
·06	·015529	·015788	·016047	·016306	·016564	·016823	·017082	·017341	·017600	·017859	·06
·07	·018117	·018376	·018635	·018894	·019153	·019412	·019670	·019929	·020188	·020447	·07
·08	·020706	·020964	·021223	·021482	·021741	·022000	·022259	·022517	·022776	·023035	·08
·09	·023294	·023553	·023811	·024070	·024329	·024588	·024847	·025106	·025364	·025623	·09
·10	·025882	·026141	·026400	·026658	·026917	·027176	·027435	·027694	·027953	·028211	·10
·11	·028470	·028729	·028988	·029247	·029505	·029764	·030023	·030282	·030541	·030800	·11
·12	·031058	·031317	·031576	·031835	·032094	·032353	·032611	·032870	·033129	·033388	·12
·13	·033647	·033905	·034164	·034423	·034682	·034941	·035200	·035458	·035717	·035976	·13
·14	·036235	·036494	·036752	·037011	·037270	·037529	·037788	·038047	·038305	·038564	·14
·15	·038823	·039082	·039341	·039599	·039858	·040117	·040376	·040635	·040894	·041152	·15
·16	·041411	·041670	·041929	·042188	·042446	·042705	·042964	·043223	·043482	·043741	·16
·17	·043999	·044258	·044517	·044776	·045035	·045294	·045552	·045811	·046070	·046329	·17
·18	·046588	·046846	·047105	·047364	·047623	·047882	·048141	·048399	·048658	·048917	·18
·19	·049176	·049435	·049693	·049952	·050211	·050470	·050729	·050988	·051246	·051505	·19
·20	·051764	·052023	·052282	·052540	·052799	·053058	·053317	·053576	·053835	·054093	·20
·21	·054352	·054611	·054870	·055129	·055387	·055646	·055905	·056164	·056423	·056682	·21
·22	·056940	·057199	·057458	·057717	·057976	·058235	·058493	·058752	·059011	·059270	·22
·23	·059529	·059787	·060046	·060305	·060564	·060823	·061082	·061340	·061599	·061858	·23
·24	·062117	·062376	·062634	·062893	·063152	·063411	·063670	·063929	·064187	·064446	·24
·25	·064705	·064964	·065223	·065481	·065740	·065999	·066258	·066517	·066776	·067034	·25
·26	·067293	·067552	·067811	·068070	·068328	·068587	·068846	·069105	·069364	·069623	·26
·27	·069881	·070140	·070399	·070658	·070917	·071176	·071434	·071693	·071952	·072211	·27
·28	·072470	·072728	·072987	·073246	·073505	·073764	·074023	·074281	·074540	·074799	·28
·29	·075058	·075317	·075575	·075834	·076093	·076352	·076611	·076870	·077128	·077387	·29
·30	·077646	·077905	·078164	·078422	·078681	·078940	·079199	·079458	·079717	·079975	·30
·31	·080234	·080493	·080752	·081011	·081269	·081528	·081787	·082046	·082305	·082564	·31
·32	·082822	·083081	·083340	·083599	·083858	·084117	·084375	·084634	·084893	·085152	·32
·33	·085411	·085669	·085928	·086187	·086446	·086705	·086964	·087222	·087481	·087740	·33
·34	·087999	·088258	·088516	·088775	·089034	·089293	·089552	·089811	·090069	·090328	·34
·35	·090587	·090846	·091105	·091363	·091622	·091881	·092140	·092399	·092658	·092916	·35
·36	·093175	·093434	·093693	·093952	·094210	·094469	·094728	·094987	·095246	·095505	·36
·37	·095763	·096022	·096281	·096540	·096799	·097058	·097316	·097575	·097834	·098093	·37
·38	·098352	·098610	·098869	·099128	·099387	·099646	·099905	·100163	·100422	·100681	·38
·39	·100940	·101199	·101457	·101716	·101975	·102234	·102493	·102752	·103010	·103269	·39
·40	·103528	·103787	·104046	·104304	·104563	·104822	·105081	·105340	·105599	·105857	·40
·41	·106116	·106375	·106634	·106893	·107151	·107410	·107669	·107928	·108187	·108446	·41
·42	·108704	·108963	·109222	·109481	·109740	·109999	·110257	·110516	·110775	·111034	·42
·43	·111293	·111551	·111810	·112069	·112328	·112587	·112846	·113104	·113363	·113622	·43
·44	·113881	·114140	·114398	·114657	·114916	·115175	·115434	·115693	·115951	·116210	·44
·45	·116469	·116728	·116987	·117245	·117504	·117763	·118022	·118281	·118540	·118798	·45
·46	·119057	·119316	·119575	·119834	·120092	·120351	·120610	·120869	·121128	·121387	·46
·47	·121645	·121904	·122163	·122422	·122681	·122940	·123198	·123457	·123716	·123975	·47
·48	·124234	·124492	·124751	·125010	·125269	·125528	·125787	·126045	·126304	·126563	·48
·49	·126822	·127081	·127339	·127598	·127857	·128116	·128375	·128634	·128892	·129151	·49
·50	·129410	·129669	·129928	·130186	·130445	·130704	·130963	·131222	·131481	·131739	·50

TABLE VIII.—Products of $S_1 \times \cdot 001$ up to $S_1 \times 1 \cdot 000$. $S_1 = \sin 15° = \cdot 25882$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·129410	·129669	·129928	·130186	·130445	·130704	·130963	·131222	·131481	·131739	·50
·51	·131998	·132257	·132516	·132775	·133033	·133292	·133551	·133810	·134069	·134328	·51
·52	·134586	·134845	·135104	·135363	·135622	·135881	·136139	·136398	·136657	·136916	·52
·53	·137175	·137433	·137692	·137951	·138210	·138469	·138728	·138986	·139245	·139504	·53
·54	·139763	·140022	·140280	·140539	·140798	·141057	·141316	·141575	·141833	·142092	·54
·55	·142351	·142610	·142869	·143127	·143386	·143645	·143904	·144163	·144422	·144680	·55
·56	·144939	·145198	·145457	·145716	·145974	·146233	·146492	·146751	·147010	·147269	·56
·57	·147527	·147786	·148045	·148304	·148563	·148822	·149080	·149339	·149598	·149857	·57
·58	·150116	·150374	·150633	·150892	·151151	·151410	·151669	·151927	·152186	·152445	·58
·59	·152704	·152963	·153221	·153480	·153739	·153998	·154257	·154516	·154774	·155033	·59
·60	·155292	·155551	·155810	·156068	·156327	·156586	·156845	·157104	·157363	·157621	·60
·61	·157880	·158139	·158398	·158657	·158915	·159174	·159433	·159692	·159951	·160210	·61
·62	·160468	·160727	·160986	·161245	·161504	·161763	·162021	·162280	·162539	·162798	·62
·63	·163057	·163315	·163574	·163833	·164092	·164351	·164610	·164868	·165127	·165386	·63
·64	·165645	·165904	·166162	·166421	·166680	·166939	·167198	·167457	·167715	·167974	·64
·65	·168233	·168492	·168751	·169009	·169268	·169527	·169786	·170045	·170304	·170562	·65
·66	·170821	·171080	·171339	·171598	·171856	·172115	·172374	·172633	·172892	·173151	·66
·67	·173409	·173668	·173927	·174186	·174445	·174704	·174962	·175221	·175480	·175739	·67
·68	·175998	·176256	·176515	·176774	·177033	·177292	·177551	·177809	·178068	·178327	·68
·69	·178586	·178845	·179103	·179362	·179621	·179880	·180139	·180398	·180656	·180915	·69
·70	·181174	·181433	·181692	·181950	·182209	·182468	·182727	·182986	·183245	·183503	·70
·71	·183762	·184021	·184280	·184539	·184797	·185056	·185315	·185574	·185833	·186092	·71
·72	·186350	·186609	·186868	·187127	·187386	·187645	·187903	·188162	·188421	·188680	·72
·73	·188939	·189197	·189456	·189715	·189974	·190233	·190492	·190750	·191009	·191268	·73
·74	·191527	·191786	·192044	·192303	·192562	·192821	·193080	·193339	·193597	·193856	·74
·75	·194115	·194374	·194633	·194891	·195150	·195409	·195668	·195927	·196186	·196444	·75
·76	·196703	·196962	·197221	·197480	·197738	·197997	·198256	·198515	·198774	·199033	·76
·77	·199291	·199550	·199809	·200068	·200327	·200586	·200844	·201103	·201362	·201621	·77
·78	·201880	·202138	·202397	·202656	·202915	·203174	·203433	·203691	·203950	·204209	·78
·79	·204468	·204727	·204985	·205244	·205503	·205762	·206021	·206280	·206538	·206797	·79
·80	·207056	·207315	·207574	·207832	·208091	·208350	·208609	·208868	·209127	·209385	·80
·81	·209644	·209903	·210162	·210421	·210679	·210938	·211197	·211456	·211715	·211974	·81
·82	·212232	·212491	·212750	·213009	·213268	·213527	·213785	·214044	·214303	·214562	·82
·83	·214821	·215079	·215338	·215597	·215856	·216115	·216374	·216632	·216891	·217150	·83
·84	·217409	·217668	·217926	·218185	·218444	·218703	·218962	·219221	·219479	·219738	·84
·85	·219997	·220256	·220515	·220773	·221032	·221291	·221550	·221809	·222068	·222326	·85
·86	·222585	·222844	·223103	·223362	·223620	·223879	·224138	·224397	·224656	·224915	·86
·87	·225173	·225432	·225691	·225950	·226209	·226468	·226726	·226985	·227244	·227503	·87
·88	·227762	·228020	·228279	·228538	·228797	·229056	·229315	·229573	·229832	·230091	·88
·89	·230350	·230609	·230867	·231126	·231385	·231644	·231903	·232162	·232420	·232679	·89
·90	·232938	·233197	·233456	·233714	·233973	·234232	·234491	·234750	·235009	·235267	·90
·91	·235526	·235785	·236044	·236303	·236561	·236820	·237079	·237338	·237597	·237856	·91
·92	·238114	·238373	·238632	·238891	·239150	·239409	·239667	·239926	·240185	·240444	·92
·93	·240703	·240961	·241220	·241479	·241738	·241997	·242256	·242514	·242773	·243032	·93
·94	·243291	·243550	·243808	·244067	·244326	·244585	·244844	·245103	·245361	·245620	·94
·95	·245879	·246138	·246397	·246655	·246914	·247173	·247432	·247691	·247950	·248208	·95
·96	·248467	·248726	·248985	·249244	·249502	·249761	·250020	·250279	·250538	·250797	·96
·97	·251055	·251314	·251573	·251832	·252091	·252350	·252608	·252867	·253126	·253385	·97
·98	·253644	·253902	·254161	·254420	·254679	·254938	·255197	·255455	·255714	·255973	·98
·99	·256232	·256491	·256749	·257008	·257267	·257526	·257785	·258044	·258302	·258561	·99
1·00	·258820										1·00

TABLE IX.—Products of $S_3 \times \cdot 001$ up to $S_3 \times 1 \cdot 000$.

$S_3 = \sin 45° = \cdot 70711.$

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000707	·001414	·002121	·002828	·003536	·004243	·004950	·005657	·006364	·00
·01	·007071	·007778	·008485	·009192	·009900	·010607	·011314	·012021	·012728	·013435	·01
·02	·014142	·014849	·015556	·016264	·016971	·017678	·018385	·019092	·019799	·020506	·02
·03	·021213	·021920	·022628	·023335	·024042	·024749	·025456	·026163	·026870	·027577	·03
·04	·028284	·028992	·029699	·030406	·031113	·031820	·032527	·033234	·033941	·034648	·04
·05	·035356	·036063	·036770	·037477	·038184	·038891	·039598	·040305	·041012	·041719	·05
·06	·042427	·043134	·043841	·044548	·045255	·045962	·046669	·047376	·048083	·048791	·06
·07	·049498	·050205	·050912	·051619	·052326	·053033	·053740	·054447	·055155	·055862	·07
·08	·056569	·057276	·057983	·058690	·059397	·060104	·060811	·061519	·062226	·062933	·08
·09	·063640	·064347	·065054	·065761	·066468	·067175	·067883	·068590	·069297	·070004	·09
·10	·070711	·071418	·072125	·072832	·073539	·074247	·074954	·075661	·076368	·077075	·10
·11	·077782	·078489	·079196	·079903	·080611	·081318	·082025	·082732	·083439	·084146	·11
·12	·084853	·085560	·086267	·086975	·087682	·088389	·089096	·089803	·090510	·091217	·12
·13	·091924	·092631	·093339	·094046	·094753	·095460	·096167	·096874	·097581	·098288	·13
·14	·098995	·099703	·100410	·101117	·101824	·102531	·103238	·103945	·104652	·105359	·14
·15	·106067	·106774	·107481	·108188	·108895	·109602	·110309	·111016	·111723	·112430	·15
·16	·113138	·113845	·114552	·115259	·115966	·116673	·117380	·118087	·118704	·119502	·16
·17	·120209	·120916	·121623	·122330	·123037	·123744	·124451	·125158	·125866	·126573	·17
·18	·127280	·127987	·128694	·129401	·130108	·130815	·131522	·132230	·132937	·133644	·18
·19	·134351	·135058	·135765	·136472	·137179	·137886	·138594	·139301	·140008	·140715	·19
·20	·141422	·142129	·142836	·143543	·144250	·144958	·145665	·146372	·147079	·147786	·20
·21	·148493	·149200	·149907	·150614	·151322	·152029	·152736	·153443	·154150	·154857	·21
·22	·155564	·156271	·156978	·157686	·158393	·159100	·159807	·160514	·161221	·161928	·22
·23	·162635	·163342	·164050	·164757	·165464	·166171	·166878	·167585	·168292	·168999	·23
·24	·169706	·170414	·171121	·171828	·172535	·173242	·173949	·174656	·175363	·176070	·24
·25	·176778	·177485	·178192	·178899	·179606	·180313	·181020	·181727	·182434	·183141	·25
·26	·183849	·184556	·185263	·185970	·186677	·187384	·188091	·188798	·189505	·190213	·26
·27	·190920	·191627	·192334	·193041	·193748	·194455	·195162	·195869	·196577	·197284	·27
·28	·197991	·198698	·199405	·200112	·200819	·201526	·202233	·202941	·203648	·204355	·28
·29	·205062	·205769	·206476	·207183	·207890	·208597	·209305	·210012	·210719	·211426	·29
·30	·212133	·212840	·213547	·214254	·214961	·215669	·216376	·217083	·217790	·218497	·30
·31	·219204	·219911	·220618	·221325	·222033	·222740	·223447	·224154	·224861	·225568	·31
·32	·226275	·226982	·227689	·228397	·229104	·229811	·230518	·231225	·231932	·232639	·32
·33	·233346	·234053	·234761	·235468	·236175	·236882	·237589	·238296	·239003	·239710	·33
·34	·240417	·241125	·241832	·242539	·243246	·243953	·244660	·245367	·246074	·246781	·34
·35	·247489	·248196	·248903	·249610	·250317	·251024	·251731	·252438	·253145	·253852	·35
·36	·254560	·255267	·255974	·256681	·257388	·258095	·258802	·259509	·260216	·260924	·36
·37	·261631	·262338	·263045	·263752	·264459	·265166	·265873	·266580	·267288	·267995	·37
·38	·268702	·269409	·270116	·270823	·271530	·272237	·272944	·273652	·274359	·275066	·38
·39	·275773	·276480	·277187	·277894	·278601	·279308	·280016	·280723	·281430	·282137	·39
·40	·282844	·283551	·284258	·284965	·285672	·286380	·287087	·287794	·288501	·289208	·40
·41	·289915	·290622	·291329	·292036	·292744	·293451	·294158	·294865	·295572	·296279	·41
·42	·296986	·297693	·298400	·299108	·299815	·300522	·301229	·301936	·302643	·303350	·42
·43	·304057	·304764	·305472	·306179	·306886	·307593	·308300	·309007	·309714	·310421	·43
·44	·311128	·311836	·312543	·313250	·313957	·314664	·315371	·316078	·316785	·317492	·44
·45	·318200	·318907	·319614	·320321	·321028	·321735	·322442	·323149	·323856	·324563	·45
·46	·325271	·325978	·326685	·327392	·328099	·328806	·329513	·330220	·330927	·331635	·46
·47	·332342	·333049	·333756	·334463	·335170	·335877	·336584	·337291	·337999	·338706	·47
·48	·339413	·340120	·340827	·341534	·342241	·342948	·343655	·344363	·345070	·345777	·48
·49	·346484	·347191	·347898	·348605	·349312	·350019	·350727	·351434	·352141	·352848	·49
·50	·353555	·354262	·354969	·355676	·356383	·357091	·357798	·358505	·359212	·359919	·50

TABLE IX.—Products of $S_3 \times \cdot 001$ up to $S_3 \times 1 \cdot 000$.

$$S_3 = \sin 45^\circ = \cdot 70711.$$

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·353555	·354262	·354969	·355676	·356383	·357091	·357798	·358505	·359212	·359919	·50
·51	·360626	·361333	·362040	·362747	·363455	·364162	·364869	·365576	·366283	·366990	·51
·52	·367697	·368404	·369111	·369819	·370526	·371233	·371940	·372647	·373354	·374061	·52
·53	·374768	·375475	·376183	·376890	·377597	·378304	·379011	·379718	·380425	·381132	·53
·54	·381839	·382547	·383254	·383961	·384668	·385375	·386082	·386789	·387496	·388203	·54
·55	·388911	·389618	·390325	·391032	·391739	·392446	·393153	·393860	·394567	·395274	·55
·56	·395982	·396689	·397396	·398103	·398810	·399517	·400224	·400931	·401638	·402346	·56
·57	·403053	·403760	·404467	·405174	·405881	·406588	·407295	·408002	·408710	·409417	·57
·58	·410124	·410831	·411538	·412245	·412952	·413659	·414366	·415074	·415781	·416488	·58
·59	·417195	·417902	·418609	·419316	·420023	·420730	·421438	·422145	·422852	·423559	·59
·60	·424266	·424973	·425680	·426387	·427094	·427802	·428509	·429216	·429923	·430630	·60
·61	·431337	·432044	·432751	·433458	·434166	·434873	·435580	·436287	·436994	·437701	·61
·62	·438408	·439115	·439822	·440530	·441237	·441944	·442651	·443358	·444065	·444772	·62
·63	·445479	·446186	·446894	·447601	·448308	·449015	·449722	·450429	·451136	·451843	·63
·64	·452550	·453258	·453965	·454672	·455379	·456086	·456793	·457500	·458207	·458914	·64
·65	·459622	·460329	·461036	·461743	·462450	·463157	·463864	·464571	·465278	·465985	·65
·66	·466693	·467400	·468107	·468814	·469521	·470228	·470935	·471642	·472349	·473057	·66
·67	·473764	·474471	·475178	·475885	·476592	·477299	·478006	·478713	·479421	·480128	·67
·68	·480835	·481542	·482249	·482956	·483663	·484370	·485077	·485785	·486492	·487199	·68
·69	·487906	·488613	·489320	·490027	·490734	·491441	·492149	·492856	·493563	·494270	·69
·70	·494977	·495684	·496391	·497098	·497805	·498513	·499220	·499927	·500634	·501341	·70
·71	·502048	·502755	·503462	·504169	·504877	·505584	·506291	·506998	·507705	·508412	·71
·72	·509119	·509826	·510533	·511241	·511948	·512655	·513362	·514069	·514776	·515483	·72
·73	·516190	·516897	·517605	·518312	·519019	·519726	·520433	·521140	·521847	·522554	·73
·74	·523261	·523969	·524676	·525383	·526090	·526797	·527504	·528211	·528918	·529625	·74
·75	·530333	·531040	·531747	·532454	·533161	·533868	·534575	·535282	·535989	·536696	·75
·76	·537404	·538111	·538818	·539525	·540232	·540939	·541646	·542353	·543060	·543768	·76
·77	·544475	·545182	·545889	·546596	·547303	·548010	·548717	·549424	·550132	·550839	·77
·78	·551546	·552253	·552960	·553667	·554374	·555081	·555788	·556496	·557203	·557910	·78
·79	·558617	·559324	·560031	·560738	·561445	·562152	·562860	·563567	·564274	·564981	·79
·80	·565688	·566395	·567102	·567809	·568516	·569224	·569931	·570638	·571345	·572052	·80
·81	·572759	·573466	·574173	·574880	·575588	·576295	·577002	·577709	·578416	·579123	·81
·82	·579830	·580537	·581244	·581952	·582659	·583366	·584073	·584780	·585487	·586194	·82
·83	·586901	·587608	·588316	·589023	·589730	·590437	·591144	·591851	·592558	·593285	·83
·84	·593972	·594680	·595387	·596094	·596801	·597508	·598215	·598922	·599629	·600336	·84
·85	·601044	·601751	·602458	·603165	·603872	·604579	·605286	·605993	·606700	·607407	·85
·86	·608115	·608822	·609529	·610236	·610943	·611650	·612357	·613064	·613771	·614479	·86
·87	·615186	·615893	·616600	·617307	·618014	·618721	·619428	·620135	·620843	·621550	·87
·88	·622257	·622964	·623671	·624378	·625085	·625792	·626499	·627207	·627914	·628621	·88
·89	·629328	·630035	·630742	·631449	·632156	·632863	·633571	·634278	·634985	·635692	·89
·90	·636399	·637106	·637813	·638520	·639227	·639935	·640642	·641349	·642056	·642763	·90
·91	·643470	·644177	·644884	·645591	·646299	·647006	·647713	·648420	·649127	·649834	·91
·92	·650541	·651248	·651955	·652663	·653370	·654077	·654784	·655491	·656198	·656905	·92
·93	·657612	·658319	·659027	·659734	·660441	·661148	·661855	·662562	·663269	· 63970	·93
·94	·664683	·665391	·666098	·666805	·667512	·668219	·668926	·669633	·670340	·671047	·94
·95	·671755	·672462	·673169	·673876	·674583	·675290	·675997	·676704	·677411	·678118	·95
·96	·678826	·679533	·680240	·680947	·681654	·682361	·683068	·683775	·684482	·685190	·96
·97	·685897	·686604	·687311	·688018	·688725	·689432	·690139	·690846	·691554	·692261	·97
·98	·692968	·693675	·694382	·695089	·695796	·696503	·697210	·697918	·698625	·699332	·98
·99	·700039	·700746	·701453	·702160	·702867	·703574	·704282	·704989	·705696	·706403	·99
1·00	·707110										1·00

TABLE X.—Products of $S_4 \times \cdot 001$ up to $S_4 \times 1.000$.

$S_4 = \sin 60° = \cdot 86603$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000866	·001732	·002598	·003464	·004330	·005196	·006062	·006928	·007794	·00
·01	·008660	·009526	·010392	·011258	·012124	·012990	·013856	·014723	·015589	·016455	·01
·02	·017321	·018187	·019053	·019919	·020785	·021651	·022517	·023383	·024240	·025115	·02
·03	·025981	·026847	·027713	·028579	·029445	·030311	·031177	·032043	·032909	·033775	·03
·04	·034641	·035507	·036373	·037239	·038105	·038971	·039837	·040703	·041569	·042435	·04
·05	·043302	·044168	·045034	·045900	·046766	·047632	·048498	·049364	·050230	·051096	·05
·06	·051962	·052828	·053694	·054560	·055426	·056292	·057158	·058024	·058890	·059756	·06
·07	·060622	·061488	·062354	·063220	·064086	·064952	·065818	·066684	·067550	·068416	·07
·08	·069282	·070148	·071014	·071880	·072747	·073613	·074479	·075345	·076211	·077077	·08
·09	·077943	·078809	·079675	·080541	·081407	·082273	·083139	·084005	·084871	·085737	·09
·10	·086603	·087469	·088335	·089201	·090067	·090933	·091799	·092665	·093531	·094397	·10
·11	·095263	·096129	·096995	·097861	·098727	·099593	·100459	·101326	·102192	·103058	·11
·12	·103924	·104790	·105656	·106522	·107388	·108254	·109120	·109986	·110852	·111718	·12
·13	·112584	·113450	·114316	·115182	·116048	·116914	·117780	·118646	·119512	·120378	·13
·14	·121244	·122110	·122976	·123842	·124708	·125574	·126440	·127306	·128172	·129038	·14
·15	·129905	·130771	·131637	·132503	·133369	·134235	·135101	·135967	·136833	·137699	·15
16	·138565	·139431	·140297	·141163	·142029	·142895	·143761	·144627	·145493	·146359	·16
·17	·147225	·148091	·148957	·149823	·150689	·151555	·152421	·153287	·154153	·155019	·17
·18	·155885	·156751	·157617	·158483	·159350	·160216	·161082	·161948	·162814	·163680	·18
·19	·164546	·165412	·166278	·167144	·168010	·168876	·169742	·170608	·171474	·172340	·19
·20	·173206	·174072	·174938	·175804	·176670	·177536	·178402	·179268	·180134	·181000	·20
·21	·181866	·182732	·183598	·184464	·185330	·186196	·187062	·187929	·188795	·189661	·21
·22	·190527	·191393	·192259	·193125	·193991	·194857	·195723	·196589	·197455	·198321	·22
·23	·199187	·200053	·200919	·201785	·202651	·203517	·204383	·205249	·206115	·206981	·23
·24	·207847	·208713	·209579	·210445	·211311	·212177	·213043	·213909	·214775	·215641	·24
·25	·216508	·217374	·218240	·219106	·219972	·220838	·221704	·222570	·223436	·224302	·25
·26	·225168	·226034	·226900	·227766	·228632	·229498	·230364	·231230	·232096	·232962	·26
·27	·233828	·234694	·235560	·23642	·237292	·238158	·239024	·239890	·240756	·241622	·27
·28	·242488	·243354	·244220	·245086	·245953	·246819	·247685	·248551	·249417	·250283	·28
·29	·251149	·252015	·252881	·253747	·254613	·255479	·256345	·257211	·258077	·258943	·29
·30	·259809	·260675	·261541	·262407	·263273	·264139	·265005	·265871	·266737	·267603	·30
·31	·268469	·269335	·270201	·271067	·271933	·272799	·273665	·274532	·275398	·276264	·31
·32	·277130	·277996	·278862	·279728	·280594	·281460	·282326	·283192	·284058	·284924	·32
·33	·285790	·286656	·287522	·288388	·289254	·290120	·290986	·291852	·292718	·293584	·33
·34	·294450	·295316	·296182	·297048	·297914	·298780	·299646	·300512	·301378	·302244	·34
·35	·303111	·303977	·304843	·305709	·306575	·307441	·308307	·309173	·310039	·310905	·35
·36	·311771	·312637	·313503	·314369	·315235	·316101	·316967	·317833	·318699	·319565	·36
·37	·320431	·321297	·322163	·323029	·323895	·324761	·325627	·326493	·327359	·328225	·37
·38	·329091	·329957	·330823	·331689	·332556	·333422	·334288	·335154	·336020	·336886	·38
·39	·337752	·338618	·339484	·340350	·341216	·342082	·342948	·343814	·344680	·345546	·39
·40	·346412	·347278	·348144	·349010	·349876	·350742	·351608	·352474	·353340	·354206	·40
·41	·355072	·355938	·356804	·357670	·358536	·359402	·360268	·361135	·362001	·362867	·41
·42	·363733	·364599	·365465	·366331	·367197	·368063	·368929	·369795	·370661	·371527	·42
·43	·372393	·373259	·374125	·374991	·375857	·376723	·377589	·378455	·379321	·380187	·43
·44	·381053	·381919	·382785	·383651	·384517	·385383	·386249	·387115	·387981	·388847	·44
·45	·389714	·390580	·391446	·392312	·393178	·394044	·394910	·395776	·396642	397508	·45
·46	·398374	·399240	·400106	·400972	·401838	·402704	·403570	·404436	·405302	·406168	·46
·47	·407034	·407900	·408766	·409632	·410498	·411364	·412230	·413096	·413962	·414828	·47
·48	·415694	·416560	·417426	·418292	·419159	·420025	·420891	·421757	·422623	·423489	·48
·49	·424355	·425221	·426087	·426953	·427819	·428685	·429551	·430417	·431283	·432149	·49
·50	·433015	·433881	·434747	·435613	·436479	·437345	·438211	·439077	·439943	·440809	·50

TABLE X.—Products of $S_4 \times \cdot 001$ up to $S_4 \times 1 \cdot 000$.

$S_4 = \sin 60° = \cdot 86603$.

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·433015	·433881	·434747	·435613	·436479	·437345	·438211	·439077	·439943	·440809	·50
·51	·441675	·442541	·443407	·444273	·445139	·446005	·446871	·447738	·448604	·449470	·51
·52	·450336	·451202	·452068	·452934	·453800	·454666	·455532	·456398	·457264	·458130	·52
·53	·458996	·459862	·460728	·461594	·462460	·463326	·464192	·465058	·465924	·466790	·53
·54	·467656	·468522	·469388	·470254	·471120	·471986	·472852	·473718	·474584	·475450	·54
·55	·476317	·477183	·478049	·478915	·479781	·480647	·481513	·482379	·483245	·484111	·55
·56	·484977	·485843	·486709	·487575	·488441	·489307	·490173	·491039	·491905	·492771	·56
·57	·493637	·494503	·495369	·496235	·497101	·497967	·498833	·499699	·500565	·501431	·57
·58	·502297	·503163	·504029	·504895	·505762	·506628	·507494	·508360	·509226	·510092	·58
·59	·510958	·511824	·512690	·513556	·514422	·515288	·516154	·517020	·517886	·518752	·59
·60	·519618	·520484	·521350	·522216	·523082	·523948	·524814	·525680	·526546	·527412	·60
·61	·528278	·529144	·530010	·530876	·531742	·532608	·533474	·534341	·535207	·536073	·61
·62	·536939	·537805	·538671	·539537	·540403	·541269	·542135	·543001	·543867	·544733	·62
·63	·545599	·546465	·547331	·548197	·549063	·549929	·550795	·551661	·552527	·553393	·63
·64	·554259	·555125	·555991	·556857	·557723	·558589	·559455	·560321	·561187	·562053	·64
·65	·562920	·563786	·564652	·565518	·566384	·567250	·568116	·568982	·569848	·570714	·65
·66	·571580	·572446	·573312	·574178	·575044	·575910	·576776	·577642	·578508	·579374	·66
·67	·580240	·581106	·581972	·582838	·583704	·584570	·585436	·586302	·587168	·588034	·67
·68	·588900	·589766	·590632	·591498	·592365	·593231	·594097	·594963	·595829	·596695	·68
·69	·597561	·598427	·599293	·600159	·601025	·601891	·602757	·603623	·604489	·605355	·69
·70	·606221	·607087	·607953	·608819	·609685	·610551	·611417	·612283	·613149	·614015	·70
·71	·614881	·615747	·616613	·617479	·618345	·619211	·620077	·620944	·621810	·622676	·71
·72	·623542	·624408	·625274	·626140	·627006	·627872	·628738	·629604	·630470	·631336	·72
·73	·632202	·633068	·633934	·634800	·635666	·636532	·637398	·638264	·639130	·639996	·73
·74	·640862	·641728	·642594	·643460	·644326	·645192	·646058	·646924	·647790	·648656	·74
·75	·649523	·650389	·651255	·652121	·652987	·653853	·654719	·655585	·656451	·657317	·75
·76	·658183	·659049	·659915	·660781	·661647	·662513	·663379	·664245	·665111	·665977	·76
·77	·666843	·667709	·668575	·669441	·670307	·671173	·672039	·672905	·673771	·674637	·77
·78	·675503	·676369	·677235	·678101	·678968	·679834	·680700	·681566	·682432	·683298	·78
·79	·684164	·685030	·685896	·686762	·687628	·688494	·689360	·690226	·691092	·691958	·79
·80	·692824	·693690	·694556	·695422	·696288	·697154	·698020	·698886	·699752	·700618	·80
·81	·701484	·702350	·703216	·704082	·704948	·705814	·706680	·707547	·708413	·709279	·81
·82	·710145	·711011	·711877	·712743	·713609	·714475	·715341	·716207	·717073	·717939	·82
·83	·718805	·719671	·720537	·721403	·722269	·723135	·724001	·724867	·725733	·726599	·83
·84	·727465	·728331	·729197	·730063	·730929	·731795	·732661	·733527	·734393	·735259	·84
·85	·736126	·736992	·737858	·738724	·739590	·740456	·741322	·742188	·743054	·743920	·85
·86	·744786	·745652	·746518	·747384	·748250	·749116	·749982	·750848	·751714	·752580	·86
·87	·753446	·754312	·755178	·756044	·756910	·757776	·758642	·759508	·760374	·761240	·87
·88	·762106	·762972	·763838	·764704	·765571	·766437	·767303	·768169	·769035	·769901	·88
·89	·770767	·771633	·772499	·773365	·774231	·775097	·775963	·776829	·777695	·778561	·89
·90	·779427	·780293	·781159	·782025	·782891	·783757	·784623	·785489	·786355	·787221	·90
·91	·788087	·788953	·789819	·790685	·791551	·792417	·793283	·794150	·795016	·795882	·91
·92	·796748	·797614	·798480	·799346	·800212	·801078	·801944	·802810	·803676	·804542	·92
·93	·805408	·806274	·807140	·808006	·808872	·809738	·810604	·811470	·812336	·813202	·93
·94	·814068	·814934	·815800	·816666	·817532	·818398	·819264	·820130	·820996	·821862	·94
·95	·822729	·823595	·824461	·825327	·826193	·827059	·827925	·828791	·829657	·830523	·95
·96	·831389	·832255	·833121	·833987	·834853	·835719	·836585	·837451	·838317	·839183	·96
·97	·840049	·840915	·841781	·842647	·843513	·844379	·845245	·846111	·846977	·847843	·97
·98	·848709	·849575	·850441	·851307	·852174	·853040	·853906	·854772	·855638	·856504	·98
·99	·857370	·858236	·859102	·859968	·860834	·861700	·862566	·863432	·864298	·865164	·99
1·00	·866030										1·00

TABLE XI.—Products of S_5 × ·001 up to S_5 × 1·000.

$S_5 = \sin 75° = ·96593.$

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·00	·000000	·000966	·001932	·002898	·003864	·004830	·005796	·006762	·007727	·008639	·00
·01	·009659	·010625	·011591	·012557	·013523	·014489	·015455	·016421	·017387	·018353	·01
·02	·019319	·020285	·021250	·022216	·023182	·024148	·025114	·026080	·027046	·028012	·02
·03	·028978	·029944	·030910	·031876	·032842	·033808	·034773	·035739	·036705	·037671	·03
·04	·038637	·039603	·040569	·041535	·042501	·043467	·044433	·045399	·046365	·047331	·04
·05	·048297	·049262	·050228	·051194	·052160	·053126	·054092	·055058	·056024	·056990	·05
·06	·057956	·058922	·059888	·060854	·061820	·062785	·063751	·064717	·065683	·066649	·06
·07	·067615	·068581	·069547	·070513	·071479	·072445	·073411	·074377	·075343	·076308	·07
·08	·077274	·078240	·079206	·080172	·081138	·082104	·083070	·084036	·085002	·085968	·08
·09	·086934	·087900	·088866	·089831	·090797	·091763	·092729	·093695	·094661	·095627	·09
·10	·096593	·097559	·098525	·099491	·100457	·101423	·102389	·103355	·104320	·105286	·10
·11	·106252	·107218	·108184	·109150	·110116	·111082	·112048	·113014	·113980	·114946	·11
·12	·115912	·116878	·117843	·118809	·119775	·120741	·121707	·122673	·123639	·124605	·12
·13	·125571	·126537	·127503	·128469	·129435	·130401	·131366	·132332	·133298	·134264	·13
·14	·135230	·136196	·137162	·138128	·139094	·140060	·141026	·141992	·142958	·143924	·14
·15	·144890	·145855	·146821	·147787	·148753	·149719	·150685	·151651	·152617	·153583	·15
·16	·154549	·155515	·156481	·157447	·158413	·159378	·160344	·161310	·162276	·163242	·16
·17	·164208	·165174	·166140	·167106	·168072	·169038	·170004	·170970	·171936	·172901	·17
·18	·173867	·174833	·175799	·176765	·177731	·178697	·179663	·180629	·181595	·182561	·18
·19	·183527	·184493	·185459	·186424	·187390	·188356	·189322	·190288	·191254	·192220	·19
·20	·193186	·194152	·195118	·196084	·197050	·198016	·198982	·199948	·200913	·201879	·20
·21	·202845	·203811	·204777	·205743	·206709	·207675	·208641	·209607	·210573	·211539	·21
·22	·212505	·213471	·214436	·215402	·216368	·217334	·218300	·219266	·220232	·221198	·22
·23	·222164	·223130	·224096	·225062	·226028	·226994	·227959	·228925	·229891	·230857	·23
·24	·231823	·232789	·233755	·234721	·235687	·236653	·237619	·238585	·239551	·240517	·24
·25	·241483	·242448	·243414	·244380	·245346	·246312	·247278	·248244	·249210	·250176	·25
·26	·251142	·252108	·253074	·254040	·255006	·255971	·256937	·257903	·258869	·259835	·26
·27	·260801	·261767	·262733	·263699	·264665	·265631	·266597	·267563	·268529	·269494	·27
·28	·270460	·271426	·272392	·273358	·274324	·275290	·276256	·277222	·278188	·279154	·28
·29	·280120	·281086	·282052	·283017	·283983	·284949	·285915	·286881	·287847	·288813	·29
·30	·289779	·290745	·291711	·292677	·293643	·294609	·295575	·296541	·297506	·298472	·30
·31	·299438	·300404	·301370	·302336	·303302	·304268	·305234	·306200	·307166	·308132	·31
·32	·309098	·310064	·311029	·311995	·312961	·313927	·314893	·315859	·316825	·317791	·32
·33	·318757	·319723	·320689	·321655	·322621	·323587	·324552	·325518	·326484	·327450	·33
·34	·328416	·329382	·330348	·331314	·332280	·333246	·334212	·335178	·336144	·337110	·34
·35	·338076	·339041	·340007	·340973	·341939	·342905	·343871	·344837	·345803	·346769	·35
·36	·347735	·348701	·349667	·350633	·351599	·352564	·353530	·354496	·355462	·356428	·36
·37	·357394	·358360	·359326	·360292	·361258	·362224	·363190	·364156	·365122	·366087	·37
·38	·367053	·368019	·368985	·369951	·370917	·371883	·372849	·373815	·374781	·375747	·38
·39	·376713	·377679	·378645	·379610	·380576	·381542	·382508	·383474	·384440	·385406	·39
·40	·386372	·387338	·388304	·389270	·390236	·391202	·392168	·393134	·394099	·395065	·40
·41	·396031	·396997	·397963	·398929	·399895	·400861	·401827	·402793	·403759	·404725	·41
·42	·405691	·406657	·407622	·408588	·409554	·410520	·411486	·412452	·413418	·414384	·42
·43	·415350	·416316	·417282	·418248	·419214	·420180	·421145	·422111	·423077	·424043	·43
·44	·425009	·425975	·426941	·427907	·428873	·429839	·430805	·431771	·432737	·433703	·44
·45	·434669	·435634	·436600	·437566	·438532	·439498	·440464	·441430	·442396	·443362	·45
·46	·444328	·445294	·446260	·447226	·448192	·449157	·450123	·451089	·452055	·453021	·46
·47	·453987	·454953	·455919	·456885	·457851	·458817	·459783	·460749	·461715	·462680	·47
·48	·463646	·464612	·465578	·466544	·467510	·468476	·469442	·470408	·471374	·472340	·48
·49	·473306	·474272	·475238	·476203	·477169	·478135	·479101	·480067	·481033	·481999	·49
·50	·482965	·483931	·484897	·485863	·486829	·487795	·488761	·489727	·490692	·491658	·50

TABLE XI.—Products of $S_5 \times \cdot 001$ up to $S_5 \times 1 \cdot 000$.

$$S_5 = \sin 75^\circ = \cdot 96593.$$

No.	·000	·001	·002	·003	·004	·005	·006	·007	·008	·009	No.
·50	·482965	·483931	·484897	·485863	·486829	·487795	·488761	·489727	·490692	·491658	·50
·51	·492624	·493590	·494556	·495522	·496488	·497454	·498420	·499386	·500352	·501318	·51
·52	·502284	·503250	·504215	·505181	·506147	·507113	·508079	·509045	·510011	·510977	·52
·53	·511943	·512909	·513875	·514841	·515807	·516773	·517738	·518704	·519670	·520636	·53
·54	·521602	·522568	·523534	·524500	·525466	·526432	·527398	·528364	·529330	·530296	·54
·55	·531262	·532227	·533193	·534159	·535125	·536091	·537057	·538023	·538989	·539955	·55
·56	·540921	·541887	·542853	·543819	·544785	·545750	·546716	·547682	·548648	·549614	·56
·57	·550580	·551546	·552512	·553478	·554444	·555410	·556376	·557342	·558308	·559273	·57
·58	·560239	·561205	·562171	·563137	·564103	·565069	·566035	·567001	·567967	·568933	·58
·59	·569899	·570865	·571831	·572796	·573762	·574728	·575694	·576660	·577626	·578592	·59
·60	·579558	·580524	·581490	·582456	·583422	·584388	·585354	·586320	·587285	·588251	·60
·61	·589217	·590183	·591149	·592115	·593081	·594047	·595013	·595979	·596945	·597911	·61
·62	·598877	·599843	·600808	·601774	·602740	·603706	·604672	·605638	·606604	·607570	·62
·63	·608536	·609502	·610468	·611434	·612400	·613366	·614331	·615297	·616263	·617229	·63
·64	·618195	·619161	·620127	·621093	·622059	·623025	·623991	·624957	·625923	·626889	·64
·65	·627855	·628820	·629786	·630752	·631718	·632684	·633650	·634616	·635582	·636548	·65
·66	·637514	·638480	·639446	·640412	·641378	·642343	·643309	·644275	·645241	·646207	·66
·67	·647173	·648139	·649105	·650071	·651037	·652003	·652969	·653935	·654901	·655866	·67
·68	·656832	·657798	·658764	·659730	·660696	·661662	·662628	·663594	·664560	·665526	·68
·69	·666492	·667458	·668424	·669389	·670355	·671321	·672287	·673253	·674219	·675185	·69
·70	·676151	·677117	·678083	·679049	·680015	·680981	·681947	·682913	·683878	·684844	·70
·71	·685810	·686776	·687742	·688708	·689674	·690640	·691606	·692572	·693538	·694504	·71
·72	·695470	·696436	·697401	·698367	·699333	·700299	·701265	·702231	·703197	·704163	·72
·73	·705129	·706095	·707061	·708027	·708993	·709959	·710924	·711890	·712856	·713822	·73
·74	·714788	·715754	·716720	·717686	·718652	·719618	·720584	·721550	·722516	·723482	·74
·75	·724448	·725413	·726379	·727345	·728311	·729277	·730243	·731209	·732175	·733141	·75
·76	·734107	·735073	·736039	·737005	·737971	·738936	·739902	·740868	·741834	·742800	·76
·77	·743766	·744732	·745698	·746664	·747630	·748596	·749562	·750528	·751494	·752459	·77
·78	·753425	·754391	·755357	·756323	·757289	·758255	·759221	·760187	·761153	·762119	·78
·79	·763085	·764051	·765017	·765982	·766948	·767914	·768880	·769846	·770812	·771778	·79
·80	·772744	·773710	·774676	·775642	·776608	·777574	·778540	·779506	·780471	·781437	·80
·81	·782403	·783369	·784335	·785301	·786267	·787233	·788199	·789165	·790131	·791097	·81
·82	·792063	·793029	·793994	·794960	·795926	·796892	·797858	·798824	·799790	·800756	·82
·83	·801722	·802688	·803654	·804620	·805586	·806552	·807517	·808483	·809449	·810415	·83
·84	·811381	·812347	·813313	·814279	·815245	·816211	·817177	·818143	·819109	·820075	·84
·85	·821041	·822006	·822972	·823938	·824904	·825870	·826836	·827802	·828768	·829734	·85
·86	·830700	·831666	·832632	·833598	·834564	·835529	·836495	·837461	·838427	·839393	·86
·87	·840359	·841325	·842291	·843257	·844223	·845189	·846155	·847121	·848087	·849052	·87
·88	·850018	·850984	·851950	·852916	·853882	·854848	·855814	·856780	·857746	·858712	·88
·89	·859678	·860644	·861610	·862575	·863541	·864507	·865473	·866439	·867405	·868371	·89
·90	·869337	·870303	·871269	·872235	·873201	·874167	·875133	·876099	·877064	·878030	·90
·91	·878996	·879962	·880928	·881894	·882860	·883826	·884792	·885758	·886724	·887690	·91
·92	·888656	·889622	·890587	·891553	·892519	·893485	·894451	·895417	·896383	·897349	·92
·93	·898315	·899281	·900247	·901213	·902179	·903145	·904110	·905076	·906042	·907008	·93
·94	·907974	·908940	·909906	·910872	·911838	·912804	·913770	·914736	·915702	·916668	·94
·95	·917634	·918599	·919565	·920531	·921497	·922463	·923429	·924395	·925361	·926327	·95
·96	·927293	·928259	·929225	·930191	·931157	·932122	·933088	·934054	·935020	·935986	·96
·97	·936952	·937918	·938884	·939850	·940816	·941782	·942748	·943714	·944680	·945645	·97
·98	·946611	·947577	·948543	·949509	·950475	·951441	·952407	·953373	·954339	·955305	·98
·99	·956271	·957237	·958203	·959168	·960134	·961100	·962066	·963032	·963978	·964964	·99
1·00	·965930										1·00

TABLE XII.—Natural Numbers to three places of Decimals corresponding to Logarithms with Indices 6, 7 and 8.

Logarithms with Index 6 or 4

Natural No.	Natural No. to 3 places of Decimals	Logarithms
·00000	·000	·0000000
·00050	·001	·6989700
·00099		·9999999

Logarithms with Index 7 or 3

Nat. No.	Nat. No. to 3 places of Decimals	Logarithms	Nat. No.	Nat. No. to 3 places of Decimals	Logarithms	Nat. No.	Nat. No. to 3 places of Decimals	Logarithms
·0010	·001	·0000000	·0045	·005	·6532125	·0085	·009	·9294189
·0015	·002	·1760913	·0055	·006	·7403627	·0095	·010	·9777236
·0025	·003	·3979400	·0065	·007	·8129134	·0099		·9999999
·0035	·004	·5440680	·0075	·008	·8750613			
·0045		·6532125	·0085		·9294189			

Logarithms with Index 8 or 2.

Nat. No.	Nat. No. to 3 places of Decimals.	Logarithms.	Nat. No.	Nat. No. to 3 places of Decimals.	Logarithms.	Nat. No.	Nat. No. to 3 places of Decimals.	Logarithms.	Nat. No.	Nat. No. to 3 places of Decimals.	Logarithms.
·0100	·010	·0000000	·0325	·033	·5118834	·0555	·056	·7442930	·0785	·079	·8948697
·0105	·011	·0211893	·0335	·034	·5250448	·0565	·057	·7520484	·0795	·080	·9003671
·0115	·012	·0606978	·0345	·035	·5378191	·0575	·058	·7596678	·0805	·081	·9057959
·0125	·013	·0969100	·0355	·036	·5502284	·0585	·059	·7671559	·0815	·082	·9111576
·0135	·014	·1303338	·0365	·037	·5622929	·0595	·060	·7745170	·0825	·083	·9164539
·0145	·015	·1613680	·0375	·038	·5740313	·0605	·061	·7817554	·0835	·084	·9216865
·0155	·016	·1903317	·0385	·039	·5854607	·0615	·062	·7888751	·0845	·085	·9268567
·0165	·017	·2174839	·0395	·040	·5965971	·0625	·063	·7958800	·0855	·086	·9319661
·0175	·018	·2430380	·0405	·041	·6074550	·0635	·064	·8027737	·0865	·087	·9370161
·0185	·019	·2671717	·0415	·042	·6180481	·0645	·065	·8095597	·0875	·088	·9420081
·0195	·020	·2900346	·0425	·043	·6283889	·0655	·066	·8162413	·0885	·089	·9469433
·0205	·021	·3117539	·0435	·044	·6384893	·0665	·067	·8228216	·0895	·090	·9518230
·0215	·022	·3324385	·0445	·045	·6483600	·0675	·068	·8293038	·0905	·091	·9566486
·0225	·023	·3521825	·0455	·046	·6580114	·0685	·069	·8356906	·0915	·092	·9614211
·0235	·024	·3710679	·0465	·047	·6674530	·0695	·070	·8419848	·0925	·093	·9661417
·0245	·025	·3891661	·0475	·048	·6766936	·0705	·071	·8481891	·0935	·094	·9708116
·0255	·026	·4065402	·0485	·049	·6857417	·0715	·072	·8543060	·0945	·095	·9754318
·0265	·027	·4232459	·0495	·050	·6946052	·0725	·073	·8603380	·0955	·096	·9800034
·0275	·028	·4393327	·0505	·051	·7032914	·0735	·074	·8662873	·0965	·097	·9845273
·0285	·029	·4548449	·0515	·052	·7118072	·0745	·075	·8721563	·0975	·098	·9890046
·0295	·030	·4698220	·0525	·053	·7201593	·0755	·076	·8779470	·0985	·099	·9934362
·0305	·031	·4842998	·0535	·054	·7283538	·0765	·077	·8836614	·0995	·100	·9978231
·0315	·032	·4983106	·0545	·055	·7363965	·0775	·078	·8893017	·0999		·9999999
·0325		·5118834	·0555		·7442930	·0785		·8948697			

TABLE XIII.—Values of N (Longitude of Moon's Ascending Node) for 0 hour Greenwich Mean time, January 1.

Value on 0 hour G. M. T., January 1st, 1880=285°·956863.
Motion per Julian year in 1880=19°·34146248.
Motion for 365 days=19°·32822387 and for one day=0°·052954.

Year	N	Year	N	Year	N	Year	N
	°		°		°		°
1850	146·1745	1875	22·6509	1900	259·1277	1925	*195·6082*
51	126·8462	76	3·3227	01	239·7995	26	*116·2801*
52	107·5180	77	343·9415	02	220·4713	27	*96·9519*
53	88·1368	78	324·6133	03	201·1431	28	*77·6237*
54	68·8086	79	305·2851	04	181·8148	29	*58·2426*
1855	49·4803	1880	285·9569	1905	162·4337	1930	*38·9144*
56	30·1521	81	266·5757	06	143·1055	31	*19·5863*
57	10·7709	82	247·2475	07	123·7773	32	*0·2581*
58	351·4427	83	227·9192	08	104·4490	33	*340·8770*
59	332·1144	84	208·5910	09	85·0679	34	*321·5488*
1860	312·7862	1885	189·2098	1910	65·7397	1935	*302·2206*
61	293·4050	86	169·8816	11	46·4115	36	*282·8925*
62	274·0768	87	150·5534	12	27·0833	37	*263·5113*
63	254·7486	88	131·2252	13	7·7021	38	*244·1832*
64	235·4203	89	111·8440	14	348·3739	39	*224·8550*
1865	216·0391	1890	92·5158	1915	329·0457	1940	*205·5268*
66	196·7109	91	73·1876	16	309·7175	41	*186·1457*
67	177·3827	92	53·8593	17	290·3363	42	*166·8175*
68	158·0544	93	34·4782	18	271·0081	43	*147·4894*
69	138·6733	94	15·1500	19	251·6799	44	*128·1612*
1870	119·3450	1895	355·8217	1920	232·3517	1945	*108·7801*
71	100·0168	96	336·4935	21	212·9705	46	*89·4519*
72	80·6886	97	317·1124	22	193·6423	47	*70·1238*
73	61·3074	98	297·7841	23	174·3141	48	*50·7956*
74	41·9792	99	278·4559	24	*164·9894*	49	*31·4145*

The values from 1924 onwards depend on the new formula in para 93. *They are shown in italics in Table XIII.*

The values for January 1 computed by the new formula were in excess of those published in the old edition of the table by 0·0034 in 1923 and 0·0042 in 1949.

The tabular values were accordingly corrected by interpolation from the above to the new values. The values may be also obtained from the N. Almanac which will usually be available in time for any particular year's computations.

TABLE XIV.—Showing the Decrement of N (Longitude of Moon's Ascending Node) since 0 hour January 1 up to Midnight of each Day throughout the Year.

Daily Motion=0°·05295392220 (new value.)

In Leap years for all dates after February 28—March 1, use a mean value between the particular day and the day following.

Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.
JAN.	°	JAN.	°	JAN.	°	JAN.	°	FEB.	°	FEB.	°
1—2	0·0265	11-12	0·5560	21-22	1·0856	31-32	1·6151	9-10	2·0917	19-20	2·6212
2—3	0·0794	12-13	0·6090	22-23	1·1385	FEB.		10-11	2·1446	20-21	2·6742
3—4	0·1324	13-14	0·6619	23-24	1·1915	1—2	1·6680	11 12	2·1976	21-22	2·7271
4—5	0·1853	14-15	0·7149	24-25	1·2444	2—3	1·7210	12-13	2·2505	22-23	2·7801
5—6	0·2383	15-16	0·7678	25-26	1·2974	3—4	1·7740	13-14	2·3035	23-24	2·8330
6—7	0·2912	16-17	0·8208	26-27	1·3503	4—5	1·8269	14-15	2·3565	24-25	2·8860
7—9	0·3442	17-18	0·8737	27-28	1·4033	5—6	1·8799	15-16	2·4094	25-26	2·9389
8—8	0·3972	18-19	0·9267	28-29	1·4562	6—7	1·9328	16-17	2·4624	26-27	2·9919
9—10	0·4501	19-20	0·9797	29-30	1·5092	7—8	1·9858	17-18	2·5153	27-28	3·0449
10—11	0·5031	20-21	1·0326	30-31	1·5621	8—9	2·0387	18-19	2·5683	28-29	3·0978

N.B.—In Table XIV. The middle of the year of observations will occur at noon or midnight according as the 29th February is included in the period of observations or not. If the *midnight* falls on a date in a common year, or before the 29th February in a leap year, then the Decrement for N as given in the Table is correct: if, however, the *midnight* falls after the 29th February in a leap year, then take the value as that given for the *succeeding date* in the Table.

TABLE XIV.—Showing the Decrement of N (Longitude of Moon's Ascending Node) since 0 hour January 1 up to Midnight of each Day throughout the year—(*continued*).

Daily Motion = $0°·05295392220$ (new value).

In Leap Years for all dates after February 28—March 1, use a mean value between the particular day and the day following.

Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.	Date	Decre.
MAR.	°	APR.	°	JUNE.	°	AUG.	°	SEPT.	°	NOV.	°
1- 2	3·1508	22-23	5·9044	12-13	8·6050	2- 3	11·3057	23-24	14·0593	13-14	16·7699
2- 3	3·2037	23-24	5·9573	13-14	8·6580	3- 4	11·3586	24-25	14·1122	14-15	16·8129
3- 4	3·2567	24-25	6·0103	14-15	8·7109	4- 5	11·4116	25-26	14·1652	15-16	16·8658
4- 5	3·3096	25-26	6·0632	15-16	8·7639	5- 6	11·4645	26-27	14·2181	16-17	16·9188
5- 6	3·3626	26-27	6·1162	16-17	8·8168	6- 7	11·5175	27-28	14·2711	17-18	16·9717
6- 7	3·4155	27-28	6·1691	17-18	8·8698	7- 8	11·5704	28-29	14·3240	18-19	17·0247
7- 8	3·4685	28-29	6·2221	18-19	8·9227	8- 9	11·6234	29-30	14·3770	19-20	17·0776
8- 9	3·5214	29-30	6·2750	19-20	8·9757	9-10	11·6763	30-31	14·4299	20-21	17·1306
9-10	3·5744	30-31	6·3280	20-21	9·0286	10-11	11·7293	OCT.		21-22	17·1835
10-11	3·6273	MAY.		21-22	9·0816	11-12	11·7822	1- 2	14·4829	22-23	17·2365
11-12	3·6803	1- 2	6·3809	22-23	9·1346	12-13	11·8352	2- 3	14·5359	23-24	17·2895
12-13	3·7333	2- 3	6·4339	23-24	9·1875	13-14	11·8882	3- 4	14·5888	24-25	17·3424
13-14	3·7862	3- 4	6·4869	24-25	9·2405	14.15	11·9411	4- 5	14·6418	25-26	17·3954
14 15	3·8392	4- 5	6·5298	25-26	9·2934	15-16	11·9941	5- 6	14·6947	26-27	17·4483
15-16	3·8921	5- 6	6·5928	26-27	9·3464	16-17	12·0470	6- 7	14·7477	27-28	17·5013
16-17	3·9451	6- 7	6·6457	27-28	9·3993	17-18	12·1000	7- 8	14·8006	28-29	17·5542
17-18	3·9980	7- 8	6·6987	28-29	9·4523	18-19	12·1529	8- 9	14·8536	29 30	17·6072
18-19	4·0510	8- 9	6·7516	29-30	9·5052	19-20	12·2059	9-10	14·9065	30-31	17·6601
19-20	4·1039	9-10	6·8046	30-31	9·5582	20-21	12·2588	10-11	14·9595	DEC.	
20 21	4·1569	10-11	6·8575	JULY.		21-22	12·3118	11-12	15·0124	1- 2	17·7131
21-22	4·2098	11-12	6·9105	1- 2	9·6111	22-23	12·3647	12-13	15·0654	2- 3	17·7660
22-23	4·2628	12-13	6·9634	2- 3	9·6641	23-24	12·4177	13-14	15·1183	3- 4	17·8190
23-24	4·3157	13-14	7·0164	3- 4	9·7170	24-25	12·4706	14-15	15·1713	4- 5	17·8719
24-25	4·3687	14-15	7·0693	4- 5	9·7700	25-26	12·5236	15-16	15·2243	5- 6	17·9249
25-26	4·4217	15-16	7·1223	5- 6	9·8230	26-27	12·5766	16-17	15·2772	6- 7	17·9779
26-27	4·4746	16-17	7·1753	6- 7	9·8759	27-28	12·6295	17-18	15·3302	7- 8	18·0308
27-28	4·5276	17 18	7·2282	7- 8	9·9289	28-29	12·6825	18-19	15·3831	8- 9	18·0838
28-29	4·5805	18-19	7·2812	8- 9	9·9818	29-30	12·7354	19-20	15·4301	9-10	18·1367
29-30	4·6335	19-20	7·3341	9-10	10·0348	30-31	12·7884	20-21	15·4890	10-11	18·1897
30-31	4·6864	20-21	7·3871	10-11	10·0877	31-32	12·8413	21-22	15·5420	11-12	18·2426
31-32	4·7394	21-22	7·4400	11-12	10·1407	SEPT.		22-23	15·5949	12-13	18·2956
APR.		22-23	7·4930	12-13	10·1936	1- 2	12·8943	23-24	15·6479	13-14	18·3485
1- 2	4·7923	23-24	7·5459	13-14	10·2466	2- 3	12·9472	24-25	15·7008	14-15	18·4015
2- 3	4·8453	24-25	7·5989	14-15	10·2995	3- 4	13·0002	25-26	15·7538	15-16	18·4544
3- 4	4·8982	25-26	7·6518	15-16	10·3525	4- 5	13·0531	26-27	15·8067	16-17	18·5074
4- 5	4·9512	26-27	7·7048	16-17	10·4054	5- 6	13·1061	27-28	15·8597	17-18	18·5603
5- 6	5·0041	27-28	7·7577	17-18	10·4584	6- 7	13·1590	28-29	15·9127	18 19	18·6133
6- 7	5·0571	28-29	7·8107	18-19	10·5114	7- 8	13·2120	29-30	15·9656	19-20	18·6663
7- 8	5·1101	29-30	7·8637	19 20	10·5643	8- 9	13·2650	30-31	16·0186	20-21	18·7192
8- 9	5·1630	30-31	7·9166	20-21	10·6173	9-10	13·3179	31-32	16·0715	21-22	18·7722
9-10	5·2160	31-32	7·9696	21-22	10·6702	10-11	13·3709	NOV.		22-23	18·8251
10-11	5·2689	JUNE		22-23	10·7232	11-12	13·4238	1- 2	16·1245	23-24	18·8781
11-12	5·3219	1- 2	8·0225	23-24	10·7761	12-13	13·4768	2- 3	16·1774	24-25	18·9310
12-13	5·3748	2- 3	8·0755	24-25	10·8291	13-14	13·5297	3- 4	16·2304	25-26	18·9840
13 14	5·4278	3- 4	8·1284	25-26	10·8820	14-15	13·5827	4- 5	16·2833	26-27	19·0369
14-15	5·4807	4- 5	8·1814	26-27	10·9350	15-16	13·6356	5- 6	16·3363	27-28	19·0899
15-16	5·5337	5- 6	8·2343	27-28	10·9879	16-17	13·6885	6- 7	16·3892	28-29	19·1428
16-17	5·5866	6- 7	8·2873	28-29	11·0409	17-18	13·7415	7- 8	16·4422	29-30	19·1958
17-18	5·6396	7- 8	8·3402	29-30	11·0938	18-19	13·7945	8- 9	16·4951	30-31	19·2488
18-19	5·6925	8- 9	8·3932	30-31	11·1468	19-20	13·8475	9-10	16·5481	31-32	19·3017
19-20	5·7455	9-10	8·4462	31-32	11·1998	20-21	13·9004	10-11	16·6011		
20-21	5·7985	10-11	8·4991	AUG.		21-22	13·9534	11-12	16·6540		
21-22	5·8514	11-12	8·5521	1- 2	11·2527	22-23	14·0063	12-13	16·7070		

If the *noon* falls in a common year or before the 29th February, in a leap year, the value to be taken from the Table is the mean between the *preceding* and *succeeding* midnights; but if the *noon* falls after the 29th February in a leap year, the mean between the values for the *two midnights immediately following* is to be taken.

TABLE XV.—Values of p_1 (Mean Longitude of Solar Perigee) for 0 hour, January 1.

p_1 for 0 hour, January 1, 1880=280°·874802.
Motion per Julian year=0°·01710693.
Motion for 365 days=0°·01709295.

Year	p_1	Year	p_1	Year	p_1	Year	p_1
	°		°		°		°
1850	280·3614	1875	280·7892	1900	281·2171	1925	*281·6506*
51	·3785	76	·8063	01	·2342	26	*·6678*
52	·3956	77	·8235	02	·2513	27	*·6850*
53	·4125	78	·8406	03	·2684	28	*·7021*
54	·4299	79	·8577	04	·2855	29	*·7194*
55	·4470	1880	·8748	05	·3027	1930	*·7366*
56	·4641	81	·8920	06	·3198	31	*·7538*
57	·4812	82	·9091	07	·3369	32	*·7709*
58	·4983	83	·9262	08	·3540	33	*·7882*
59	·5154	84	·9433	09	·3711	34	*·8054*
1860	·5325	85	·9604	1910	·3882	35	*·8225*
61	·5497	86	·9775	11	·4054	36	*·8397*
62	·5668	87	·9946	12	·4225	37	*·8570*
63	·5839	88	281·0117	13	·4396	38	*·8741*
64	·6010	89	·0289	14	·4567	39	*·8913*
65	·6181	1890	·0460	15	·4738	1940	*·9081*
66	·6352	91	·0631	16	·4909	41	*·9257*
67	·6523	92	·0802	17	·5081	42	*·9429*
68	·6694	93	·0973	18	·5252	43	*·9602*
69	·6866	94	·1144	19	·5423	44	*·9773*
1870	·7037	95	·1315	1920	·5594	45	*·9945*
71	·7208	96	·1485	21	·5766	46	*282·0117*
72	·7379	97	·1658	22	·5937	47	*·0290*
73	·7550	98	·1829	23	·6108	48	*·0461*
74	·7721	99	·2000	24	*·6395*	49	*·0634*

The values from 1924 onwards depend on the new formula in para 95. *They are shown in italics in Table XV.*

The values for Jan. 1st computed by the new formula were in excess of those published in the old edition of the table by 0·0055 in 1923, 0·0064 in 1936 and 0·0075 in 1949.

The tabular values were accordingly corrected by interpolation from the above to the new values. These values may also be obtained from the N. Almanac which will usually be available in time for any particular year's computations.

TABLE XVI.—Increment of p_1 since 0 hour, January 1, for certain Days of the Year.

Motion for 1 day =0°·00004706845 (new formula).

Date		Increment	Date		Increment	Date		Increment	Date		Increment
		°			°			°			°
Jan.	10	0·00042	Apr.	10	0·00464	July.	9	0·00885	Oct.	7	0·01307
,,	20	·00089	,,	20	·00510	,,	19	·00932	,,	17	·01353
,,	30	·00136	,,	30	·00557	,,	29	·00979	,,	27	·01400
Feb.	9	·00183	May	10	·00604	Aug.	8	·01026	Nov.	6	·01447
,,	19	·00229	,,	20	·00651	,,	18	·01072	,,	16	·01494
Mar.	1	·00276	,,	30	·00698	,,	28	·01119	,,	26	·01541
,,	11	·00323	June	9	·00745	Sept.	7	·01166	Dec.	6	·01588
,,	21	·00370	,,	19	·00791	,,	17	·01213	,,	16	·01634
,,	31	·00417	,,	29	·00838	,,	27	·01260	,,	26	·01681

TABLE XVII.—Values of I, ν and ξ, corresponding to N.

N	I	ν	ξ	N	N	I	ν	ξ	N	N	I	ν	ξ	N
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0·0	28·602	0·000	0·000	360·0	30·5	28·005	5·564	5·017	329·5	60·5	26·351	10·131	9·196	299·5
0·5	28·602	0·094	0·084	359·5	31·0	27·985	5·651	5·095	329·0	61·0	26·316	10·194	9·255	299·0
1·0	28·601	0·188	0·169	359·0	31·5	27·965	5·736	5·173	328·5	61·5	26·280	10·256	9·313	298·5
1·5	28·600	0·281	0·253	358·5	32·0	27·945	5·822	5·251	328·0	62·0	26·245	10·318	9·370	298·0
2·0	28·599	0·375	0·337	358·0	32·5	27·925	5·907	5·328	327·5	62·5	26·209	10·379	9·427	297·5
2·5	28·598	0·468	0·421	357·5	33·0	27·904	5·992	5·406	327·0	63·0	26·173	10·440	9·484	297·0
3·0	28·596	0·562	0·506	357·0	33·5	27·884	6·077	5·482	326·5	63·5	26·137	10·500	9·539	296·5
3·5	28·594	0·656	0·590	356·5	34·0	27·862	6·162	5·559	326·0	64·0	26·101	10·560	9·595	296·0
4·0	28·591	0·749	0·674	356·0	34·5	27·841	6·246	5·636	325·5	64·5	26·064	10·619	9·650	295·5
4·5	28·589	0·843	0·758	355·5	35·0	27·819	6·330	5·712	325·0	65·0	26·027	10·677	9·705	295·0
5·0	28·585	0·936	0·842	355·0	35·5	27·797	6·414	5·788	324·5	65·5	25·990	10·735	9·759	294·5
5·5	28·582	1·030	0·926	354·5	36·0	27·775	6·497	5·864	324·0	66·0	25·953	10·793	9·812	294·0
6·0	28·578	1·123	1·010	354·0	36·5	27·752	6·580	5·939	323·5	66·5	25·916	10·849	9·865	293·5
6·5	28·574	1·217	1·094	353·5	37·0	27·729	6·663	6·015	323·0	67·0	25·878	10·906	9·918	293·0
7·0	28·570	1·310	1·178	353·0	37·5	27·706	6·745	6·090	322·5	67·5	25·841	10·961	9·970	292·5
7·5	28·565	1·403	1·262	352·5	38·0	27·682	6·828	6·164	322·0	68·0	25·803	11·016	10·021	292·0
8·0	28·560	1·496	1·346	352·0	38·5	27·658	6·909	6·239	321·5	68·5	25·765	11·070	10·072	291·5
8·5	28·555	1·590	1·430	351·5	39·0	27·634	6·991	6·313	321·0	69·0	25·726	11·124	10·123	291·0
9·0	28·549	1·683	1·514	351·0	39·5	27·610	7·072	6·387	320·5	69·5	25·688	11·177	10·173	290·5
9·5	28·543	1·776	1·598	350·5	40·0	27·585	7·153	6·461	320·0	70·0	25·649	11·230	10·222	290·0
10·0	28·537	1·869	1·681	350·0	40·5	27·560	7·234	6·534	319·5	70·5	25·610	11·282	10·271	289·5
10·5	28·530	1·962	1·765	349·5	41·0	27·535	7·314	6·608	319·0	71·0	25·571	11·333	10·319	289·0
11·0	28·523	2·054	1·848	349·0	41·5	27·510	7·394	6·680	318·5	71·5	25·532	11·383	10·366	288·5
11·5	28·516	2·147	1·932	348·5	42·0	27·484	7·473	6·753	318·0	72·0	25·493	11·433	10·414	288·0
12·0	28·508	2·240	2·015	348·0	42·5	27·458	7·553	6·825	317·5	72·5	25·453	11·482	10·460	287·5
12·5	28·500	2·332	2·099	347·5	43·0	27·432	7·631	6·897	317·0	73·0	25·413	11·531	10·506	287·0
13·0	28·492	2·424	2·182	347·0	43·5	27·405	7·710	6·969	316·5	73·5	25·374	11·579	10·551	286·5
13·5	28·483	2·517	2·265	346·5	44·0	27·378	7·788	7·040	316·0	74·0	25·334	11·626	10·596	286·0
14·0	28·475	2·609	2·348	346·0	44·5	27·351	7·866	7·111	315·5	74·5	25·293	11·673	10·640	285·5
14·5	28·465	2·701	2·431	345·5	45·0	27·324	7·943	7·182	315·0	75·0	25·253	11·719	10·684	285·0
15·0	28·456	2·793	2·514	315·0	45·5	27·296	8·020	7·253	314·5	75·5	25·213	11·764	10·726	284·5
15·5	28·446	2·885	2·596	344·5	46·0	27·268	8·097	7·323	314·0	76·0	25·172	11·809	10·769	284·0
16·0	28·436	2·977	2·679	344·0	46·5	27·240	8·173	7·392	313·5	76·5	25·131	11·852	10·811	283·5
16·5	28·425	3·068	2·762	343·5	47·0	27·212	8·249	7·462	313·0	77·0	25·090	11·895	10·852	283·0
17·0	28·414	3·160	2·844	343·0	47·5	27·183	8·324	7·531	312·5	77·5	25·049	11·938	10·892	282·5
17·5	28·403	3·251	2·927	342·5	48·0	27·154	8·399	7·600	312·0	78·0	25·008	11·980	10·932	282·0
18·0	28·392	3·342	3·009	342·0	48·5	27·125	8·474	7·668	311·5	78·5	24·966	12·021	10·971	281·5
18·5	28·380	3·433	3·091	341·5	49·0	27·095	8·548	7·736	311·0	79·0	24·925	12·061	11·009	281·0
19·0	28·368	3·524	3·173	341·0	49·5	27·066	8·622	7·804	310·5	79·5	24·883	12·100	11·047	280·5
19·5	28·356	3·615	3·255	340·5	50·0	27·036	8·695	7·871	310·0	80·0	24·841	12·139	11·084	280·0
20·0	28·343	3·705	3·337	340·0	50·5	27·006	8·768	7·938	309·5	80·5	24·800	12·177	11·121	279·5
20·5	28·330	3·796	3·418	339·5	51·0	26·975	8·841	8·005	309·0	81·0	24·757	12·214	11·157	279·0
21·0	28·317	3·886	3·500	339·0	51·5	26·944	8·913	8·071	308·5	81·5	24·715	12·251	11·192	278·5
21·5	28·303	3·976	3·581	338·5	52·0	26·913	8·985	8·137	308·0	82·0	24·673	12·287	11·227	278·0
22·0	28·289	4·066	3·662	338·0	52·5	26·882	9·056	8·203	307·5	82·5	24·631	12·322	11·261	277·5
22·5	28·275	4·156	3·743	337·5	53·0	26·851	9·127	8·268	307·0	83·0	24·588	12·356	11·294	277·0
23·0	28·260	4·245	3·824	337·0	53·5	26·819	9·197	8·333	306·5	83·5	24·545	12·389	11·326	276·5
23·5	28·245	4·335	3·905	336·5	54·0	26·787	9·267	8·397	306·0	84·0	24·503	12·422	11·358	276·0
24·0	28·230	4·424	3·985	336·0	54·5	26·755	9·336	8·461	305·5	84·5	24·460	12·454	11·389	275·5
24·5	28·215	4·513	4·066	335·5	55·0	26·723	9·405	8·525	305·0	85·0	24·417	12·485	11·419	275·0
25·0	28·199	4·602	4·146	335·0	55·5	26·690	9·474	8·588	304·5	85·5	24·374	12·515	11·449	274·5
25·5	28·183	4·690	4·226	334·5	56·0	26·657	9·543	8·651	304·0	86·0	24·331	12·545	11·478	274·0
26·0	28·166	4·779	4·306	334·0	56·5	26·624	9·609	8·713	303·5	86·5	24·287	12·573	11·506	273·5
26·5	28·149	4·867	4·386	333·5	57·0	26·591	9·676	8·775	303·0	87·0	24·244	12·601	11·533	273·0
27·0	28·132	4·955	4·466	333·0	57·5	26·557	9·743	8·836	302·5	87·5	24·200	12·628	11·560	272·5
27·5	28·115	5·043	4·545	332·5	58·0	26·523	9·809	8·897	302·0	88·0	24·157	12·654	11·586	272·0
28·0	28·097	5·130	4·624	332·0	58·5	26·489	9·874	8·958	301·5	88·5	24·113	12·680	11·611	271·5
28·5	28·079	5·218	4·703	331·5	59·0	26·455	9·939	9·018	301·0	89·0	24·070	12·704	11·635	271·0
29·0	28·061	5·305	4·782	331·0	59·5	26·421	10·004	9·078	300·5	89·5	24·026	12·728	11·659	270·5
29·5	28·043	5·391	4·861	330·5	60·0	26·386	10·068	9·137	300·0	90·0	23·982	12·751	11·681	270·0
30·0	28·024	5·478	4·939	330·0										

N.B.—I is always positive. When N is between 0° and 180°, ν and ξ are positive; when N is between 180° and 360°, ν and ξ are negative.

TABLE XVII.—Values of *I*, *ν* and *ξ*, corresponding to *N*—(*Continued*).

N	*I*	*ν*	*ξ*	*N*	*N*	*I*	*ν*	*ξ*	*N*	*N*	*I*	*ν*	*ξ*	*N*
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
90·5	23·938	12·772	11·704	269·5	120·5	21·283	12·295	11·378	239·5	150·5	19·135	7·745	7·223	209·5
91·0	23·894	12·793	11·725	269·0	121·0	21·241	12·254	11·342	239·0	151·0	19·108	7·635	7·120	209·0
91·5	23·850	12·814	11·745	268·5	121·5	21·199	12·211	11·304	238·5	151·5	19·082	7·523	7·017	208·5
92·0	23·806	12·833	11·765	268·0	122·0	21·157	12·168	11·266	238·0	152·0	19·056	7·411	6·913	208·0
92·5	23·761	12·851	11·784	267·5	122·5	21·115	12·123	11·226	237·5	152·5	19·030	7·298	6·809	207·5
93·0	23·717	12·869	11·802	267·0	123·0	21·073	12·078	11·186	237·0	153·0	19·005	7·184	6·703	207·0
93·5	23·673	12·886	11·819	266·5	123·5	21·032	12·031	11·144	236·5	153·5	18·981	7·069	6·596	206·5
94·0	23·628	12·901	11·835	266·0	124·0	20·990	11·983	11·101	236·0	154·0	18·956	6·953	6·488	206·0
94·5	23·584	12·916	11·851	265·5	124·5	20·949	11·933	11·057	235·5	154·5	18·933	6·836	6·380	205·5
95·0	23·539	12·930	11·866	265·0	125·0	20·908	11·883	11·012	235·0	155·0	18·909	6·718	6·270	205·0
95·5	23·495	12·943	11·880	264·5	125·5	20·867	11·831	10·965	234·5	155·5	18·886	6·599	6·160	204·5
96·0	23·450	12·955	11·893	264·0	126·0	20·826	11·778	10·918	234·0	156·0	18·863	6·480	6·049	204·0
96·5	23·406	12·966	11·905	263·5	126·5	20·786	11·724	10·869	233·5	156·5	18·841	6·359	5·937	203·5
97·0	23·361	12·976	11·916	263·0	127·0	20·746	11·669	10·820	233·0	157·0	18·819	6·238	5·824	203·0
97·5	23·316	12·985	11·927	262·5	127·5	20·705	11·612	10·769	232·5	157·5	18·798	6·116	5·710	202·5
98·0	23·271	12·994	11·936	262·0	128·0	20·666	11·555	10·717	232·0	158·0	18·777	5·993	5·596	202·0
98·5	23·227	13·001	11·945	261·5	128·5	20·626	11·496	10·664	231·5	158·5	18·756	5·869	5·480	201·5
99·0	23·182	13·007	11·953	261·0	129·0	20·586	11·436	10·610	231·0	159·0	18·736	5·744	5·364	201·0
99·5	23·137	13·013	11·960	260·5	129·5	20·547	11·374	10·554	230·5	159·5	18·716	5·618	5·247	200·5
100·0	23·092	13·017	11·966	260·0	130·0	20·508	11·312	10·498	230·0	160·0	18·697	5·492	5·130	200·0
100·5	23·047	13·021	11·971	259·5	130·5	20·469	11·248	10·440	229·5	160·5	18·678	5·365	5·011	199·5
101·0	23·003	13·023	11·975	259·0	131·0	20·430	11·184	10·382	229·0	161·0	18·660	5·237	4·892	199·0
101·5	22·958	13·024	11·979	258·5	131·5	20·392	11·118	10·322	228·5	161·5	18·642	5·109	4·773	198·5
102·0	22·913	13·025	11·981	258·0	132·0	20·353	11·050	10·261	228·0	162·0	18·624	4·980	4·652	198·0
102·5	22·868	13·024	11·983	257·5	132·5	20·315	10·982	10·199	227·5	162·5	18·607	4·850	4·531	197·5
103·0	22·823	13·023	11·983	257·0	133·0	20·278	10·912	10·135	227·0	163·0	18·591	4·719	4·409	197·0
103·5	22·778	13·020	11·983	256·5	133·5	20·240	10·841	10·071	226·5	163·5	18·575	4·588	4·287	196·5
104·0	22·734	13·017	11·982	256·0	134·0	20·203	10·769	10·005	226·0	164·0	18·559	4·456	4·164	196·0
104·5	22·689	13·012	11·979	255·5	134·5	20·166	10·696	9·939	225·5	164·5	18·544	4·323	4·040	195·5
105·0	22·644	13·006	11·976	255·0	135·0	20·129	10·622	9·871	225·0	165·0	18·529	4·190	3·916	195·0
105·5	22·599	13·000	11·972	254·5	135·5	20·092	10·546	9·802	224·5	165·5	18·515	4·056	3·791	194·5
106·0	22·554	12·992	11·967	254·0	136·0	20·056	10·469	9·732	224·0	166·0	18·501	3·922	3·665	194·0
106·5	22·510	12·983	11·961	253·5	136·5	20·020	10·391	9·660	223·5	166·5	18·487	3·787	3·539	193·5
107·0	22·465	12·974	11·954	253·0	137·0	19·984	10·312	9·588	223·0	167·0	18·475	3·651	3·413	193·0
107·5	22·420	12·963	11·946	252·5	137·5	19·949	10·232	9·515	222·5	167·5	18·462	3·515	3·286	192·5
108·0	22·376	12·951	11·937	252·0	138·0	19·913	10·150	9·440	222·0	168·0	18·450	3·379	3·158	192·0
108·5	22·331	12·938	11·927	251·5	138·5	19·878	10·068	9·364	221·5	168·5	18·439	3·242	3·030	191·5
109·0	22·287	12·924	11·916	251·0	139·0	19·844	9·984	9·287	221·0	169·0	18·428	3·104	2·902	191·0
109·5	22·242	12·909	11·904	250·5	139·5	19·809	9·899	9·209	220·5	169·5	18·417	2·966	2·773	190·5
110·0	22·198	12·892	11·891	250·0	140·0	19·775	9·813	9·130	220·0	170·0	18·407	2·828	2·644	190·0
110·5	22·153	12·875	11·877	249·5	140·5	19·742	9·725	9·050	219·5	170·5	18·398	2·689	2·514	189·5
111·0	22·109	12·857	11·862	249·0	141·0	19·708	9·637	8·969	219·0	171·0	18·388	2·550	2·384	189·0
111·5	22·065	12·837	11·846	248·5	141·5	19·675	9·547	8·887	218·5	171·5	18·380	2·410	2·254	188·5
112·0	22·021	12·817	11·829	248·0	142·0	19·642	9·457	8·803	218·0	172·0	18·372	2·270	2·123	188·0
112·5	21·976	12·795	11·811	247·5	142·5	19·610	9·365	8·719	217·5	172·5	18·364	2·130	1·992	187·5
113·0	21·932	12·772	11·792	247·0	143·0	19·577	9·272	8·633	217·0	173·0	18·357	1·989	1·860	187·0
113·5	21·888	12·748	11·772	246·5	143·5	19·545	9·177	8·546	216·5	173·5	18·350	1·848	1·729	186·5
114·0	21·845	12·723	11·750	246·0	144·0	19·514	9·082	8·458	216·0	174·0	18·344	1·707	1·597	186·0
114·5	21·801	12·697	11·728	245·5	144·5	19·483	8·986	8·370	215·5	174·5	18·338	1·566	1·464	185·5
115·0	21·757	12·670	11·705	245·0	145·0	19·452	8·888	8·280	215·0	175·0	18·333	1·424	1·332	185·0
115·5	21·713	12·642	11·681	244·5	145·5	19·421	8·790	8·189	214·5	175·5	18·328	1·282	1·199	184·5
116·0	21·670	12·612	11·655	244·0	146·0	19·391	8·690	8·097	214·0	176·0	18·324	1·140	1·067	184·0
116·5	21·627	12·581	11·629	243·5	146·5	19·361	8·589	8·004	213·5	176·5	18·320	0·998	0·934	183·5
117·0	21·583	12·550	11·601	243·0	147·0	19·332	8·487	7·910	213·0	177·0	18·317	0·856	0·801	183·0
117·5	21·540	12·517	11·572	242·5	147·5	19·302	8·384	7·814	212·5	177·5	18·315	0·713	0·667	182·5
118·0	21·497	12·483	11·543	242·0	148·0	19·273	8·280	7·718	212·0	178·0	18·312	0·571	0·534	182·0
118·5	21·454	12·447	11·512	241·5	148·5	19·245	8·175	7·621	211·5	178·5	18·311	0·428	0·401	181·5
119·0	21·411	12·411	11·480	241·0	149·0	19·217	8·069	7·523	211·0	179·0	18·309	0·286	0·267	181·0
119·5	21·368	12·373	11·447	240·5	149·5	19·189	7·962	7·424	210·5	179·5	18·309	0·143	0·133	180·5
120·0	21·326	12·335	11·413	240·0	150·0	19·162	7·854	7·324	210·0	180·0	18·308	0·000	0·000	180·0

N.B.—*I* is always positive. When *N* is between 0° and 180°, *ν* and *ξ* are positive; when *N* is between 180° and 360°, *ν* and *ξ* are negative.

TABLE XVIII. (1)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tides M_2, N, 2N, ν, MS, 2SM and Luni-Solar fortnightly.

$$\text{Argument } 1/f = \frac{\text{Cos}^4 \frac{1}{2}\,\omega\ \text{Cos}^4 \frac{1}{2} i}{\text{Cos}^4 \frac{1}{2} I}$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′30″	0·96354		1·03784		23°·5	0·99630		1·00371	
18·4	·96403		·03731		·6	·99703	73	·00298	73
·5	·96458	55	·03672	59	·7	·99775	72	·00225	73
·6	·96513	55	·03613	59	·8	·99849	74	·00152	73
·7	·96569	56	·03553	60	·9	·99922	73	·00078	74
·8	·96624	55	·03494	59	24·0	·99996	74	·00004	74
·9	·96680	56	·03434	60	·1	1·00070	74	0·99930	74
19·0	·96736	56	·03374	60	·2	·00145	75	·99855	75
·1	·96793	57	·03313	61	·3	·00220	75	·99781	74
·2	·96850	57	·03252	61	·4	·00296	76	·99705	76
·3	·96908	58	·03191	61	·5	·00372	76	·99629	76
·4	·96965	57	·03129	62	·6	·00448	76	·99553	76
·5	·97023	58	·03068	61	·7	·00525	77	·99478	75
·6	·97082	59	·03006	62	·8	·00601	76	·99402	76
·7	·97141	59	·02943	63	·9	·00679	78	·99326	76
·8	·97199	58	·02882	61	25·0	·00757	78	·99249	77
·9	·97259	60	·02819	63	·1	·00835	78	·99172	77
20·0	·97318	59	·02756	63	·2	·00913	78	·99095	77
·1	·97379	61	·02692	64	·3	·00993	80	·99017	78
·2	·97439	60	·02628	64	·4	·01072	79	·98939	78
·3	·97500	61	·02564	64	·5	·01151	79	·98862	77
·4	·97561	61	·02500	64	·6	·01231	80	·98783	79
·5	·97622	61	·02436	64	·7	·01312	81	·98705	78
·6	·97684	62	·02371	65	·8	·01393	81	·98626	79
·7	·97746	62	·02306	65	·9	·01474	81	·98547	79
·8	·97808	62	·02241	65	26·0	·01556	82	·98468	79
·9	·97871	63	·02175	66	·1	·01638	82	·98388	80
21·0	·97935	64	·02109	66	·2	·01721	83	·98308	80
·1	·97999	64	·02042	67	·3	·01803	82	·98229	79
·2	·98062	63	·01976	66	·4	·01887	84	·98148	81
·3	·98127	65	·01909	67	·5	·01970	83	·98068	80
·4	·98191	64	·01842	67	·6	·02054	84	·97988	80
·5	·98256	65	·01775	67	·7	·02138	84	·97907	81
·6	·98321	65	·01707	68	·8	·02223	85	·97826	81
·7	·98387	66	·01639	68	·9	·02308	85	·97744	82
·8	·98453	66	·01572	67	27·0	·02394	86	·97662	82
·9	·98519	66	·01503	69	·1	·02480	86	·97580	82
22·0	·98586	67	·01434	69	·2	·02566	86	·97498	82
·1	·98653	67	·01365	69	·3	·02653	87	·97416	82
·2	·98721	68	·01296	69	·4	·02740	87	·97333	83
·3	·98788	67	·01226	70	·5	·02828	88	·97250	83
·4	·98857	69	·01157	69	·6	·02916	88	·97167	83
·5	·98925	68	·01087	70	·7	·03005	89	·97083	84
·6	·98994	69	·01016	71	·8	·03093	88	·97000	83
·7	·99063	69	·00945	71	·9	·03183	90	·96916	84
·8	·99133	70	·00875	70	28·0	·03272	89	·96832	84
·9	·99203	70	·00803	72	·1	·03363	91	·96747	85
23·0	·99273	70	·00732	71	·2	·03453	90	·96662	85
·1	·99344	71	·00661	71	·3	·03544	91	·96577	85
·2	·99415	71	·00589	72	·4	·03635	91	·96492	85
·3	·99486	71	·00516	73	·5	·03727	92	·96407	85
·4	·99558	72	·00444	72	·6	·03819	92	·96321	86
·5	·99630	72	·00371	73	28°36′ 6″	·03821		·96320	

TABLE XVIII. (2)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tides O and Q. Also used in determining $1/f$ for the Tide M_1.

$$\text{Argument } 1/f = \frac{\sin \omega \cos^2 \frac{1}{2}\omega \cos^4 \frac{1}{2}i}{\sin I \cos^2 \frac{1}{2} I}.$$

Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I	Values of I	$1/f$	Differences for 0°.1 of I	f	Differences for 0°.1 of I
18° 18′ 30″	1·24126		0·80563		23°·5	0·99434		1·00569	
18·4	·23563		·80932		·6	·99073	361	·00936	367
·5	·22951	612	·81333	401	·7	·98715	358	·01303	367
·6	·22350	601	·81734	401	·8	·98360	355	·01667	364
·7	·21753	597	·82135	401	·9	·98009	351	·02032	365
·8	·21163	590	·82534	399	24·0	·97660	349	·02396	364
·9	·20581	582	·82933	399	·1	·97316	344	·02758	362
19·0	·20003	578	·83331	398	·2	·96974	342	·03121	363
·1	·19435	568	·83729	398	·3	·96635	339	·03482	361
·2	·18870	565	·84126	397	·4	·96300	335	·03843	361
·3	·18313	557	·84523	397	·5	·95966	334	·04204	361
·4	·17762	551	·84919	396	·6	·95637	329	·04563	359
·5	·17214	548	·85314	395	·7	·95310	327	·04921	358
·6	·16676	538	·85708	394	·8	·94986	324	·05280	359
·7	·16142	534	·86102	394	·9	·94666	320	·05636	350
·8	·15614	528	·86496	394	25·0	·94347	319	·05992	356
·9	·15092	522	·86889	393	·1	·94033	314	·06347	355
20·0	·14573	519	·87281	392	·2	·93720	313	·06702	355
·1	·14064	509	·87672	391	·3	·93410	310	·07055	353
·2	·13557	507	·88062	390	·4	·93103	307	·07408	353
·3	·13056	501	·88453	391	·5	·92798	305	·07761	353
·4	·12561	495	·88842	389	·6	·92497	301	·08112	351
·5	·12069	492	·89231	389	·7	·92198	299	·08463	351
·6	·11585	484	·89619	388	·8	·91902	296	·08812	349
·7	·11105	480	·90006	387	·9	·91608	294	·09161	349
·8	·10629	476	·90393	387	26·0	·91316	292	·09510	349
·9	·10159	470	·90779	386	·1	·91028	288	·09857	347
21·0	·09692	467	·91164	385	·2	·90742	286	·10203	346
·1	·09233	459	·91549	385	·3	·90458	284	·10549	346
·2	·08776	457	·91933	384	·4	·90177	281	·10894	345
·3	·08325	451	·92316	383	·5	·89897	280	·11238	344
·4	·07878	447	·92698	382	·6	·89622	275	·11581	343
·5	·07434	444	·93080	382	·7	·89347	275	·11924	343
·6	·06997	437	·93461	381	·8	·89076	271	·12265	341
·7	·06563	434	·93842	381	·9	·88806	270	·12605	341
·8	·06133	430	·94222	380	27·0	·88538	268	·12946	340
·9	·05708	425	·94601	379	·1	·88273	265	·13285	339
22·0	·05286	422	·94979	378	·2	·88010	263	·13623	338
·1	·04870	416	·95357	378	·3	·87750	260	·13961	338
·2	·04457	413	·95734	377	·4	·87491	259	·14297	336
·3	·04047	410	·96111	377	·5	·87234	257	·14634	337
·4	·03643	404	·96486	375	·6	·86981	253	·14968	334
·5	·03241	402	·96861	375	·7	·86729	252	·15302	334
·6	·02845	396	·97235	374	·8	·86478	251	·15636	334
·7	·02451	394	·97608	373	·9	·86231	247	·15968	332
·8	·02061	390	·97981	373	28·0	·85985	246	·16299	331
·9	·01675	386	·98353	372	·1	·85741	244	·16530	331
23·0	·01292	383	·98725	372	·2	·85499	242	·16961	331
·1	·00915	377	·99095	370	·3	·85260	239	·17289	328
·2	·00539	376	·99464	369	·4	·85022	238	·17617	328
·3	·00167	372	·99834	370	·5	·84785	237	·17945	328
·4	0·99799	368	1·00202	368	·6	·84551	234	·18272	327
·5	·99434	365	·00569	367	28° 36′ 6″	·84547		·18277	

TABLE XVIII. (3)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide J; and for determining k_1, used in the preparation of Table XVIII. (6).

$$\text{Argument } 1/f = \frac{\text{Sin } w \text{ Cos } w \ (1 - \tfrac{3}{2} \text{ Sin}^2 i)}{\text{Sin } I \text{ Cos } I}.$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·20958		0·82673		23°·5	0·98648		1·01371	
18·4	·20442		·83028		·6	·98329	319	·01700	329
·5	·19882	560	·83416	388	·7	·98013	316	·02028	328
·6	·19332	550	·83801	385	·8	·97700	313	·02355	327
·7	·18785	547	·84186	385	·9	·97391	309	·02680	325
·8	·18246	539	·84570	384	24·0	·97083	308	·03005	325
·9	·17714	532	·84953	383	·1	·96780	303	·03328	323
19·0	·17185	529	·85335	382	·2	·96479	301	·03650	322
·1	·16667	518	·85715	380	·3	·96182	297	·03970	320
·2	·16152	515	·86095	380	·4	·95888	294	·04289	319
·3	·15643	509	·86474	379	·5	·95595	293	·04608	319
·4	·15141	502	·86851	377	·6	·95308	287	·04924	316
·5	·14642	499	·87228	377	·7	·95022	286	·05240	316
·6	·14152	490	·87603	375	·8	·94738	284	·05554	314
·7	·13666	486	·87978	375	·9	·94459	279	·05867	313
·8	·13186	480	·88351	373	25·0	·94181	278	·06179	312
·9	·12712	474	·88724	373	·1	·93907	274	·06489	310
20·0	·12240	472	·89095	371	·2	·93635	272	·06798	309
·1	·11777	463	·89465	370	·3	·93366	269	·07106	308
·2	·11318	459	·89834	369	·4	·93100	266	·07412	306
·3	·10863	455	·90201	367	·5	·92835	265	·07718	306
·4	·10415	448	·90568	367	·6	·92575	260	·08021	303
·5	·09970	445	·90934	366	·7	·92316	259	·08323	302
·6	·09532	438	·91298	364	·8	·92060	256	·08625	302
·7	·09097	435	·91662	364	·9	·91807	253	·08925	300
·8	·08667	430	·92024	362	26·0	·91555	252	·09224	299
·9	·08243	424	·92385	361	·1	·91307	248	·09520	296
21·0	·07821	422	·92746	361	·2	·91061	246	·09816	296
·1	·07407	414	·93105	359	·3	·90818	243	·10111	295
·2	·06995	412	·93463	358	·4	·90577	241	·10404	293
·3	·06589	406	·93819	356	·5	·90337	240	·10696	292
·4	·06187	402	·94175	356	·6	·90101	236	·10986	290
·5	·05787	400	·94530	355	·7	·89867	234	·11276	290
·6	·05395	392	·94883	353	·8	·89635	232	·11563	287
·7	·05004	391	·95235	352	·9	·89406	229	·11849	286
·8	·04619	385	·95586	351	27·0	·89178	228	·12135	286
·9	·04238	381	·95935	349	·1	·88954	224	·12419	284
22·0	·03859	379	·96284	349	·2	·88730	224	·12702	283
·1	·03487	372	·96631	347	·3	·88510	220	·12982	280
·2	·03117	370	·96978	347	·4	·88292	218	·13262	280
·3	·02751	366	·97323	345	·5	·88075	217	·13540	278
·4	·02390	361	·97667	344	·6	·87861	214	·13817	277
·5	·02031	359	·98010	343	·7	·87649	212	·14092	275
·6	·01678	353	·98351	341	·8	·87439	210	·14366	274
·7	·01326	352	·98691	340	·9	·87231	208	·14638	272
·8	·00980	346	·99031	340	28·0	·87024	207	·14910	272
·9	·00636	344	·99368	337	·1	·86821	203	·15179	269
23·0	·00295	341	·99705	337	·2	·86619	202	·15448	269
·1	0·99960	335	1·00040	335	·3	·86420	199	·15715	267
·2	·99627	333	·00375	335	·4	·86222	198	·15980	265
·3	·99298	329	·00708	333	·5	·86025	197	·16245	265
·4	·98972	326	·01040	332	·6	·85831	194	·16509	264
·5	·98648	324	·01371	331	28°· 36′ 6″	·85828		·16513	

TABLE XVIII. (4)—Values of 1/f and f corresponding to various values of I, to be used in computing H and R for the Tide Mf.

Argument $1/f = \frac{\text{Sin}^2\omega \text{Cos}^4\frac{1}{2}i}{\text{Sin}^2 I}$

Values of I	1/f	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	1/f	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·59903		0·62538		23°·5	0·99238		1·00768	
18·4	·58377		·63145		·6	·98450	788	·01579	811
·5	·56720	1657	·63808	663	·7	·97667	783	·02392	813
·6	·55107	1613	·64476	668	·8	·96895	772	·03207	815
·7	·53508	1599	·65146	670	·9	·96134	761	·04026	819
·8	·51937	1571	·65820	674	24·0	·95378	756	·04846	820
·9	·50394	1543	·66496	676	·1	·94638	740	·05670	824
19·0	·48865	1529	·67175	679	·2	·93903	735	·06495	825
·1	·47375	1490	·67858	683	·3	·93179	724	·07324	829
·2	·45898	1477	·68544	686	·4	·92463	716	·08155	831
·3	·44448	1450	·69232	688	·5	·91753	710	·08988	833
·4	·43021	1427	·69924	692	·6	·91058	695	·09825	837
·5	·41607	1414	·70618	694	·7	·90367	691	·10663	838
·6	·40229	1378	·71316	698	·8	·89685	682	·11504	841
·7	·38862	1367	·72017	701	·9	·89012	673	·12348	844
·8	·37519	1343	·72720	703	25·0	·88344	668	·13193	845
·9	·36198	1321	·73427	707	·1	·87689	655	·14043	850
20·0	·34888	1310	·74136	709	·2	·87039	650	·14893	850
·1	·33610	1278	·74849	713	·3	·86398	641	·15747	854
·2	·32343	1267	·75564	715	·4	·85764	634	·16603	856
·3	·31096	1247	·76283	719	·5	·85135	629	·17461	858
·4	·29871	1225	·77004	721	·6	·84518	617	·18323	862
·5	·28655	1216	·77727	723	·7	·83904	614	·19186	863
·6	·27468	1187	·78455	728	·8	·83300	604	·20051	865
·7	·26291	1177	·79185	730	·9	·82702	598	·20920	869
·8	·25133	1158	·79919	734	26·0	·82109	593	·21789	869
·9	·23993	1140	·80655	736	·1	·81527	582	·22663	874
21·0	·22862	1131	·81392	737	·2	·80949	578	·23538	875
·1	·21758	1104	·82135	743	·3	·80378	571	·24415	877
·2	·20663	1095	·82879	744	·4	·79814	564	·25296	881
·3	·19584	1079	·83626	747	·5	·79254	560	·26177	881
·4	·18523	1061	·84376	750	·6	·78704	550	·27062	885
·5	·17470	1053	·85128	752	·7	·78158	546	·27948	886
·6	·16441	1029	·85885	757	·8	·77619	539	·28838	890
·7	·15420	1021	·86644	759	·9	·77086	533	·29729	891
·8	·14414	1006	·87405	761	27·0	·76557	529	·30622	893
·9	·13424	990	·88169	764	·1	·76037	520	·31519	897
22·0	·12441	983	·88935	766	·2	·75521	516	·32417	898
·1	·11480	961	·89706	771	·3	·75010	511	·33318	901
·2	·10527	953	·90478	772	·4	·74506	504	·34221	903
·3	·09588	939	·91254	776	·5	·74006	500	·35125	904
·4	·08663	925	·92032	778	·6	·73514	492	·36033	908
·5	·07745	918	·92812	780	·7	·73025	489	·36942	909
·6	·06847	898	·93596	784	·8	·72542	483	·37853	911
·7	·05955	892	·94382	786	·9	·72065	477	·38767	914
·8	·05077	878	·95171	789	28·0	·71591	474	·39683	916
·9	·04211	866	·95963	792	·1	·71125	466	·40602	919
23·0	·03352	859	·96756	793	·2	·70662	463	·41521	919
·1	·02511	841	·97554	798	·3	·70205	457	·42444	923
·2	·01677	834	·98354	800	·4	·69752	453	·43369	925
·3	·00854	823	·99156	802	·5	·69303	449	·44295	926
·4	·00043	811	·99962	806	·6	·68859	444	·45224	929
·5	0·99238	805	1·00768	806	28° 36′ 6″	·68852		·45239	

TABLE XVIII. (5)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide Mm.

Argument $1/f = \dfrac{(1-\frac{3}{2}\,\mathrm{Sin}^2 w)\,(1-\frac{3}{2}\,\mathrm{Sin}^2 i)}{1-\frac{3}{2}\,\mathrm{Sin}^2 I}$.

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
8° 18′ 30″	0·88401		1·13121		23° ·5	0·98904		1·01108	
18·4	·88550		·12931		·6	·99155	251	·00853	255
·5	·88713	163	·12723	208	·7	·99407	252	·00597	256
·6	·88879	166	·12512	211	·8	·99660	253	·00341	256
·7	·89046	167	·12301	211	·9	·99917	257	·00083	258
·8	·89215	169	·12090	211	24·0	1·00174	257	0·99826	257
·9	·89384	169	·11877	213	·1	·00435	261	·99567	259
19·0	·89554	170	·11665	212	·2	·00697	262	·99308	259
·1	·89727	173	·11450	215	·3	·00962	265	·99048	260
·2	·89900	173	·11234	216	·4	·01229	267	·98786	262
·3	·90076	176	·11018	216	·5	·01498	269	·98524	262
·4	·90252	176	·10800	218	·6	·01770	272	·98261	263
·5	·90430	178	·10582	218	·7	·02044	274	·97998	263
·6	·90611	181	·10362	220	·8	·02319	275	·97734	264
·7	·90792	181	·10142	220	·9	·02598	279	·97469	265
·8	·90974	182	·09922	220	25·0	·02878	280	·97203	266
·9	·91159	185	·09699	223	·1	·03162	284	·96936	267
20·0	·91344	185	·09476	223	·2	·03447	285	·96669	267
·1	·91531	187	·09252	224	·3	·03735	288	·96400	269
·2	·91719	188	·09028	224	·4	·04025	290	·96131	269
·3	·91910	191	·08803	225	·5	·04317	292	·95862	269
·4	·92102	192	·08576	227	·6	·04613	296	·95591	271
·5	·92295	193	·08348	228	·7	·04910	297	·95320	271
·6	·92491	196	·08119	229	·8	·05210	300	·95048	272
·7	·92687	196	·07890	229	·9	·05513	303	·94776	272
·8	·92885	198	·07660	230	26·0	·05818	305	·94502	274
·9	·93085	200	·07429	231	·1	·06127	309	·94227	275
21·0	·93286	201	·07197	232	·2	·06437	310	·93952	275
·1	·93490	204	·06964	233	·3	·06751	314	·93677	275
·2	·93695	205	·06730	234	·4	·07067	316	·93400	277
·3	·93902	207	·06494	236	·5	·07385	318	·93123	277
·4	·94110	208	·06259	235	·6	·07707	322	·92845	278
·5	·94319	209	·06023	236	·7	·08031	324	·92566	279
·6	·94532	213	·05785	238	·8	·08359	328	·92286	280
·7	·94746	214	·05546	239	·9	·08689	330	·92006	280
·8	·94960	214	·05308	238	27·0	·09021	332	·91726	280
·9	·95178	218	·05067	241	·1	·09358	337	·91444	282
22·0	·95397	219	·04825	242	·2	·09696	338	·91162	282
·1	·95618	221	·04583	242	·3	·10037	341	·90878	284
·2	·95840	222	·04341	242	·4	·10382	345	·90594	284
·3	·96064	224	·04098	243	·5	·10729	347	·90310	284
·4	·96290	226	·03853	245	·6	·11081	352	·90025	285
·5	·96518	228	·03608	245	·7	·11434	353	·89739	286
·6	·96748	230	·03362	246	·8	·11791	357	·89453	286
·7	·96980	232	·03115	247	·9	·12152	361	·89165	288
·8	·97214	234	·02866	249	28·0	·12514	362	·88878	287
·9	·97450	236	·02617	249	·1	·12881	367	·88589	289
23·0	·97687	237	·02368	249	·2	·13250	369	·88300	289
·1	·97926	239	·02118	250	·3	·13624	374	·88011	289
·2	·98167	241	·01867	251	·4	·14000	376	·87720	291
·3	·98412	245	·01614	253	·5	·14378	378	·87429	291
·4	·98657	245	·01361	253	·6	·14763	385	·87137	292
·5	·98904	247	·01108	253	28° 36′ 6″	·14769		·87132	

TABLE XVIII. (6)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide K_1.

$$\text{Argument } 1/f = \frac{1{\cdot}46407 \times k_1}{\left\{1 + (0{\cdot}46407 \times k_1)^2 + 0{\cdot}92814 k_1 \text{Cos}\nu\right\}^{\frac{1}{2}}} \quad \text{where } k_1 = \frac{\text{Sin}\,\omega\ \text{Cos}\,\omega\ (1 - \frac{3}{2}\text{Sin}^2 i)}{\text{Sin}\,I\ \text{Cos}\,I}$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·13424		0·88165		23°·5	0·99619	224	1·00383	226
18·4	·13142	307	·88385	240	·6	·99395	223	·00609	226
·5	·12835	303	·88625	238	·7	·99172	222	·00835	227
·6	·12532	303	·88863	240	·8	·98950	220	·01062	224
·7	·12229	302	·89103	241	·9	·98730	220	·01286	226
·8	·11927	299	·89344	239	24 ·0	·98510	217	·01512	225
·9	·11628	299	·89583	241	·1	·98293	217	·01737	225
19·0	·11329	294	·89824	237	·2	·98076	214	·01962	222
·1	·11035	295	·90061	240	·3	·97862	213	·02184	224
·2	·10740	292	·90301	239	·4	·97649	214	·02408	224
·3	·10448	290	·90540	239	·5	·97435	208	·02632	221
·4	·10158	291	·90779	240	·6	·97227	210	·02853	222
·5	·09867	287	·91019	238	·7	·97017	209	·03075	222
·6	·09580	286	·91257	239	·8	·96808	206	·03297	220
·7	·09294	283	·91496	238	·9	·96602	205	·03517	221
·8	·09011	282	·91734	238	25 ·0	·96396	202	·03738	219
·9	·08729	283	·91972	240	·1	·96194	203	·03957	220
20·0	·08446	278	·92212	237	·2	·95991	201	·04177	218
·1	·08168	279	·92449	239	·3	·95790	200	·04395	218
·2	·07889	275	·92688	237	·4	·95590	199	·04613	219
·3	·07614	274	·92925	237	·5	·95391	196	·04832	216
·4	·07340	273	·93162	238	·6	·95195	196	·05048	217
·5	·07067	271	·93400	237	·7	·94999	195	·05265	215
·6	·06796	269	·93637	236	·8	·94804	192	·05480	215
·7	·06527	269	·93873	238	·9	·94612	193	·05695	216
·8	·06258	265	·94111	235	26 ·0	·94419	191	·05911	215
·9	·05993	266	·94346	237	·1	·94228	189	·05126	213
21·0	·05727	262	·94583	235	·2	·94039	187	·06339	211
·1	·05465	262	·94818	236	·3	·93852	186	·06550	213
·2	·05203	259	·95054	235	·4	·93666	187	·06763	213
·3	·04944	257	·95289	234	·5	·93479	184	·06976	211
·4	·04687	258	·95523	236	·6	·93295	183	·07187	210
·5	·04429	254	·95759	234	·7	·93112	182	·07397	211
·6	·04175	256	·95993	235	·8	·92930	180	·07608	209
·7	·03919	250	·96228	233	·9	·92750	180	·07817	209
·8	·03669	250	·96461	233	27 ·0	·92570	177	·08026	207
·9	·03419	251	·96694	235	·1	·92393	178	·08233	209
22·0	·03168	246	·96929	232	·2	·92215	173	·08442	205
·1	·02922	245	·97161	232	·3	·92042	175	·08647	206
·2	·02677	246	·97393	233	·4	·91867	174	·08853	206
·3	·02431	242	·97626	232	·5	·91693	172	·09059	205
·4	·02189	241	·97858	231	·6	·91521	170	·09264	204
·5	·01948	238	·98089	230	·7	·91351	170	·09468	204
·6	·01710	239	·98319	231	·8	·91181	168	·09672	202
·7	·01471	237	·98550	231	·9	·91013	167	·09874	203
·8	·01234	235	·98781	230	28 ·0	·90846	166	·10077	201
·9	·00999	235	·99011	230	·1	·90680	165	·10278	201
23·0	·00764	230	·99241	228	·2	·90515	162	·10479	198
·1	·00534	231	·99469	229	·3	·90353	163	·10677	200
·2	·00303	228	·99698	228	·4	·90190	162	·10877	200
·3	·00075	229	·99926	228	·5	·90028	160	·11077	198
·4	0·99846	227	1·00154	229	·6	·89868		·11275	
·5	·99619		·00383		28° 36′ 6″	·89865		·11279	

TABLE XVIII. (7)—Values of $1/f$ and f corresponding to various values of I, to be used in computing H and R for the Tide K_2.

$$\text{Argument } 1/f = \frac{1{\cdot}46407 \times k_2}{\left\{1 + (0{\cdot}46407 \times k_2)^2 + 0{\cdot}92814 k_2 \operatorname{Cos} 2\nu\right\}^{\frac{1}{2}}} \quad \text{where } k_2 = \frac{\operatorname{Sin}^2\omega(1-\frac{3}{2}\operatorname{Sin}^2 i)}{\operatorname{Sin}^2 I}.$$

Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I	Values of I	$1/f$	Differences for 0°·1 of I	f	Differences for 0°·1 of I
18° 18′ 30″	1·33764		0·74759		23°·5	1·01137		0·98876	
18·4	·33173		·75090		·6	·00566	571	·99437	561
·5	·32518	655	·75462	372	·7	0·99996	570	1·00004	567
·6	·31871	647	·75832	370	·8	·99429	567	·00574	570
·7	·31218	653	·76209	377	·9	·98865	564	·01148	574
·8	·30564	654	·76591	382	24·0	·98304	561	·01725	577
·9	·29915	649	·76974	383	·1	·97748	556	·02303	578
19·0	·29257	658	·77365	391	·2	·97194	554	·02887	584
·1	·28608	649	·77756	391	·3	·96643	551	·03474	587
·2	·27953	655	·78154	398	·4	·96095	548	·04064	590
·3	·27301	652	·78554	400	·5	·95548	547	·04659	595
·4	·26650	651	·78958	404	·6	·95008	540	·05254	595
·5	·25993	657	·79369	411	·7	·94469	539	·05855	601
·6	·25345	648	·79780	411	·8	·93932	537	·06459	604
·7	·24693	652	·80197	417	·9	·93401	531	·07065	606
·8	·24042	651	·80618	421	25·0	·92871	530	·07677	612
·9	·23394	648	·81041	423	·1	·92347	524	·08287	610
20·0	·22740	654	·81473	432	·2	·91823	524	·08905	618
·1	·22096	644	·81903	430	·3	·91303	520	·09526	621
·2	·21448	648	·82340	437	·4	·90788	515	·10147	621
·3	·20800	648	·82782	442	·5	·90273	515	·10775	628
·4	·20157	643	·83224	442	·6	·89765	508	·11403	628
·5	·19510	647	·83675	451	·7	·89257	508	·12036	633
·6	·18872	638	·84124	449	·8	·88754	503	·12671	635
·7	·18228	644	·84582	458	·9	·88254	500	·13309	638
·8	·17590	638	·85042	460	26·0	·87755	499	·13954	645
·9	·16952	638	·85505	463	·1	·87263	492	·14596	642
21·0	·16313	639	·85975	470	·2	·86772	491	·15245	649
·1	·15682	631	·86444	469	·3	·86284	488	·15896	651
·2	·15049	633	·86919	475	·4	·85801	483	·16549	653
·3	·14418	631	·87399	480	·5	·85317	484	·17210	661
·4	·13790	628	·87881	482	·6	·84842	475	·17867	657
·5	·13161	629	·88370	489	·7	·84366	476	·18531	664
·6	·12539	622	·88858	488	·8	·83893	473	·19200	669
·7	·11915	624	·89354	496	·9	·83425	468	·19868	668
·8	·11296	619	·89851	497	27·0	·82959	466	·20542	674
·9	·10678	618	·90352	501	·1	·82499	460	·21214	672
22·0	·10060	618	·90860	508	·2	·82039	460	·21894	680
·1	·09449	611	·91366	506	·3	·81583	456	·22575	681
·2	·08838	611	·91880	514	·4	·81131	452	·23258	683
·3	·08230	608	·92396	516	·5	·80679	452	·23948	690
·4	·07624	606	·92916	520	·6	·80234	445	·24636	688
·5	·07019	605	·93442	526	·7	·79789	445	·25330	694
·6	·06421	598	·93966	524	·8	·79351	438	·26023	693
·7	·05823	598	·94498	532	·9	·78912	439	·26723	700
·8	·05326	597	·95033	535	28·0	·78477	435	·27425	702
·9	·04635	591	·95570	537	·1	·78048	429	·28127	702
23·0	·04043	592	·96114	544	·2	·77619	429	·28834	707
·1	·03457	586	·96658	544	·3	·77193	426	·29546	712
·2	·02873	584	·97207	549	·4	·76772	421	·30257	711
·3	·02292	581	·97759	552	·5	·76351	421	·30974	717
·4	·01714	578	·98315	556	·6	·75934	417	·31693	719
·5	·01137	577	·98876	561	28° 36′ 6″	·75928		·31703	

TABLE XIX.—Values of ν' corresponding to I, to determine initial argument of Tide K_1.

$$\text{Tan } \nu' = \frac{\text{Sin } \nu}{\text{Cos } \nu + 0.46407 \times k_1} \text{ where } k_1 = \frac{\text{Sin } \omega \text{ Cos } \omega \left(1 - \tfrac{3}{2} \text{Sin}^2 i\right)}{\text{Sin } I \text{ Cos } I}.$$

I	ν'	Differences for 0°.1 of I	I	ν'	Differences for 0°.1 of I	I	ν'	Differences for 0°.1 of I	I	ν'	Differences for 0°.1 of I	I	ν'	Differences for 0°.1 of I
° ′ ″	°	°	°	°	°	°	°	°	°	°	°	°	°	°
18 18 30	0.000		20.4	7.363	0.124	22.5	8.819	0.021	24.6	8.567	0.049	26.7	6.672	0.145
18.4	1.557	0.958	.5	7.487	.112	.6	8.840	.019	.7	8.518	.053	.8	6.527	.155
.5	2.515	.534	.6	7.599	.109	.7	8.859	.016	.8	8.465	.057	.9	6.372	.158
.6	3.049	.491	.7	7.708	.103	.8	8.875	.011	.9	8.408	.059	27.0	6.214	.172
.7	3.540	.406	.8	7.811	.096	.9	8.886	.010	25.0	8.349	.065	.1	6.042	.177
.8	3.946	.358	.9	7.907	.093	23.0	8.896	.004	.1	8.284	.068	.2	5.865	.187
.9	4.304	.334	21.0	8.000	.085	.1	8.900	.001	.2	8.216	.071	.3	5.678	.200
19.0	4.638	.286	.1	8.085	.081	.2	8.901	.001	.3	8.145	.076	.4	5.478	.206
.1	4.924	.274	.2	8.166	.077	.3	8.900	.006	.4	8.069	.078	.5	5.272	.228
.2	5.198	.251	.3	8.243	.070	.4	8.894	.008	.5	7.991	.085	.6	5.044	.237
.3	5.449	.231	.4	8.313	.068	.5	8.886	.013	.6	7.906	.088	.7	4.807	.256
.4	5.680	.223	.5	8.381	.061	.6	8.873	.015	.7	7.818	.092	.8	4.551	.280
.5	5.903	.199	.6	8.442	.059	.7	8.858	.019	.8	7.726	.097	.9	4.271	.292
.6	6.102	.193	.7	8.501	.053	.8	8.839	.022	.9	7.629	.100	28.0	3.979	.344
.7	6.295	.180	.8	8.554	.049	.9	8.817	.025	26.0	7.529	.107	.1	3.635	.372
.8	6.475	.169	.9	8.603	.047	24.0	8.792	.029	.1	7.422	.111	.2	3.263	.427
.9	6.644	.163	22.0	8.650	.040	.1	8.763	.032	.2	7.311	.116	.3	2.836	.553
20.0	6.807	.149	.1	8.690	.038	.2	8.731	.036	.3	7.195	.123	.4	2.283	.614
.1	6.956	.144	.2	8.728	.034	.3	8.695	.040	.4	7.072	.125	.5	1.669	1.642
.2	7.100	.136	.3	8.762	.029	.4	8.655	.041	.5	6.947	.136	.6	0.027	
.3	7.236	.127	.4	8.791	.028	.5	8.614	.047	.6	6.811	.139	28°36′ 6″	.000	
.4	7.363		.5	8.819		.6	8.567		.7	6.672				

N. B.—In the above table ν' is positive when N is between 0° and 180°, and negative when N is between 180° and 360°; thus it is necessary to observe what is the value of N, because I is always positive.

TABLE XX.—Values of $2\nu''$ corresponding to I, to determine initial argument of Tide K_2.

$$\text{Tan } 2\nu'' = \frac{\text{Sin } 2\nu}{\text{Cos } 2\nu + 0{\cdot}46407 \times k_2} \text{ where } k_2 = \frac{\text{Sin}^2\,\omega\,(1 - \frac{3}{2}\,\text{Sin}^2\,i)}{\text{Sin}^2\,I}.$$

I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I	I	$2\nu''$	Differences for 0°·1 of I
° ′ ″	°	°	°	°	°	°	°	°	°	°	°	°	°	°
18 18 30	0·000		20·4	13·963	0·265	22·5	17·422	0·072	24·6	17·463	0·078	26·7	13·931	0·291
18·4	2·810	1·738	·5	14·228	·246	·6	17·494	·068	·7	17·385	·086	·8	13·640	·309
·5	4·548	0·983	·6	14·474	·240	·7	17·562	·059	·8	17·299	·095	·9	13·331	·319
·6	5·531	·907	·7	14·714	·227	·8	17·621	·052	·9	17·204	·099	27·0	13·012	·348
·7	6·438	·758	·8	14·941	·216	·9	17·673	·047	25·0	17·105	·113	·1	12·664	·359
·8	7·196	·674	·9	15·157	·210	23·0	17·720	·035	·1	16·992	·117	·2	12·305	·383
·9	7·870	·632	21·0	15·367	·194	·1	17·755	·032	·2	16·875	·128	·3	11·922	·409
19·0	8·502	·549	·1	15·561	·189	·2	17·787	·023	·3	16·747	·136	·4	11·513	·423
·1	9·051	·528	·2	15·750	·179	·3	17·810	·016	·4	16·611	·142	·5	11·090	·470
·2	9·579	·488	·3	15·929	·168	·4	17·826	·012	·5	16·469	·157	·6	10·620	·491
·3	10·067	·454	·4	16·097	·164	·5	17·838	·000	·6	16·312	·162	·7	10·129	·529
·4	10·521	·438	·5	16·261	·149	·6	17·838	·004	·7	16·150	·172	·8	9·600	·583
·5	10·959	·399	·6	16·410	·145	·7	17·834	·012	·8	15·978	·184	·9	9·017	·609
·6	11·358	·388	·7	16·555	·136	·8	17·822	·020	·9	15·794	·189	28·0	8·408	·720
·7	11·746	·364	·8	16·691	·126	·9	17·802	·024	26·0	15·605	·207	·1	7·688	·780
·8	12·110	·346	·9	16·817	·122	24·0	17·778	·036	·1	15·398	·213	·2	6·908	·901
·9	12·456	·336	22·0	16·939	·109	·1	17·742	·040	·2	15·185	·226	·3	6·007	1·166
20·0	12·792	·310	·1	17·048	·105	·2	17·702	·049	·3	14·959	·239	·4	4·841	1·300
·1	13·102	·303	·2	17·153	·097	·3	17·653	·056	·4	14·720	·245	·5	3·541	3·483
·2	13·405	·286	·3	17·250	·088	·4	17·597	·061	·5	14·475	·268	·6	0·058	
·3	13·691	·272	·4	17·338	·084	·5	17·536	·073	·6	14·207	·276	28°36′6″	·000	
·4	13·963		·5	17·422		·6	17·463		·7	13·931				

N. B.—In the above table $2\nu''$ is positive when N is between 0° and 180°, and negative when N is between 180° and 360°; thus it is necessary to observe what is the value of N, because I is always positive.

TABLE XXI.—Showing Scales, Time corrections and Differences in height between M.S.L. and datum of soundings used on tide-predicting machine, etc.—

Name of Port	Scale	M= Scale factor	C= $\frac{M}{12 \cdot 7}$	Log. of M	A_0= Difference in height between M.S.L. and datum of soundings	Time used	Longitude of Port	Correction to L.M.T. to obtain Standard Time	Correction applied to S_2	Remarks.
PART I—(*Western Ports.*)					Feet					
1. Suez	2″ =1 foot	25·40	2	1·4048	3·73	Local Mean	32° 33′E.	Nil	Nil	
2. Perim	2″ = ,,	25·40	2	1·4048	4·48	,,	43° 25′E.	,,	,,	
3. Aden	2″ = ,,	25·40	2	1·4048	4·25	,,	44° 59′E.	,,	,,	
4. Maskat	1½″= ,,	19·05	1·5	1·2799	4·77	,,	58° 36′E.	,,	,,	
5. Basrah. R.	3″ = ,,	38·10	3	1·5809	7·03	Irāq Standard*	47° 51′E.	m s −11 24	+ 5°·70	* Irāq Standard Time or mean of the meridian of 45° 0′ E.
6. Bushire	2″ = ,,	25·40	2	1·4048	2·88	Local Mean	50° 45′E.	Nil	Nil	
7. Karāchi	1″ = ,,	12·70	1	1·1038	5·21	Indian Standard†	66° 58′E.	m s +62 08	−31°·07	† Indian Standard Time or mean of the meridian of 82° 30′ E.
8. Okha Pt. & Bet Harbour	1″ = ,,	12·70	1	1·1038	6·67	,,	69° 05′E.	+53 40	−26°·83	
9. Porbandar	1½″= ,,	19·05	1·5	1·2799	5·92	,,	69° 37′E.	+51 32	−25°·77	
10. Port Albert Victor ...	1″ = ,,	12·70	1	1·1038	5·78	,,	71° 32′E.	+43 52	−21°·93	
11. Bhāvnagar	½″ = ,,	6·35	0·5	0·8028	19·74	,,	72° 09′E.	+41 24	−20°·70	
12. Bombay (Apollo Bandar)	1″ = ,,	12·70	1	1·1038	8·23	,,	72° 50′E.	+38 40	−19°·33	
13. Marmagao	2″ = ,,	25·40	2	1·4048	3·52	,,	73° 48′E.	+34 48	−17°·40	
14. Kārwār	2″ = ,,	25·40	2	1·4048	3·70	,,	74° 06′E.	+33 36	−16°·80	
15. Beypore	3″ = ,,	38·10	3	1·5809	2·88	,,	75° 48′E.	+26 48	−13°·40	
16. Cochin	4″ = ,,	50·80	4	1·7059	1·91	,,	76° 15′E.	+25 00	−12°·50	
17. Tuticorin	4″ = ,,	50·80	4	1·7059	1·86	,,	78° 09′E.	+17 24	− 8°·70	
18. Minicoy	3″ = ,,	38·10	3	1·5809	3·17	,,	73° 03′E.	+37 48	−18°·90	
19. Pāmban Pass	4″ = ,,	50·80	4	1·7059	1·33	,,	79° 12′E.	+13 12	− 6°·60	
PART II—(*Eastern Ports.*)										
20. Colombo	4″ =1 foot	50·80	4	1·7059	1·24	,,	79° 51′E.	+10 36	− 5°·30	
21. Galle	4″ = ,,	50·80	4	1·7059	1·11	,,	80° 13′E.	+ 9 08	− 4°·57	
22. Trincomalee	4″ = ,,	50·80	4	1·7059	1·03	,,	81° 13′E.	+ 5 08	− 2°·57	
23. Negapatam	4″ = ,,	50·80	4	1·7059	1·11	,,	79° 51′E.	+10 36	− 5°·30	
24. Madras	3″ = ,,	38·10	3	1·5809	1·94	,,	80° 18′E.	+ 8 48	− 4°·40	
25. Cocanāda	2″ = ,,	25·40	2	1·4048	2·84	,,	82° 15′E.	+ 1 00	− 0°·50	
26. Vizagapatam	2″ = ,,	25·40	2	1·4048	2·61	,,	83° 17′E.	− 3 08	+ 1°·57	
27. False Point	1½″= ,,	19·05	1·5	1·2799	5·06	,,	86° 47′E.	−17 08	+ 8°·57	
28. Dublat (Sāgar Island). R.	4″ = ,,	50·80	4	1·7059	9·56	Calcutta Mean	88° 08′E.	Nil	Nil	
29. Diamond Harbour. R. ...	4″ = ,,	50·80	4	1·7059	8·94	,,	88° 11′E.	,,	,,	
30. Kidderpore. R. ...	4″ = ,,	50·80	4	1·7059	10·69	,,	88° 20′E.	,,	,,	
31. Chittagong. R. ...	4″ = ,,	50·80	4	1·7059	6·87	Indian Standard	91° 50′E.	−37 20	+18°·67	
32. Akyab	1½″= ,,	19·05	1·5	1·2799	4·16	Burma Standard‡	92° 54′E.	+18 24	− 9°·20	‡ Burma Standard Time or mean of the meridian of 97° 30′ E.
33. Diamond Island ...	1½″= ,,	19·05	1·5	1·2799	4·69	,,	94° 17′E.	+12 52	− 6°·43	
34. Bassein R. §	1½″= ,,	19·05	1·5	1·2799	5·30	,,	94° 47′E.	+10 52	− 5°·43	
35. Elephant Point. R. ...	3″ = ,,	38·10	3	1·5809	12·01	,,	96° 18′E.	+ 4 48	− 2°·40	
36. Rangoon. R.	4″ = ,,	50·80	4	1·7059	10·25	,,	96° 10′E.	+ 5 20	− 2°·67	
37. Amherst. R.	4″ = ,,	50·80	4	1·7059	10·06	,,	97° 34′E.	− 0 16	+ 0°·13	
38. Moulmein. R.	4″ = ,,	50·80	4	1·7059	5·86	,,	97° 37′E.	− 0 28	+ 0°·23	
39. Mergui	¾″ = ,,	9·525	0·75	0·9788	9·16	,,	98° 36′E.	− 4 24	+ 2°·20	
40. Port Blair	2″ = ,,	25·40	2	1·4048	3·60	,,	92° 46′E.	+18 56	− 9°·47	

N.B.—The letter R after a port denotes it is a Riverain Port. § Riverain Port but worked as an open sea port, machine times receiving corrections.

TABLE XXII.—Showing corrections to be applied to the phase angles Z or $(360° - \zeta)$ for 28th Dec. of any year in order to obtain the values on the first of each month in the following year. It will be found useful for computing the phase angles at the beginning of any month so as to enable the tidal machine to be set up and restarted from any particular month in the event of any accidental stoppage. This will be referred to again in due course in Chapter III, which deals with the Tidal machine.

Tides	Speed per mean solar hour	$\Delta\zeta$ for 96 hours = 4 days on 1 Jan.	840 hours = 35 days on 1 Feb.	1512 hours = 63 days on 1 March	2256 hours = 94 days on 1 April	2976 hours = 124 days on 1 May	3720 hours = 155 days on 1 June	4440 hours = 185 days on 1 July	5184 hours = 216 days on 1 Aug.	5928 hours = 247 days on 1 Sept.	6648 hours = 277 days on 1 Oct.	7392 hours = 308 days on 1 Nov.	8112 hours = 338 days on 1 Dec.	8856 hours = 369 days on 1 Jan.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°
$2M_2K_1$	42·9271398	+161·0	+58·8	+105·8	+3·6	−48·8	−151·0	+156·5	+54·3	−47·9	−100·4	+157·4	+105·0	+2·8
2SM	31·0158958	+97·5	+133·4	+96·0	+131·9	+143·3	+179·1	+190·6	−133·6	−97·8	−86·3	−50·5	−39·1	−3·2
MS	58·9841042	−97·5	−133·4	−96·0	−131·9	−143·3	−179·1	+169·4	+133·6	+97·8	+86·3	+50·5	+39·1	+3·2
J	15·5854433	+56·2	+131·8	+165·2	−119·2	−57·7	+17·8	+79·4	+154·9	−129·5	−68·0	+7·6	+69·1	+144·7
Q	13·3986609	−153·7	+94·9	+98·8	−12·6	−85·6	+163·0	+90·1	−21·3	−132·7	+154·3	+42·9	−30·1	+218·5
T	29·9589314	−3·9	−34·5	−62·1	−92·7	−122·2	−152·8	+177·7	+147·1	+116·5	+87·0	+56·4	+26·9	−3·7
L	29·5284788	−45·3	−36·1	+7·1	+16·2	+36·8	+45·9	+66·4	+75·6	+84·8	+105·3	+114·5	+135·0	+144·2
ν	28·5125830	−142·8	+190·6	−89·0	−115·6	−106·6	−133·2	−124·1	−150·8	+182·6	+191·7	+165·0	+174·1	+147·4
μ	27·9682084	+164·9	+93·3	+167·9	+96·3	+73·4	+1·7	−21·2	−92·8	+195·5	+172·6	+101·0	+78·1	+6·5
K_2	30·0821372	+7·9	+69·0	+124·2	+185·3	−115·6	−54·4	+4·7	+65·8	+126·9	+186·0	−112·8	−53·7	+7·4
P	14·9589314	−3·9	−34·5	−62·1	−92·7	−122·2	−152·8	+177·7	+147·1	+116·5	+87·0	+56·4	+26·9	−3·7
N	28·4397296	−149·8	+129·4	+160·9	+80·0	+36·6	−44·2	−87·6	+191·6	+110·7	+67·3	−13·5	−56·9	+222·2
M_2N	57·4238338	+112·7	−4·0	+64·8	−51·8	−106·7	+136·7	+81·8	−34·8	−151·5	+153·6	+37·0	−17·9	+225·5
M_2K_1	44·0251728	−93·6	−98·9	−33·9	−39·2	−21·1	−26·4	−8·2	−13·5	−18·8	−0·7	−5·9	+12·2	+6·9
η (Sa)	0·0410686	+3·9	+34·5	+62·1	+92·7	+122·2	+152·8	+182·3	−147·1	−116·5	−87·0	−56·4	−26·9	+3·7
2η (SSa)	0·0821372	+7·9	+69·0	+124·2	+185·3	−115·6	−54·4	+4·7	+65·8	+126·9	+186·0	−112·8	−53·7	+7·4
M_4	57·9682084	+164·9	+93·3	+167·9	+96·3	+73·4	+1·7	−21·2	−92·8	+195·5	+172·6	+101·0	+78·1	+6·4
M_6	86·9523126	+67·4	−40·1	+71·9	−35·6	−69·9	+182·6	+148·3	+40·8	−66·7	−101·0	+151·5	+117·2	+9·7
S_1	15·0000000	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
M_2	28·9841042	−97·5	−133·4	−96·0	−131·9	−143·3	+180·9	+169·4	+133·6	+97·8	+86·3	+50·5	+39·1	+3·2
S_2	30·0000000	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
K_1	15·0410686	+3·9	+34·5	+62·1	+92·7	+122·2	+152·8	+182·3	−147·1	−116·5	−87·0	−56·4	−26·9	+3·7
O	13·9430356	−101·5	+192·1	−158·1	+135·5	+94·5	+28·1	−12·9	−79·3	−145·7	+173·3	+106·9	+65·9	−0·5
2N	27·8953548	+158·0	+32·1	+57·8	−68·1	−143·4	+90·7	+15·4	−110·5	+123·7	+48·3	−77·5	−152·9	+81·3

Survey of India

THE TIDES

CHAPTER II

Tidal Observations

1. In order to obtain data for harmonic analysis, it is necessary, in the first place, to determine the heights of water at any port above some fixed mark or datum for every instant of time, for a more or less extended period, in order to obtain values of tidal constants for the purposes of prediction of tides, by means of harmonic analysis.

Practically the determination of the tidal heights at any station enables zeros of level to be fixed for purposes of survey, and affords data for the calculation of the rise and fall of the tides at a future period. The tide tables prepared by means of these data subserve the purposes of navigation.

Tide-poles.

2. Tidal heights can be read direct on a graduated tide-pole erected in the sea, but as the latter is continually being disturbed by waves, observations taken on a pole are not very accurate.

Self-Registering Tide-gauges.

The measurements can be better obtained by means of a Self-Registering Tide-Gauge of which there are several patterns. These exhibit the heights of the tides in a graphical form by means of a pencil, driven by the rising and falling water with the help of suitable mechanical contrivances, marking a sheet of paper rolled round a drum driven by clock-work. The period during which the gauges are allowed to work is five years for minor stations, as this is considered sufficient to give a fair representation of the tidal oscillations at any place, and permanently at other stations, or at least as long as the general tidal operations last, and certainly not less than nineteen years, 18·6 years being the period of revolution of the moon's nodes, which results in creating a certain tide which is expected to give valuable information with regard to the rigidity of the earth. It is of the utmost importance that as few interruptions as possible may occur in the observations, and when they do occur that they may be of short duration; otherwise the method of interpolation employed in filling up the breaks fails, and a more complicated and less satisfactory one has to be adopted.

Selection of a site for a tide-gauge.

3. The choice of a site for the erection of a tide-gauge depends so much on local circumstances that a careful reconnaissance of the fore-shore is a necessary preliminary to the selection of the best of the generally limited number of suitable positions.

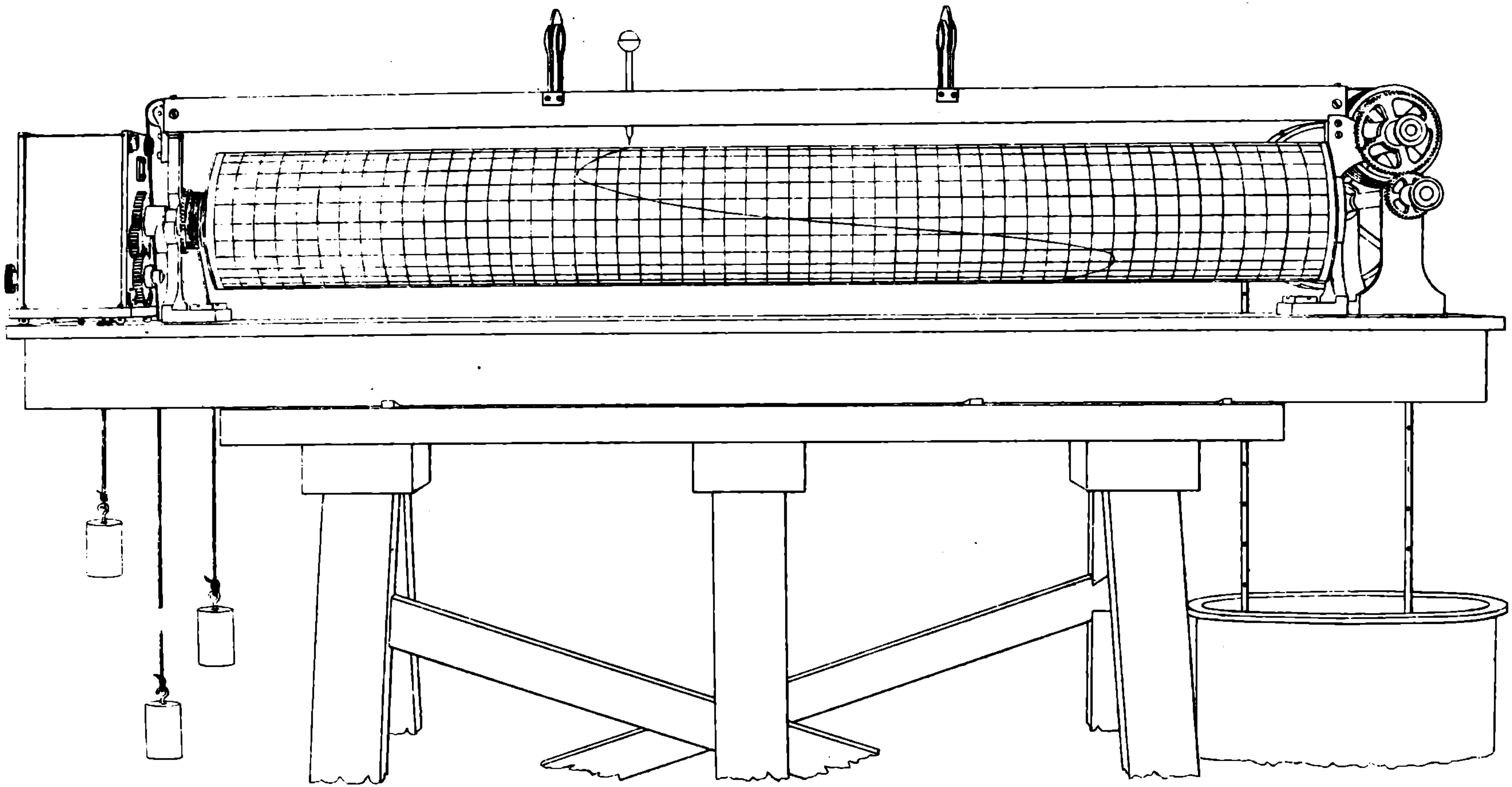
The gauge should be placed so as to obtain a fair representation of the tidal oscillations of the surrounding area, and to secure this it is necessary, (1) that the sea should have direct communication with the gauge, and not approach it through tortuous channels, (2) that the spot chosen should be sheltered from heavy weather, and (3) that there should be at least 5 feet at low-water at the cylinder. For example, a good position would be the end of a pier or jetty, or the wall of a dock. It must, however, be pointed out that a position in a cove or in a minor bay at the head of a large bay, though it may apparently answer the above requirements, is not a good one for a tidal observatory, as experience shows that, at stations where the range is small, (as in the south of India), the tidal curves, recorded at such a site, often present a zig-zag appearance all along the rise and fall.* The irregularities are certainly not caused by rough or lumpy water ; because it has frequently been noticed that they were being registered inside the float cylinder at times when the surface of the water outside was perfectly smooth, and no swell or ripple was apparent to the eye. There seems to be a slow throbbing or pulsatory action going on in such localities, during both rise and fall, which the eye does not readily detect; for instance, during a rising tide the recording pencil will remain stationary, sometimes for nearly five minutes, and then gradually fall for two or three minutes to an extent representing 2 or 3 inches in actual fall of tide, then again remain stationary for a few minutes, and afterwards move up on the rise. This will be repeated at intervals during the entire rising of the tide, and the same thing will recur in reversed order during the fall of the tide. In tidal rivers no such peculiarities have as yet been met with.

When a station has been selected near deep water, a vertical cylinder is fixed in the water in such a way as to admit it only through holes small enough to annul wave-motion and large enough to cause no sensible retardation of its rise and fall in the cylinder.

4. At several observatories communication between the cylinder and the sea was obtained by means of a connecting pipe, as explained in

* This phenomenon, however, is not confined to coves, for at Madras and Bushire, both exposed positions, the diagrams are most irregular.

NEWMAN'S PATTERN
SELF-REGISTERING TIDE GAUGE,
Constructed for the Government of India.



Chapter IV Part I, G. T. Survey Vol. XVI. This method is not to be recommended, as the pipe is liable to get blocked.

The tide-gauge which has recently been installed at Basrah by the Port Authorities, was made by Messrs Glenfield and Kennedy. It is worked on this principle of having a pipe connection with the river, which as stated above, is not a satisfactory one. Its diagrams are moreover on too small a scale for accurate readings.

5. The gauge in use at Prince's Dock, Bombay, is now only maintained by the Port Authorities for their own requirements. A full description of this pattern of instrument is given on page 16 Chapter III. Part 1, G.T. Survey Vol. XVI. This gauge has not worked satisfactorily in the past having too frail working parts and too complicated mechanism. The float end band tends to twist and the paper gets torn by the recording pencil. There are no adjustments for the height of the latter, and the diagram is on too small a scale. Measurements cannot be taken while the instrument is working, and breaks in registration are not noticed till the diagram is removed.

NEWMAN'S PATTERN TIDE-GAUGE

6. The pattern of tide-gauge used at the 7 working ports, viz: Aden, Karāchi, Apollo-Bandar (Bombay), Madras, Kidderpore, Rangoon and Moulmein, is known as Newman's pattern. Its distinguishing characteristic is that it is provided with a very long drum, whereby the curves are delineated on comparatively large scales alterable at will. It is superior to the other patterns and a description of it is given below.

General description.—To facilitate the detailed description, it will be best to begin with a general account of the instrument.

The motion due to the rise and fall of the water is directly communicated to a float partially immersed in it, and in order that the float may be freed as far as possible from wave action, it is surrounded by a cylinder into which the admission of the water is so regulated, that there is no sensible retardation between its rise and fall in the cylinder and outside. The float resting on the water inside the cylinder rises and falls with the tide, and to the float is attached a copper 'band' which passes over a wheel called the 'stud-wheel'.

The rise and fall of the 'float' communicates motion to the 'stud-wheel' by means of the 'band,' and the 'stud-wheel' in turn, by means of a projecting axle on which is fastened a 'toothed-wheel,' communicates motion to another 'toothed-wheel.' On the same axis as the latter, and consequently moving with it, is another wheel round which a flexible 'chain' is passed, one end of which is attached to the wheel and the other to the 'pencil.' The 'chain' is kept taut by a 'counterpoise weight' to ensure the 'pencil' following the movements of the 'float.'

The 'pencil' moves longitudinally along a cylindrical 'drum' touching the surface with its point; the 'drum' revolves once in 24 hours by means of 'clock-work' at the opposite end to the float. The 'drum' is supported on a cast-iron 'bed-plate' and the whole instrument on wooden trestles.

The Cylinder.

7. The size of the cylinder varies at different places, but it is generally 24 inches in internal diameter, and is usually made of thin iron plate, in sections of from 4 to 8 or even 10 feet in length, with angle-iron flanges at each end for bolting the lengths together: the bottom of the cylinder should be closed with an iron plate, while the top reaches to the floor of the observatory, or preferably a little above it, so as to be clear of the dust when the floor is swept.

The bottom of the cylinder should rest on a concrete block, to which it should be securely bolted, where water is likely to be rough. In rivers, or sheltered positions it may be suspended from stout beams fixed in the observatory floor. In both cases it should be well braced down to low-water level to the adjacent piles, with iron rods.

If the cylinder rests on the ground, the best inlet for the water is through a number of holes $\frac{1}{4}$ to $\frac{1}{2}$ an inch in diameter, near the bottom of the cylinder below the lowest level of spring-tides, but at the same time well clear of the ground on which the cylinder rests.

If it does not rest on the ground, the bottom of the cylinder is the best place for the holes.

The float band and stud-wheel.

8. The float is a cylindrical hollow copper vessel 1 foot in diameter and 9 inches deep and is of such density that it will just sink, if unsupported. The band is a copper ribbon, about 1 inch wide, perforated with holes about $2\frac{1}{2}$ inches apart.

It is attached by means of thumb screws and a plate to the head of an upright rod, (or pillar), which carries 3 small rollers so arranged that they bear on their upper surface a disc attached by three uprights to a plate soldered to the float. The rod passes through the disc, its lower end being pivoted into the plate, so that the arrangement forms a kind of swivel and prevents the band being twisted, which is most important.

The stud-wheel is of brass, about 9½ inches in diameter, with a rim an inch wide: it has studs of the same diameter as the holes in the band, placed in the rim at intervals also of about 2½ inches, so that when the band is passed round the wheel the studs exactly fit into the holes, thus ensuring the revolution of the wheel as the float rises and falls.

The band is cut to such a length that it passes over the stud-wheel and about 6 feet beyond, when the float is in its lowest position in the cylinder. To the end of the band, as a counterpoise to the float, a weight is attached, and from its bottom a copper chain is suspended, which theoretically should be equal in weight, length for length, to the copper band. The other end of the chain is attached to a hook below the float, so as to form with the band a sort of endless chain passing over the stud-wheel and reaching to the bottom of the cylinder. This contrivance is introduced in order that the pull on the float shall be constant, otherwise a systematic error is introduced between rising and falling water. The counterpoise weight should be such as to give a decided preponderance, of say 3 or 4 lbs, on the float side; but when once adjusted it should not be altered, without noting the fact in the inspection book. When the whole system of float, band and counterpoise weight is hanging in position in the cylinder, there should be 3 or 4 inches space between the float and cylinder on the one side, and the counterpoise weight and cylinder on the other.

9. The bed-plate is of cast-iron about 7 feet long, 1 foot broad and ¾ of an inch thick, the upper surface being carefully planed. Underneath this plate and cast in the same piece with it, is a web or frame work, 4 inches high, which extends to within an inch of the edge of the upper plate both at the sides and at the ends, and has diagonals or stiffeners.

The bed-plate.

10. The web rests on a wooden trestle the top of which is 5 feet long, 1 foot broad and 2 inches thick : the legs are splayed and firmly braced. The trestle is placed longitudinally in the observatory and touching the top of the cylinder at one end, being thus in such a position as to bring the stud-wheel almost over the centre of the cylinder.

Trestle.

The axle of the stud-wheel is supported on two uprights fixed to the bed-plate. The axle is about 8 inches long and carries at its other end a toothed-wheel which is in gearing with another toothed-wheel. The latter is fixed on an axle supported by two arms fastened to one of the uprights which support the drum.

The toothed-wheels.

11. The toothed-wheels are constructed in couples so as to enable the working scale on the tidal diagram to be varied at pleasure, from the natural or full scale to that of $\frac{1}{8}$th, according to the range of the tide. Six couples are supplied with each gauge, giving scales of $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$; and as the drum is 5 feet long, any tide whose range does not exceed 30 feet can be safely registered. At Bhāvnagar where the range is over 36 feet a $\frac{1}{10}$ scale was employed. In practice the couple is selected which enables the tidal curves to be exhibited on the largest scale possible.

The chain wheel.

12. The axle which carries the toothed-wheel also carries a chain wheel, round which the chain regulating the motion of the pencil-holder winds, so that the motion of the water is communicated to the pencil. This wheel is about 5 inches in diameter and its rim is spirally grooved to receive the chain which winds around it without overlapping.

The friction-rollers.

13. At about 10 inches from each end of the bed-plate there are brass uprights and on each of these a pair of friction-rollers is fixed, the pins on which the rollers turn being screwed into the uprights some 5 inches above the bed-plate.

The drum.

14. The drum which is 5 feet 3 inches long and exactly 24 inches in circumference revolves between the uprights: it is composed of sheet-brass and is made as nearly as possible a true circular cylinder. Axles project from each end and rest on the friction-rollers: one axle is elongated and, passing through an oblique slot in the upright, carries a toothed-wheel which gears with the driving clock. The position of the drum is horizontal and it carries the paper on which the tidal curve is registered.

Two grooves about $\frac{1}{16}$ of an inch deep are cut round the drum near each end and exactly 5 feet apart and a third is cut midway between the two first. The groove at the clock end is generally adopted as the *zero-line* of the gauge. When the paper is fixed on the drum, the zero, middle and end lines can be indicated on it by rubbing over the grooves with a hard pencil. The paper is nearly 5 feet 3 inches long and extends well beyond the extreme grooves.

The paper is wrapped around the drum and held in position by clips, the edges of the paper being pasted together along the length of the drum.

Arrangements for the pencil.

15. Parallel to the drum and fixed above it to the brass uprights are two bars of solid brass drawn to angle shape and between them moves a slide carrying the pencil-holder, in such a position that the pencil is exactly over the axis of the drum. The pencil-holder slide, which is a double T-shape, moves along the bars and is pushed towards one of them by means of a spring, so that it has no lateral motion. The upper flanges of the parallel bars are gripped between the springs and the upper plate of the slide for extra smoothness of motion.

The bars are prevented from buckling or having lateral motion by two arched stiffeners which are screwed on to the outer sides of the bars and allow the pencil-holder slide to pass through them.

The chain and counterpoise weight.

16. To each end of the upper plate of the pencil-slide, hooks are fixed: to the one nearest the float, a flexible chain, (see para 6), is fastened and carried from thence round the chain-wheel, to which the end of it is made fast. The loop itself forms a swivel on a cylindrical capstan-headed screw which works into the upper plate of the slide and by means of which the pencil can be set exactly to the zero of the gauge for height.

To the other loop a silver wire or a piece of whip-cord is tied which passes over a rimmed pulley at the top of the driving clock and has a weight of about 5 lbs attached to its other end. As the pencil-slide moves between the bars, the weight rises and falls and a sufficient space must be allowed for its drop.

The pencil-holder.

17. The pencil-holder is a small tube which screws into the slide and is adjusted so that it almost touches the paper on the drum. It is made to contain common leads which are pressed down on the paper from above by a weight of two or three ounces, placed in a cup provided for the purpose.

The clock.

18. The driving clocks in use are of two kinds. The 'regulator clock' with English lever escapement, (gold hair-spring), and a similar clock with a pendulum. The latter does not work well in positions where the gauge is liable to slight vibrations, in such cases the 'regulator clocks' should be employed.

The movements are boxed in by movable brass slides and the oil-cups are protected by bushes. The drum is driven by a toothed-wheel of the clock gearing with another on the axle of the drum. The arrangement for connecting and disconnecting the drum and the clock is as follows :—A clamping screw with a milled head is connected with an interior arbor or spindle, so that when the screw is clamped the driving toothed-wheel of the clock is not movable on the arbor, but when the screw is released the wheel can turn freely, and thus permit the drum to be placed in any position required. The drum can thus be set so that the pencil shows the correct time on the diagram and the clamping of the screw then brings the two toothed-wheels into connection with the rest of the wheel work of the clock.

The back-lash weight.

19. In order to prevent any back-lash which may exist between the gearing of the clock and the drum, a cord carrying a weight of about 5 ℔s, is attached to and encircles a barrel on the axle of the drum and passes over a pulley on the bed-plate. This barrel carries a pawl which drops into a ratchet-wheel on the drum: it also carries a crown-wheel which gears with a bevel-pinion. This pinion turns freely in a socket fastened to the upright which supports the clock end of the drum. The outer end of the pinion is square so that the key which winds the clock fits it. In winding the pinion the barrel is also turned, thus winding up the weight which prevents the back-lash, while any backward motion is stopped by the pawl and ratchet on the drum.

Ellipticity of the drum.

20. As the drum for the diagram may not be quite circular, the position of the pencil, as the clock shows the exact hour at four different times of the day, is marked on the diagram which is afterwards re-divided in accordance with these marks.

The auxiliary instruments.

21. No system of tidal observations can be considered complete which does not contain a continuous record of the atmospheric conditions at the station. Consequently at every tidal station, when observations were in progress, (with the exception of Bombay where the information was supplied from Colaba), the following auxiliary instruments were maintained :—a self-registering aneroid barometer to measure the momentary variations of pressure, and a standard mercurial barometer to check the aneroid from time to time and enable its index

error to be determined; also a self-registering anemometer indicating the velocity and direction of the wind at every moment, a maximum and a minimum thermometer and a rain-gauge.

These auxiliary instruments have since been removed from all the observatories. A description of the instruments, however, is given below for the information and assistance of officers inspecting at any new station at which it may be desirable to take observations in future.

Self-registering aneroid.

22. Two classes of self-registering aneroids are generally used viz:—those made by Légé and Co., and by Richard Frères, of Paris. All new instruments are of the latter type, which is much the simplest. A description of both is given below.

In Légé's self-registering aneroid barometer there are seven vacuum-chambers or boxes coupled together; the top one is attached to a screw used for setting the metallic registering pointer and to the lowest is fixed a fork with hardened steel knife-bearings. On these bearings rests, by means of knife-edges, a lever which connects the balancing-spring with the vacuum-boxes, being pivoted on other knife-edges midway between the attaching points. The vacuum-boxes and balancing-spring are placed on a brass frame.

The balancing-spring is a spiral one hooked at the bottom to the lower edge of the lever and at the top to a screw working in the upper part of the frame, by which its pull in connection with the lever is so adjusted that the reading of the instrument corresponds to that of the mercurial barometer under the particular atmospheric conditions existing at the time of its first adjustment. This adjustment, made by the maker, ought not to be altered unless the instrument has to be taken to pieces.

The amount of motion produced by the variation of the atmospheric pressure on the boxes is multiplied by the lever above-mentioned and then again by a second lever which is supported on two uprights and counterpoised. These two levers are connected together by a steel rod pointed at both ends and pivoted in conical holes out of which it is prevented from slipping by means of forks.

Projecting from the clock is a third lever of the same length as the second and attached to it by a joint whose length is half the height of the recording barrel. The joint is movable ,and at its centre a metallic pointer is fixed for the purpose of marking on the prepared paper on the barrel; the pointer can be adjusted to press more or less heavily as required. The mode of marking is as follows:—Attached to the back of the third lever there is a sliding-piece which is pulled by the clock movement three times per hour and by this motion the joint is twisted and the pointer pressed against the paper.

To the brass frame, on which the vacuum-boxes and balancing-spring are placed, there is fixed a steel tube and on this the revolving drum pivots, being maintained in position by a nut screwing in the top of the tube. The tube is hollow to admit of a turnscrew being inserted to set the recording pointer to agree with the mercurial barometer. At the bottom of the revolving drum there is a toothed crown-wheel which gears with a pinion driven by the clock. The drum revolves in 8½ days. The recording barrel, on which the specially prepared paper is fixed, rests by its own weight on the revolving drum and has a knob on the top with a hole through it of the same size as the hollow in the steel tube. The several parts are fitted on a substantial brass plate, about 21 inches long and 6 inches wide, screwed on to a board an inch thick and the whole instrument is fixed in a neat case with a glass front, the top and front being made to open on hinges.

The Richard instrument is of much simpler construction, and much less liable to get out of order. The recording pen is placed at the extremity of a long aluminium style, which is connected with the vacuum-box by means of a system of multiplying levers. The pen records the pressure on a brass drum, round which the diagram paper is wound, for which purpose a special ink is used. The drum is revolved by clock-work once in 8 days. The following instructions are supplied by Richard Frères with the instrument:—

(*a*) Open the case of the instrument.

(*b*) Fit the pen on the style: bring it away from the cylinder by means of the pivoted pin and turn the cylinder to the left so that the spring binder fixing the paper on the drum is on the *left side of the pen.*

(*c*) Take off the spring binder.

(*d*) Place the key in the opening which is closed by the milled edge brass button and wind up by turning the key to the left, (the other opening, closed by a sliding plate is only used for regulating the escapement and must not be left open). The cylinder should be maintained firmly with one hand while winding up with the other.

(*e*) Place the paper round the drum, the right end under the left one and in such a way that the latter be in a line with the two slots in which the spring binder fits, so that, when this one is fixed on again, the two ends of the paper are pressed equally and without passing the binder. Care must be taken to lay the paper perfectly flat and to let its lower edge rest on the projection at the bottom of the cylinder.

(*f*) Put the ink in the pen without filling it up. No ink must be allowed to remain on the style, especially if it is made of aluminium, as it would become corroded. If the ink overflows, the pen must be taken off, dipped in water and allowed to dry; the style must also be washed and dried, a drop of oil placed on the thin end of the style and the pen re-fitted.

(*g*) Turn the cylinder round its axis so as to make the time correspond with a clock, and if there is a standard instrument, make sure that the indication of the self-recording one is exact; if there is any difference, turn to the right or left, as may be required, the square nut which is placed underneath the case.

(*h*) Push back the pivoted pin which kept the pen away from the paper and give the pen a slight up and down motion to make sure that it writes.

In order to trace a regular diagram, the pressure of the pen on the paper must be *very slight*; to ensure this, *the instrument must be tilted forward* to an angle of about 30° to 45°. When in that position, the pen ought to lose contact with the cylinder, if it does not, the pressure is regulated by means of the milled edge knob placed at the broadest part of the style, the elasticity of which is sufficient to give the necessary pressure.

(*i*) Close the case of the instrument.

Cleaning the Pen.—When the pen is dirty, let it remain for some time in clean water and wipe it with a piece of thin linen or a fine brush. This is needed only once in every three or four months. Generally, if the pen ceases writing, it is sufficient to take it off and slip a piece of *thin paper* between its two blades.

Ordinary ink must never be used, as the admixture of a single drop with the special ink would decompose the latter.

Important Notice.—The pen-bearing style must always be *in front* of the pivoted pin which serves to keep it away from the cylinder. If, on receiving the instrument, it is found that during the transit the style has got *between* the pin and the cylinder it must be lifted up and placed again in front of the pin.

The self-registering anemometer.

23. The self-registering anemometer is fitted with Robinson's cups and steering-vanes and is about half the size of Beckley's standard anemometer. A special long protecting tube is fixed on the cross-bars carrying the cups to prevent dust or rain blowing into the bearings. The steering-vanes are made as large as possible with a sharper angle than usual, and the screw working the wheel of the indicating shaft has a quicker thread, so as to make the motion of the vane still more rapid.

The recording instrument has larger crown-wheels than usual, so as to produce an easier gearing. The driving-barrel of the clock movement is drilled through its centre and a shaft passed through the hole having at one end a pinion gearing with the reading barrel, and, at the other, in front of the clock, a milled-headed screw. This arrangement allows the registering drum to be easily set, for by loosening the screw the shaft becomes free and after the drum is set to correct time, the clock connection is made by clamping the screw.

To prevent the back-lash of the recording barrel, the pinion on the centre spindle of the driving-barrel of the clock in gearing with the recording barrel is double; one part of this pinion is fixed on to the spindle and the other is loose but attached to the fixed part by means of a circular wire spring, and, this being pressed back when gearing, the double pinion causes the tooth of the recording drum, which is in gear, to be clamped on both sides and thus prevents lost motion.

The clock has a good lever escapement with gold hair-spring and is driven by a weight; a dial showing minutes is fitted on to it.

The recording is done by the usual spiral metallic pencil marking on specially prepared paper. Spare parts are occasionally required, especially for the worm gearing in the small box just below the cups.

Sometimes it is necessary to place the anemometer at a distance from the tidal observatory in order to obtain a site freely exposed to the wind from every direction.

The observatory.

24. A tidal observatory is constructed of wood and is usually so made as to be readily taken to pieces and put together again in order to be removed and re-erected.

It is about 12 feet by 9 feet in plan and 12 feet high in the middle up to the ridge, from which the roof slopes down to the sides which are 8 feet high.

The internal fittings consist of a cupboard placed on the wall for keeping records and supporting the self-registering aneroid; a shelf near the top of the observatory for the recording gear of the self-registering anemometer and a table on which to spread out the tidal diagrams.

The graduated staff.

25. Outside the observatory and attached to a pile of the pier or to the wharf wall, a graduated staff is fixed vertically, in such a position as to be easily read, so that a comparison of the level of the water outside and inside the cylinder may be readily made by simultaneous readings of the pencil on the diagram and of the water on the graduated staff.

Bench-marks

26. Two or three bench-marks are laid down in the vicinity of every tidal observatory and connected with the bed-plate by first-class levelling *vide* Levelling chapter. They are either cut in the dock-wall, or on the steps of some neighbouring building, or are cubical blocks of masonry, about 3½ feet each way, containing a large stone imbedded on the upper face. The stone is inscribed to show that it is a bench-mark of the Great Trigonometrical Survey, and the year on which it was laid down is also given. In the centre of this stone, a square depression of 5-inch side and ½ inch deep is cut and nicely smoothed, its size being just sufficient to allow the levelling-staff to turn freely in it.

To set up and start the self-registering tide gauge.

27. The trestle is first put in position, longitudinally in the observatory. The tide-gauge is next set up on the trestle touching the float cylinder at one end in such a position that the centre of the float, the band and counterpoise weight shall all be in a diametral plane of the float-cylinder, and also so that the float and counterpoise weight shall each be about 3 or 4 inches from the sides of the cylinder.

The trestle is then wedged up so that its top is nearly level, and its legs are firmly screwed to the floor. After this the bed-plate is levelled, longitudinally and transversely, by driving wedges in between the web and the top of the trestle. The extreme range of the tide having been found roughly, either by observation on the staff or from local information, the various parts of the instrument are tested, to see that they work freely and that there is enough drop, at extreme tides, for the counterpoise weight attached to the pencil traveller.

The working zero of the gauge should be at least 18 inches below the lowest low-water on record.

The wheels to govern the scale of the diagram are now placed in gear with the float-wheel, a trial diagram put on, and the instrument approximately adjusted so that at half-tide the pencil will be at the centre of the drum. Sometimes the zero of the gauge is made to correspond with some particular level which has been taken as the datum for local surveys and the instrument is adjusted accordingly, :—*e.g.* when soundings are being taken in the vicinity and the times noted, the gauge readings at those times may be made to indicate the amounts to be subtracted from the soundings to find the distances of the bed of the sea below the datum.

Whatever be the approximate adjustments, careful measurements must next be taken to determine the distance of the water below the surface of the bed-plate when the pencil is on the zero of the gauge. In making these measurements, called 'zero measurements,' a special apparatus is employed.

Zero measurements.

28. A flat strip of brass with a right-angled band is fixed by two or three counter-sunk screws to the top of an ebony scale, divided into tenths, hundredths and five thousandths of a foot, so that when the flat piece of brass lies on the top of the bed-plate, the scale hangs vertically down; care is taken to have the under surface of the flat piece, which rests on the top of the bed-plate, corresponding exactly to the zero mark of the ebony scale.

A small circular wooden disc, 3 inches in diameter and ½ an inch thick, bevelled at its edge from the bottom towards the top, is attached by means of a brass slip fixed on the top of the disc, to the end of a Chesterman's steel tape. The slip is made of two brass plates, about 2½ inches long and 1 inch broad, one fixed vertically nearly at the centre of the disc, and the other attached to the first by four screws at its corners, so as to be removed at pleasure. The tape is held between the plates, and, when the screws are clamped, the disc is suspended by the tape. The disc is slightly loaded with lead, so that, when hanging from the tape, its under surface is quite horizontal. The distance from the bottom surface of the disc to the top of the clip, is exactly 3 inches; the 3-inch mark on the tape should, therefore, be made to correspond exactly with the top of the clip, so that the bottom surface of the disc may correspond to zero on the tape. Steel tapes should be used when possible, but, if for any reason only an ordinary metallic tape is available, it should be tested by applying it to a standard levelling staff, the disc arrangement being held suspended all the time, so as to have the tape in tension under the same condition as it would be when in use.

The measurements are taken as follows:—The ebony scale is suspended from the bed-plate above the float-cylinder, and the disc is lowered into the cylinder, care being taken to keep the tape close to the side of the scale; this is done by running off more tape than is required, passing it across the bed-plate and holding the tape on the top, so that it can be paid out easily and in very minute increments of length. When the disc is seen to be close to the surface of the water, warning is given to the clerk who is standing by, ready to mark the exact position of the pencil on the barrel. The lowering is continued very carefully, and actual contact with the water is noticed by the disc causing a tremor on the surface; at the moment of contact a signal is made to the clerk who marks on the diagram the position of the centre of the lead pencil, and the distance in feet is read off the tape, while the tenths, hundredths and thousandths are read off the scale. This distance is entered in the inspection book in one column: the measurement of the position of the pencil above the zero-cut on the drum is then carefully made, by using another scale, and entered in the next column. The latter entry is multiplied by the denominator of the fraction indicating the working scale of the gauge and entered in a third column. The sum of the entries in the first and third columns gives a value of the distance of the water from the bed-plate with the pencil on the zero-line, or, in other words, the distance of the zero of the gauge from the bed-plate.

As a rule, twenty such measurements are made and the mean taken. The measurements have to be taken both during a rising and a falling tide, (when it is well on the rise or fall), and the mean of the two sets is the adopted value of the zero below bed-plate: this eliminates the influence of the lost motion or back-lash between the two toothed-wheels connecting the stud-wheel and chain-wheel and also the error arising from looseness of the pencil in the pencil-holder. If the value of the distance of the zero below bed-plate agrees with the true zero, previously fixed upon in regard to the datum, there is nothing more to be done; but if not, the chain must be lengthened or shortened by means of the milled-headed screw until the agreement is complete; reference lines are then painted on the band at the level of the top of the bed-plate, when the pencil is at each of 3 grooves in the drum, and the gauge may then be started.

The float-end of the bed-plate should now be connected by careful spirit levelling with the bench-marks and with the graduated staff, and, if necessary, the graduations of the latter altered so that its zero may correspond with that of the gauge.

It merely remains to set up and start the meteorological instruments and the tidal observatory is in complete working order.

Duties of the clerk in charge.

29. For each observatory there is a clerk provided, who is generally one of the writers in the Port Office and receives a small increase of pay for the observatory duties; but in some cases special men had to be engaged as in the case of Port Blair.* Printed instructions are given to the clerk in charge concerning his work which should be carried out as follows:—

Hours of visiting the observatory.

30. The observatory should be visited each day at 7 and 10 A. M. and at 4 and 6 P. M. except on Sundays when two visits are considered sufficient; and also twice a month at some other hour to change the diagram.

The tide-gauge

31. The tide-gauge clock must be wound up twice a week and the back-lash weight every evening. The positions of the pencil on the barrel should be marked by a circle of ink round the pencil, on each visit to the observatory, at the exact hour, and the date of the month written alongside, so that, should the drum not be truly circular, the diagram can be re-divided by means of these hour marks. The preceding day's curve should also be inked in with one of the coloured inks supplied. Simultaneous readings of the position of the pencil on the

* The observatory at Port Blair is now closed.

diagram and of certain white lines marked on the float-band should be taken once daily and the values to 2 places of decimals entered in the report, so as to make sure that the band has not been displaced. The heights of the tidal curve for each hour of the preceding 24 hours must be carefully measured off the diagram to 2 places of decimals and entered in the report.

Tide-gauge clock

32. The clock of the tide-gauge must be compared daily with the gun or time-ball, or with a watch previously taken to the telegraph office, where standard time can be correctly obtained at 4 P. M. If the clock is in error by over a minute, the error of the clock should be noted in the report with a remark as to whether it has been corrected or allowed to stand.

The reading of the pencil on the diagram and the height of the water at the same moment on the graduated staff (if there is one) should be taken once a day and entered in the report. If there is an unmistakable difference of two-tenths of a foot when the water is calm, the cylinder should be flushed out and the communication holes examined. The diagram must have the zero, mid and end lines marked by rubbing a hard pencil over the grooves cut in the drum at these points. A line about half an inch long should be rubbed at the hours of 10 A. M., 2 P. M. and midnight in a part of the paper not marked by any curve, if possible. The marking should be done on the day after the diagram is put on and again on the day it is taken off, and the date entered against each set of marks.

Changing diagram.

33. The diagram must be changed once a fortnight, when the tide has well turned so as to make sure of getting the highest and lowest tides. The change is made as follows :—The new diagram is numbered, dated, and has a narrow slip one inch long cut out in readiness for setting the zero-line: it is damped all over with clean water and paste applied to the overlap. The hour and date that the work was stopped is noted in ink on the old diagram and the pencil-holder is taken off. The diagram on the drum is next cut carefully along the 12 o'clock line, the back-lash weight removed, the clips unscrewed and the diagram taken off, carefully rolled up and put aside. The drum and the clock are then disconnected and the new diagram put on the drum by first making the 12 o'clock line of the paper agree with that marked on the drum and the zero of the diagram with that of the drum; this can be done by means of the small slits that

have been cut out of the diagram. The clips are now screwed down and the drum turned round by hand till the outer edge of the diagram comes in contact with the 12 o'clock line and the height lines meet. The pencil-holder must then be fixed and the clock and drum clamped, care being taken that the pencil is over that part of the diagram which corresponds, as nearly as possible in time, with the time of the clock. The back-lash weight is carefully and slowly put on and the hour of commencing work noted on the diagram. It only remains to regulate the position exactly for time. This is done by selecting any convenient hour, say noon, and at one minute before, unclamping the clock-drum connecting-screw and, when the second hand shows the complete hour exactly, bringing the centre of the pencil exactly over the hour-line of the diagram and clamping very firmly, otherwise the clock may fail to drive the drum.

Stoppage of clock.

34. Any remarks regarding the stoppage of the clock, or in fact anything unusual, must be noted on the diagram and in the daily report. If the clock should stop, the weight must be removed from the pencil-cup and the pencil slightly raised; the clock-drum connecting-screw must then be unclamped, the drum being held so that the back-lash weight does not run down, revolved by hand so as to bring the pencil over that part of the diagram which corresponds as nearly as possible to about 5 minutes in advance of the correct time, and the pencil weight replaced: the clock must then be started and stopped again, when it shows the first exact hour after starting, the pencil should be brought exactly over the hour-line, the clock and the drum firmly clamped and the clock re-started when the exact hour is shown by the watch.

Displacement of band.

35. If by any chance the band should come off the stud-wheel, it should be replaced carefully by turning the wheel until the pencil is on the zero-line of the diagram and fitting the whole of the band marked with paint on the stud similarly marked; as the marks are made when the zero-line marked on the float-band is brought to agree precisely with the bed-plate, on the wheel being released, the pencil will assume its proper position.

Breakage of chain or wire.

36. Should the chain between the pencil-traveller and the float end break, the pencil-holder must be removed from the traveller, the counterpoise weight detached and the two pieces of the chain taken out and

re-rivetted. The ends must then be attached to the wheel as before, and the stud-wheel turned by hand till the 2·5 line painted on the band is on a level with the bed-plate. If the deviation of the pencil from the 2·5 line on the barrel is small, it can be set right by the adjusting screw attached to the pencil-holder; but if the deviation is more than the screw admits of, one of the change-wheels will have to be taken off, turned one or two cogs and refixed, the final adjustment being made by the screw attached to the holder.

During the whole of the operations great care must be taken that the band does not kink.

In the case of a pipe connected tide-gauge, the stop-cock must be opened every day at high-water or near as is convenient, but not if the level of the water is nearly the same as the height of the stop-cock.

Aneroid barometer.

37. The aneroid and mercurial barometers and the thermometers attached thereto should be read daily at 7 and 10 A.M., and at 4 and 6 P.M.

In the case of Légé's aneroid should the clock stop, the hand must be gently turned round till it points to the proper time as shown by the tide-gauge clock. The barrel of the diagram must then be turned until the pencil points to the proper time, but in doing this great care must be taken, otherwise the gold thread in Adie's pattern or the marker in Légé's may be broken. In the former, it should not be attempted if the clock hands are between 5 minutes to an hour and 10 minutes past the hour as the pencil marker suspended to the gold thread is at these times, either pressing or close to the barrel; and in the latter, it may be best done at 5 minutes past a full hour.

Richard's aneroid has no clock face. The point of the pen must be made to correspond on the diagram as nearly as possible with the time indicated by the tide-gauge clock.

The aneroid clock must be wound up every Monday morning and may be regulated by stopping for a few minutes if fast or pushing the minute hand forward if slow; but for one or two minutes' error it need not be altered.

The aneroid diagrams should be carefully numbered and changed every Monday morning, and the sheets should be carefully inked as they are taken off and put away.

38. The anemometer clock must be wound up every morning by pulling the cord with the small weight and thus raising the heavy weight close to the bed-plate.

Anemometer.

The anemometer diagram must be changed daily at 7 A.M. and the diagrams dated and numbered, the hour being recorded as put on at such a time, and taken off at such a time. The number of miles of wind for the last 24 hours must be entered in the report, and is obtained by counting the number of velocity lines and multiplying by 10. The diagrams must be carefully inked in daily.

The instruments should be all oiled occasionally; and in the case of the anemometer, if the direction of the wind has been steadily from one point for many days without altering, as in the S.W. monsoon, the fans of the direction gear should be turned with the hand until the vane has made one or two complete revolutions.

39. The daily reports must be made up in duplicate and one copy sent by post to the head office. Anything unusual must be marked on the diagram and noted on the back of the report, and if anything emergent is required to be done, the port officer must telegraph to the officer in charge of the tidal party.

Reports.

40. As a rule an inspection is made once a year, but sometimes oftener. Of course if any interruption has taken place, such as the removal of instruments by the port officer for safety on account of a cyclone, as has occurred more than once, or for the settlement of the observatory, thus necessitating a temporary suspension of the observations pending repairs, then an inspection should be made as soon as possible after the information has been received.

Inspection of a tidal observatory. General remarks.

The substructure of the observatory should be carefully inspected at every visit, especially when the cabin stands on piles. The verticality of the float cylinder should be tested. If the cylinder, from any cause, is no longer vertical, the float, at low-water, may come in contact with the side of the cylinder and the friction thus caused will give a false record on the diagram.

When it is intended to make an inspection, the first thing to be done is to test the accuracy of the one-foot graduations of the Chesterman's tape with which measurements for determination of working zero will be made, especially when a metallic tape is used.

The inspecting officer ought to carry in his inspection box, a copy of this hand-book and of the current tide-tables for Indian ports, and the necessary scales, measuring tape, and other instruments required at an inspection.

The inspecting officer, accompanied by a mechanic, to dismantle, clean, repair if necessary, and refit the instruments, attends to the following points when making an inspection; and at the time of inspection writes in the observatory inspection book, under appropriate heads, a report, a copy of which is forwarded to the head office. The usual heads of the report are :—

General remarks.
Bench-marks.
Details of levelling.
Self-registering tide-gauge.
Details of determination of working zero.
Auxiliary instruments.

The general remarks should contain an account of the working of all the instruments since the last inspection, and should draw attention to the manner in which the observatory clerk performs his duties, and to anything else requiring special notice.

Connection of bed-plate, bench-marks and staff.

41. On arrival at the tidal observatory, it is necessary in the first place to ascertain if any settlement of the tide-gauge has taken place, by connecting the float end of its bed-plate by spirit-levelling of precision with the bench-mark of reference, which in its turn should be similarly connected with the other bench-marks, and with the graduated staff, in order to test the accuracy of the zero of the latter. The report should give both the results and the details of the levelling, and should mention the condition in which each bench-mark and the staff is found; it should also mention whether the bed-plate is level both longitudinally and transversely.

Zero measurements and pencil and clock comparisons

42. Before cleaning the gauge, a set of measurements for the determination of working zero at a rising tide and another at a falling tide, each set to consist generally of not less than 20 measurements, should be taken; their mean will give a value of the distance of the working zero below the bed-plate. This eliminates the influence of the lost motion, or back-lash, between the toothed-wheels connecting the stud-wheel with the sheave for the wire of the pencil-slide; it also cancels the error arising from looseness of the pencil in the

pencil-holder. If it should be necessary to continue these measurements, another pair of sets should be taken before proceeding with the inspection. These measurements for determination of the working zero should be made when the tide is well on the rise and fall, and not when it is almost high or low-water. The method of taking them has been previously explained in para. 28 and, in entering them in the inspection report, it should be stated that they were made *before cleaning* the gauge.

Concurrently with the measurements for the determination of the working zero, mentioned in the preceding paragraph, a comparison of the reading of the pencil on the drum with the reading of the bed-plate on the float-band, and with the reading of the graduated staff, should be made both at a rising and falling tide, and entered in the report.

Before cleaning the gauge, its clock should be compared with the telegraph or gun time by the inspecting officer, as a check on the previous recent comparisons entered by the observatory clerk in his daily reports.

Balance of gauge and setting of pencil.

43. The balance of the gauge ought also to be tested before beginning to clean the instrument. This may be done with sufficient accuracy by raising the float completely out of the water by gently turning the stud-wheel, and then taking the reading of a spring-balance hooked, for the purpose, to one of the holes of the band on the counterpoise side. The reading of the spring-balance will give the preponderance of the float. There ought to be a decided preponderance of, say, 3 or 4 ℔s. on the float side, but the weight when once adjusted should require no alteration, as it would affect the value of the zero-line. Should the preponderance be found to have increased, it points to a probable flaw in the float sufficient to have admitted an influx of water or to a break in the counterpoise chain on the side of the counterpoise weight.

After testing the balance of the gauge, and before cleaning it, the 2·5 painted line of reference on the band should be brought to the level of the bed-plate, when the pencil should be exactly on the 2.5 or mid-line of the drum (not diagram). If the pencil be found out of this position, the discrepancy should be measured and noted.

Dismantling gauge.

44. Then, the float and band should be raised into the observatory for examination and measurement, and the time noted. This is the first step in the dismantling of the gauge, preparatory to cleaning it. The total length of

the band should be measured, also the distances from the painted 2·5 line and from the painted 0 line upon it to its junction with the float. It is often found necessary to replace an old and worn band by a new one. It is well to carefully mark off the position of the 2·5 line, from the old band on the new one, with reference to the float, so that the new band can be put on in exactly the same relative position as the old one. During the inspection of the Bhāvnagar tidal observatory in December 1887, it was found that the readings of the band and pencil agreed, but the measurements for determination of the working zero differed largely from what they ought to have been. The measurements and examination of the band disclosed that it had broken close to the float, and it was found that the observatory clerk had tried to conceal the breakage by attaching the float to the band at the place where the breakage had occurred. The float should be closely examined and, if any water is found in it, it should be repaired or renewed. In such a case it is interesting if the quantity of water which found its way into the float can be ascertained. The influx may have been sufficient to alter the balance of the instrument and raise the working zero. As an instance of this it was found that water had entered the float of the Dublat tide-gauge in November 1882, in sufficient quantity to raise the working zero 0·14 of a foot.

The dismantling of the gauge should now be completed, and it should be thoroughly cleaned and oiled where necessary, special mention being made in the report as to whether the driving clock requires cleaning or not. The reference lines painted on the band and stud-wheel at the last inspection should not as yet be obliterated. It should be mentioned specially whether or not the driving-clock required cleaning.

Refitting gauge.

45. After the several parts of the gauge are cleaned, they should be refitted carefully, the band being replaced so that when its painted 2·5 line of reference is level with the bed-plate, the pencil shall be at, or close to, the 2·5 line on the drum. The working of the band on the stud-wheel, while the pencil is being moved along the drum from its zero to the highest line of the diagram, should be tested; and it is best if this can be done at low-water so as to reduce as much as possible the chance of the band kinking. The working should be smooth, and each hole of the band should fit over the studs freely. Any hole found too tight may be enlarged slightly with a file.

The bed-plate should be made level both longitudinally and transversely if necessary, and if this operation be found, by spirit-levelling, to have altered the level of the bed-plate relatively to that of the bench-mark of reference, the alteration of level should be recorded.

46. The refitted gauge being now clean, level, and connected by levelling with the bench-mark of reference, is still unadjusted. To ascertain the amount of adjustment required, measurements for determination of the working zero at rising and falling tides should now be taken and recorded as having been made *after cleaning* the gauge. If a combination of the results of these measurements with the final level of the bed-plate make the distance below the bench-mark of reference of the working zero, thus obtained, to differ from the distance of the true zero below the same bench-mark by a quantity appreciable on the scale of the diagram, the position of the pencil must be adjusted until the working zero coincides with the true zero. It is usual to take one more pair of sets of measurements for determination of the working zero (which should be registered as having been made *after cleaning* the gauge) as a final test of the perfect adjustment of the instrument.

Adjusting gauge.

The gauge being in adjustment, the reference lines painted on the band at last inspection should be compared with the pencil readings on the drum, and if the former are found out of position, they may now be obliterated and new lines substituted for them, special care being taken in the painting of the streaks, the ***upper edges*** of which mark where the readings of the bed-plate on the band correspond with the readings of the pencil on the engraved lines of the drum.

47. The dismantling, cleaning, and refitting of the auxiliary instruments proceed hand in hand with the similar duties in connection with the self-registering tide-gauge. The auxiliary instruments are :—

Auxiliary instruments.

Standard mercurial barometer.
Self-registering aneroid barometer.
Maximum and minimum thermometer.
Anemometer.
Rain-gauge.

Of these only the self-registering aneroid barometer and the anemometer require dismantling, cleaning, and refitting, and these operations are carried out as described in the following paragraphs.

Aneroid barometer.

48. The aneroid should be compared with the mercurial barometer and its clock should be rated.

The position of the pencil-marker on the diagram should be made to agree with the reading shown on the dial, and adjusted, if necessary, by the screw; the diagrams should be examined to see if the marker is working freely; if they show a straight line, *i.e.*, no rises at 10 o'clock nor depressions at 4 o'clock, then the marker will be moving stiffly and requires cleaning.

Thermometer comparisons should be made between that on the aneroid and that attached to the mercurial barometer.

The clerk should be made to read the aneroid and mercurial barometers and the thermometer, and to set the maximum and minimum thermometers.

The diagrams should be examined, and the clerk told if the inking in has been properly done or not; the supply of blank diagrams should be noted, to see that there are plenty for future work.

Anemometer.

49. The direction of the vane with the wind, and the marking of the direction on the barrel should be tested. The upper part of the instrument should be oiled and the cups so marked that they cannot be put wrong.

The diagrams should be examined and the clock looked at, to see if new catgut or anything else is required. If the diagrams are faintly marked, the bearing of the helices should be looked at—they should be quite free. If not, their edges should be cautiously cleaned with a piece of fine sand paper.

Miscellaneous duties before closing inspection.

50. Before ending his inspection, the inspecting officer should see the observatory clerk make an accurate comparison of the tide-gauge clock, which, like all the other clocks in the observatory, should be set to standard time, as the tide-tables are now published in terms of standard time. The comparison may be made with the telegraph or gun-time, allowance being made for the difference between local and standard time where necessary, and the result entered in the report.

The clerk must also show that, in addition to being able to rate the clock, he knows how to bring it to correct time when it is fast or slow, according to his printed instructions. A comparison of the reading of the pencil on the drum with that of the band at the bed-plate, and with the level of the water on the graduated staff should be made and recorded after the tide-gauge has been put into adjustment. The inspecting officer should see that a conspicuous note is contained in the observatory report book, for the information not only of the observatory clerk but of the local official appointed to supervise him and superintend the working of the observatory, to the effect that, whenever any interruption in the working of the tide-gauge takes place, owing, *e.g.*, to the stoppage of the driving clock, hourly readings should be taken on the diagram by day and night during the interruption. If for any reason these readings can not be taken, then hourly readings of the graduated staff (the zero of which should agree with that of the gauge) should be taken by day and night and entered in the daily reports. Should this amount of frequency be unattainable, then it is indispensable that readings at high and low-water should be taken day and night and registered in the daily reports. If the cause of the interruption be of so serious a nature as to render necessary the removal of the instruments from the observatory, the promptest information should be sent to the officer in charge of the tidal operations to enable him to arrange for an inspection at the earliest possible date. The inspecting officer ends his inspection by taking a note of whatever diagrams, ink, books, pencils or other necessaries are required to be sent to the observatory.

Preparation of tide diagrams for reduction.

51. The tidal diagrams are examined and prepared for reduction in the head-quarters office in the following manner:— Vertical lines in red ink are drawn through each set of the points which have been marked by the clerk of the observatory, showing the position of the pencil when the clock indicated the exact hours of 7 A.M., 10 A.M., 4 P.M., and 6 P.M., irrespective of whether there was a clock error or not; using these lines as bases, intermediate hour lines are then drawn, which, when they cut the curve, show the position of the pencil when the clock indicated each exact hour. The daily reports are next examined to see if there are any clock errors amounting to 3 minutes or more, as compared with telegraphic time or gun signal; if there are, then (×'s) crosses in red ink are made *on* the tidal curves, the amount of error, fast or slow, being measured from the red lines which have just been drawn. The limit

of 3 minutes' error has been adopted because $\frac{1}{20}$ of an inch is the smallest distance which can be conveniently and accurately laid down in measuring along the curve, and $\frac{1}{20}$ inch = 3 minutes.

If the clock is fast, the cross is placed in *advance* of the hour-line; if slow, then *behind* the vertical time-line. Thus, suppose the clock 4 minutes *fast* at 2 P.M. the cross (×) is placed between the 2 and 3 P.M. lines at $\frac{1}{15}$ of an inch from the 2 P.M. line; if, however, the clock was *slow* by 4 minutes, then the cross (×) is put between the 1 and 2 P.M. lines at $\frac{1}{15}$ of an inch from the 2. P.M. line. This error is distributed proportionately to the time which has elapsed since the clock was last corrected. As a rule, however, there is rarely any correction of this kind required, for when the clocks are properly attended to, errors of over 30 seconds are at once corrected by the clerks.

Interrupted curves or non-recorded curves caused by the stoppage of the clock, or other suspension of the tidal registration, are carefully filled in by drawing a curve in dotted lines exactly between the two contiguous instrumental curves.

The zero-lines, to which all the measurements for height are referred, are now laid down as indicated in the rules below, in which the terms 'true zero', 'working zero,' 'accepted value of true zero,' and 'adopted level of bed-plate' have the following meanings :—

True zero.

52. The true zero is that which has been adopted in determining the datum-line for heights in the tide-tables. Its relative level with regard to the bench-mark of reference is fixed. As a rule, the zero corresponds to that originally adopted when the gauge was started, and its distance below the bed-plate was determined when the level of the bed-plate with regard to the bench-mark was fixed.

Working zero.

53. The working zero is the level of the water with reference to the bed-plate, corresponding to the pencil being on the zero-groove cut on the drum. In starting the instrument the working zero of course corresponds to the true zero, but from various causes the instrument may get out of adjustment, and its working zero may be altered. The position of the working zero on the diagram is always marked by the clerk rubbing over the groove cut in the drum with a hard pencil. In general at an inspection, the working zero is made to agree with the true zero by adjusting the instrument.

54. The accepted value of the true zero is the distance of the true zero from the bed-plate, which was determined when the bed-plate was fixed as regards its relative level with the bench-mark of reference.

Accepted value of true zero.

55. The adopted level of the bed-plate means the level of the bed-plate with reference to the bench-mark, which has been adopted in determining the true zero; as a rule this will correspond to the level obtained when the observations commenced.

Adopted level of bed-plate.

56. The inspection book must first of all be examined to see if the bed-plate has altered in level relatively to the bench-mark. If there is any difference from the adopted level exceeding ·02 of a foot, a correction will have to be applied on this account. The measurements for the determination of the *working zero* at the various inspections are next examined. If no alteration has been made in the adjustment of the gauge during the inspection, then the whole of the sets of measurements should be grouped, and the mean value would represent the distance of the working zero from the bed-plate on the day of the inspection.

Rules for fixing true zero on diagrams.

If an adjustment has been made during an inspection, then those measurements for determination of zero before and after adjustment must be grouped separately, and the means respectively applied to the preceding and the following diagrams.

In treating the diagrams for any period between two inspections, the distance of the working zero from the bed-plate must be taken as the mean of the values obtained at the inspections.

The following are the cases which may occur and the ways of adjusting for them :—

I. *Bed-plate settled below adopted level.*—The true zero will have to be placed above the working zero at a distance proportioned to the amount of the settlement in accordance with the scale of the diagram; hence the measurements from the true zero will be *less* than from the working zero.

II. *Bed-plate raised above adopted level.*—In this case the true zero will be placed below the working zero.

III. *Bed-plate unaltered and working zero at greater distance from bed-plate than accepted value for true zero.*—The true zero in this case will be placed *above* the working zero.

IV. *Bed-plate unaltered and working zero at less distance from bed-plate than accepted value for true zero.*—In this case the true zero will be placed *below* the working zero.

V. *Bed-plate settled and working zero at greater distance from bed-plate than true zero.*—In this case the true zero would be placed *above* the working zero at a distance equal *to the sum of the corrections* on account of each event.

VI. *Bed-plate settled and working zero at less distance from bed-plate than true zero.*—If the correction for settlement is the *greater of the two* the true zero will be placed *above* the working zero, and if the correction on account of the difference of zero-measurements was the greater, then the true zero should be placed *below* the working zero. Obviously the amount in each case would be the difference of the two corrections.

VII. *Bed-plate raised and distance of working zero from bed-plate less than that accepted for true zero.*—In this case the true zero would be placed *below* the working zero at a distance equal to the sum of the two corrections.

VIII. *Bed-plate raised and distance of working zero from bed-plate greater, etc.*—The true zero should be placed *below* the working zero if the correction on account of the raising is the greater, and *above* if the latter correction is the greater. The distance between the two zeros is the difference of the corrections.

N.B.—If the determination of the true shows that the working zero comes within 0·005 of the true zero *on the diagram*, then no correction is considered necessary, and the working zero is used as the line of reference in measuring the heights from the diagram. What is meant by being within 0·005 *on the diagram* is the actual difference between the true and working zeros reduced to scale.

Water getting into the float or a break in the counterpoise chain on the side of the counterpoise weight would have the effect of making the working zero *nearer* the bed-plate than the value formerly obtained, and this would have to be treated under IV, VI, or VII, according as the bed-plate had remained unaltered, had settled, or had been raised.

A break in the counterpoise weight on the float side.—This has the effect of lowering the working zero and is treated under III, V, or VIII, according as the bed-plate is unaltered, has settled or has been raised.

A kink in the band.—If this occurred, and zero-measurements were taken, it would have the effect of showing the zero so determined as being nearer the bed-plate than it would be if the kink were removed, and if the band righted itself in the course of working, the determination for zero at next inspection would be at a greater distance from the bed-plate than formerly.

Information to be recorded in the book entitled 'Determination of the True Zero on the Diagram' should be somewhat as follows:—

(1) Level of bed-plate with reference to B. M. unaltered, or settled by . . . . or raised by . . . .

No correction necessary, or correction under rule equal to . . . has been applied to all diagrams from . . . . to . . . .

(2) Distance of working zero from bed-plate at . . . . of inspection of . . . . 192 }	feet	= . . . . .
Distance of ditto ditto at . . . . . of inspection of . . . . 192 }		= value from . . to . . . .

Correction on account of (1) or (2) or (1) and (2) = ... applied, and the true zero has been placed . . . . $\frac{\text{above}}{\text{below}}$ the working zero from . . . . to . . . .

No other inspection having taken place, the value of the working zero at the inspection of . . . 192 , as given above, has been used in determining the true zero for the remainder of the diagrams, and for these diagrams the true zero has been placed . . . $\frac{\text{above}}{\text{below}}$ the working zero in accordance with rule No. . . . . . .

Cases may occur which will have to be specially treated. All ordinary cases are here dealt with.

Intermediate lines, generally about 6 inches apart, are now laid down in red ink parallel to the true zero-line to facilitate the measurements. These are made with paper scales differently divided, according to the scale which may be adopted for the tidal diagram in each instance.

Survey of India

THE TIDES

CHAPTER III

The Tide Predicting Machine

1. This machine was constructed by Messrs Légé and Co. for the Indian Government under the supervision of Mr. E. Roberts in 1879 on principles suggested by Lord Kelvin. It was brought to India in October 1921 and erected at the office of the Superintendent of the Trigonometrical Survey, Dehra Dun.

(A full description of this instrument is given in Chapter VIII, Volume XVI of the G. T. Survey of India).

Its object is to predict the tides for any port for which the tidal constituents have been found from the harmonic analysis from tide gauge observations, not merely to predict the times and heights of high and low-water, but the depths of water at any and every instant, showing thereby a continuous curve, for a year or any number of years in advance. As already explained in para 16 of Chapter I, the prediction of tides depends on the re-composition, or synthesis, of the partial constituent waves into which the aggregate tide wave has been resolved by harmonic analysis. The machine has been designed so as to avoid the labour which would be necessary to obtain the results by direct computation.

Principle of Machine.

2. Its mechanism depends on the following principles by which simple harmonic motions can be compounded in one line.

If any number of pulleys be so placed that a cord passing from a fixed point half round each of them has its free parts all in parallel lines, and if their centres be moved with simple harmonic motion of any ranges and any periods in lines parallel to those lines, the unattached end of the cord moves with a complex harmonic motion equal to twice the sum of the given simple harmonic motions.

As the movement of the pen is equal to double of the amplitude owing to the cord passing over the pulleys, the factor used in computations are so arranged that they allow for this, and give half.

If therefore a hanging pen-box, consisting of a brass ink-bottle with a pen, be attached to the cord, the pen will trace a continuous curve on a long band of paper moved horizontally across the line of motion of the pen by a vertical cylinder geared to the revolving shafts of the machine.

By this method a curve is obtained giving the heights and times of the tides for every day in the year from which measurements can be made horizontally and vertically for the times and heights to be recorded in the tide-tables.

Heights also can be read direct by attaching a wooden scale graduated to feet and inches in front of the pen guide, and merely estimating the heights of high and low-water as a fixed line on the pen-box at the same level as the pen reaches the top and bottom of its run. This method is very tiring to the eye and neck, and is also liable to error through parallax when reading the scale, and is therefore not recommended.

Reverse curve.

3. As at times there is a difficulty in estimating to which precise point the horizontal or time measurements for high or low-water on these curves should be made when they become flattened, the machine can also be set (by changing the phase angles of the components by 90°) to run a reverse curve, on which the intersections of the curve on the mean-sea-level line represent the times of high and low-water.

Chronograph method.

4. For time predictions a new Chronograph method has been introduced, involving the use of an electrical contact made by a small wheel substituted for the pen on the pen-box on a suitable contact strip fixed at the mean-sea-level. This contact is electrically recorded on a chronograph drum, which is revolved by means of the S_1 mean solar diurnal component on the machine.

The machine being set to run a reverse curve, as above, a chrono-time chart is obtained, on which each separate line, of about 22 inches in length, represents a day's motion, and the clicks thereon represent alternately the times of high or low-water. The chrono. drum is set to commence and end at noon standard time for each particular day, and the noon line is recorded by means of a click made by recording a contact on the drum at the commencement and end of a year, or any other period of observations. If a line be ruled between the first and last click so made, the noon line is obtained, and by sub-divisions the 6 hour lines are ruled in from which the measurements are taken.

By this method it is possible to measure the predicted times of high and low-water more accurately on the greater length of paper allotted to each day viz :—22 inches of paper as against 6 inches on

ROBERTS'S TIDE-PREDICTING MACHINE.

CONSTRUCTED BY MESSRS. A. LÉGÉ AND CO., LONDON.

FIG.1.

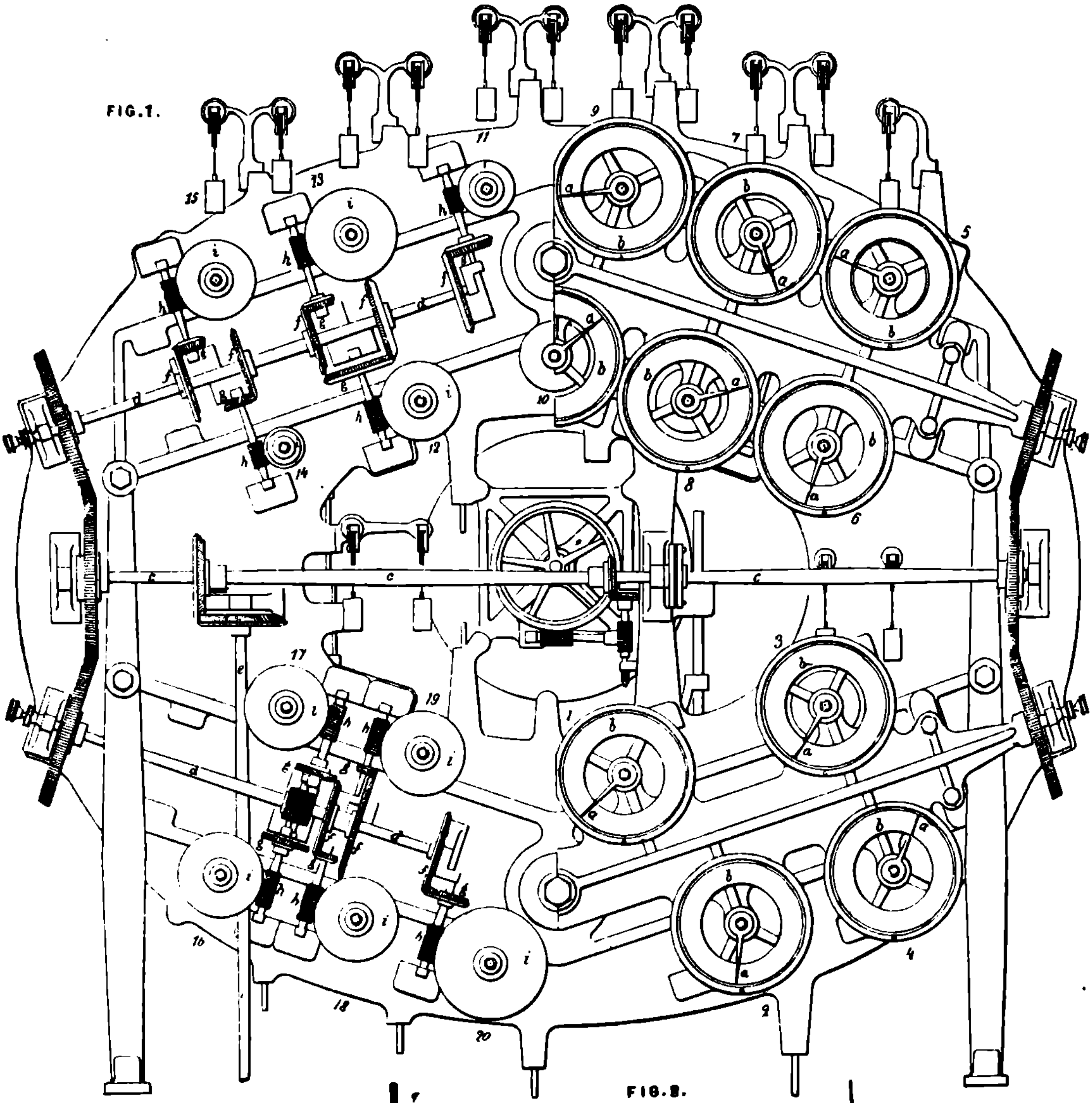


FIG.2.

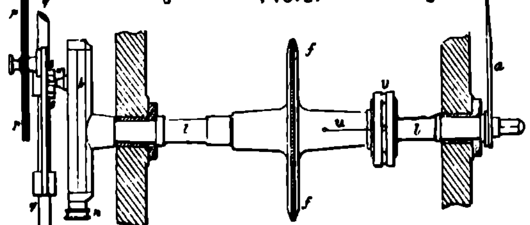
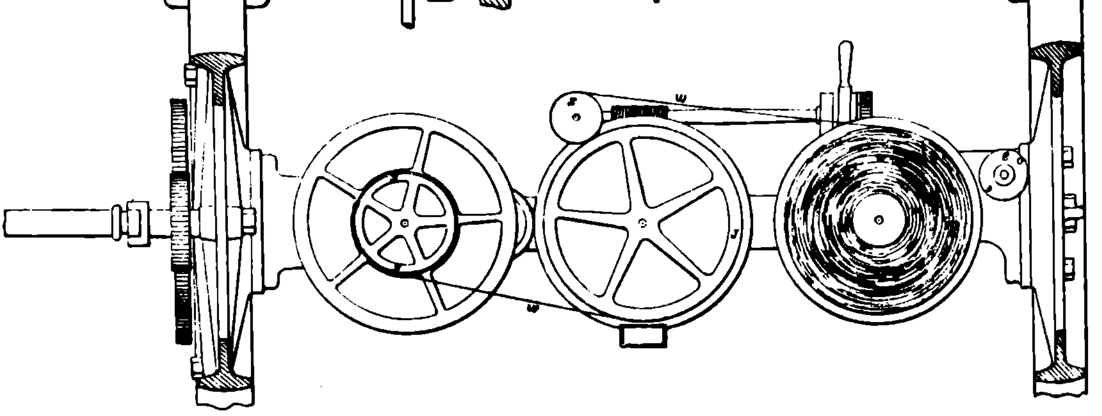


FIG.3.



the ordinary tidal curve. Moreover the exact times of high and low-water are directly registered and have not to be estimated. It is hoped therefore that, by these means, the time predictions may be improved.

This renders the ordinary system of checking proofs, (and the measurements and copy from which they have resulted), quite simple by the method of reading the differences of times of alternate high waters, as the differences, which are obtained from these more exact measurements, are smooth, and no smoothing of times is necessary, as used to be the case with the old method.

5. A further development of the chronograph method has lately been suggested by Dr. J. de Graaff Hunter M. A., Sc. D., F. Inst. P., who has devised a similar arrangement with multiple contacts corresponding to successive values of ordinate, which records the main character of the ordinary height-time curve. This arrangement indicates the times the predicted height of water reaches the various selected heights and forms in itself an ideal tide-table, such as is mentioned in Darwin's Tides, 2nd Edition, p. 200. Copies of such a table could be vandyked or reproduced for issue to navigators, who would have to read off the information for themselves. For the present however we are concerned with the preparation of ordinary tide-tables giving the heights and times of high and low-water only. These can be obtained, precisely as explained in para 4, from a chrono-chart, without the necessity for running a separate height chart, the whole record being obtained on 2 pieces of paper each only 22″ in length, as against 100 to 200 feet of paper required for the old 3″ and 6″ height-time charts.

6. We will now more particularly describe the machine with reference to the illustrations. The machine consists of a plate of metal of oval form measuring about 3 ft. 8 in. wide by 3 ft. deep, supported on two standards. Upon the plate supporting the crank axes of the different movements are fitted the whole of the guide pillars and bars of the parallel slides—hereafter described. At the back of the plate, and distant from it some 6 in., are bolted two skeleton plates—Fig. 1—carrying the other ends of the crank axes, which are provided with pointers *a* for setting, and also dials *b* divided into degrees, or to 360ths of the period of the tide component. Between the plates are a horizontal main shaft *c*, and four oblique shafts *d*, turning in the same time, the oblique shafts being driven through the main shaft, two from each end.

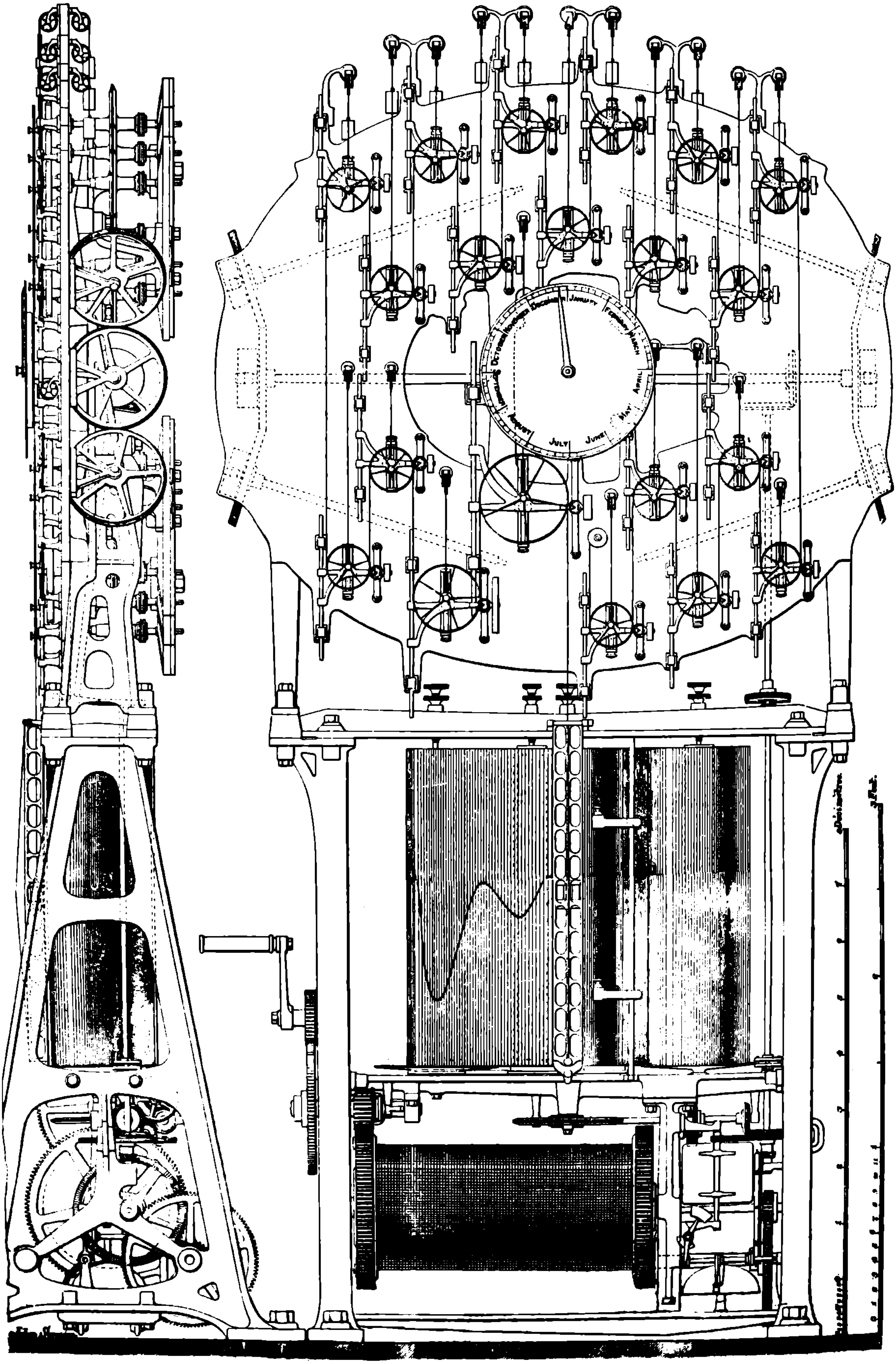
7. The machine is driven by an electromotor, the main shaft receiving its motion from the driving gear through the approximately vertical shafts *e* pinned together. Rivetted to the oblique shafts are the bevel wheels *f*, which are geared with other bevel wheels *g*, provided on their axes with endless screws *h* working into wheels *i* on the crank axes of the several components. The clock-driving gear also gives motion to the centre recording barrel *j*—Fig. 3—and through it to the receiving drum barrel. The tracing point moves vertically up and down in the frame immediately in front of the recording barrel.

8. The chief difficulty in the construction of the machine is the finding, within reasonable limits, of proportions which represent with sufficient accuracy the periods of the respective tide components, in order that the machine may be used for a considerable period of prediction—say for twelve months' tides. Very great success has been attained in this respect in the present instrument. For instance, the error of the period of the chief component—the mean lunar semidiurnal—relatively to the mean solar semidiurnal, is inappreciable during a whole year's predictions, amounting to about 0·10 deg. only in a period of fifty years. The greatest deviation from strict accuracy is 0·37 deg. after a run representing twelve months. This is, however, one of the smaller components, and insensible in its effect. This part of the design may therefore be regarded as practically perfect.

9. A crank *k*, Fig. 2, is fitted to the axis *l* of each component. A sliding piece, carrying a steel guiding pin *m* for setting, is fitted in each crank. The guiding pin is thrown out from the centre of the crank axis by means of a fine-cut screw and micrometer head *n*. The requisite distance of the throw of the guiding pin must be previously determined by the proper analysis of tidal observations of the port for which the predictions are required. It may be here remarked that a year's tidal observations will yield fairly good constants, but a longer period of observations is very desirable. A horizontal crosshead *o* carries at its centre a very light and well-balanced pulley *p*. The crosshead is fitted at one end with an adjustable steel rod *q*, moving freely in two pillar guides, drilled out nearly their entire length to reduce the touching parts to a minimum. The opposite end of the crosshead carries a projecting fork, which travels with freedom on either side of a narrow, flat brass bar supported on pillars. The steel rod of the crosshead is

ROBERTS'S TIDE PREDICTING MACHINE.

CONSTRUCTED BY MESSRS. A. LÉGÉ AND CO., LONDON.



balanced by an adjustable sliding weight, fitted in continuation of the projecting fork, so that the centre of gravity of the crosshead and guide is in a vertical through the centre of the axis of the pulley. The crosshead thus balanced is counterpoised by a cord and weight passing over pulleys at some distance above it, in a vertical through its centre of gravity and the axis of the pulley. At the back of the crosshead are fitted two parallel steel jaws *s*, the lower one adjustable, in order that the distance between them can be regulated. Both the brass bar guide and the steel rod guide are divided to millimetres; the brass bar guide for approximate and the steel rod guide for the accurate adjustment of the throw of the crank pin, for which purpose the upper pillar guide of the steel rod is furnished with a vernier.

10. The head of the micrometer *n* is also divided and may be used with the divisions of the brass bar guide. The pulley crosshead is movable on its steel rod for the perfect adjustment of the pulley about the centre of motion of the crank axis. A fine toothed wheel *f* is fitted on a slotted cone *u*, which can be clamped to the crank axis by means of a screw nut *v*. This contrivance is necessary, because each of the components requires to be set in its proper position, previously determined by calculation at the commencement before starting the machine. The setting dials *b* are toothed round their outer edges and movable round their centres by a pinion for setting.

11. A fine flexible wire fixed to a large screw-head, a little to the right hand at the bottom of the date-dial in the centre of the machine—see the illustration—passes alternately under and over the pulleys of the lower and upper series of components, till, leaving the large pulley, it hangs vertically above the ink-recorder slide. The wire carries at its free end an ink-bottle fitted with a fine brass recording pen. The ink-recorder travels in a geometrical slide, and is suspended to give just sufficient pressure to ensure contact with the paper of the recording barrel. The recording barrel is fitted with brass pins at equidistant intervals, to form the time indications on the paper by perforation.

The time lines are ruled up by hand through the machine perforations, when the traced paper has been removed from the machine, a whole year's rulings only occupying about an hour and a half.

12. An index for setting the recording barrel to time is fitted behind the framework near the upper axis of the barrel. The paper, which

is continuous, and supplied from a reel *w*, Fig. 3, passes round two grooved rollers *x* at the back of the recording barrel, and is held in position whilst the pins enter the paper, and after receiving the traced curves, is wound round the receiving barrel *y*. The receiving barrel rests on toothed driving wheels, which are driven by the recording barrel, and by friction turns and slips to accommodate itself to receive the recorded paper.

13. The machine used to be run when in England by means of a small water motor and a falling weight. As an electromotor is now used the lower part of the machine viz :—the cord barrel, clockwork gearing, warning bell etc ; are no longer utilised but they are left in position, as their weight serves to stabilise the machine.

14. The setting of the machine for the prediction of any port for which the tide components are known is as follows :—The dials *b* are first turned so that the epoch or time of maximum is exactly under or above the highest or lowest point, according as the component is situated on the upper or lower row of components.

The cranks *k* are thus set vertically—and the setting of the amplitudes or rather half amplitudes of the components is carried out with all the screwheads of the micrometer boxes on the front of the machine turned downwards, the slotted cone *v* of the wheel *f*, on the axle *l*, having been first released—and the guide pin *m* thrown out to its proper range according to scale required to represent the half amplitude of the component.

The setting of the dial pointers on the back of the machine having been determined previously by calculation for the time of starting, the dial pointers are set and the slotted cones are tightened up. The recording barrel is then set to time and the wheelwork set in motion. The complete setting only occupies a few minutes.

15. The date dial in the centre is to show the progress of the record, which can be marked occasionally to facilitate the entry of the dates after the record has been removed from the machine. Noon, midnight, &c., are distinguished from the perforations of the other hours of the day by a few supplementary pins. Two speeds of travel can be given to the paper, viz, 1 in. and ½ in. per hour. A fixed vertical rod near the ink-recorder slide carries ruling pens for the tracing of base lines, such as docksills, river bars, or mean tide levels. If desired the paper can thus be ruled its entire depth to represent feet, metres, &c., as it passes through the machine.

In practice it has been usual to mark two horizontal lines only—one representing Indian spring low water mark as defined in the tidal reductions and the other mean sea level.

16. An idea of the saving effected by the machine may be gathered from the fact that it would take a computer not less than 5 or 6 months to obtain the results by computation which can be obtained by measurement in a few days from the curves run on the machine. The machine actually runs off the tidal curve for one port for height and time in about 4 hours. If height and time are done separately 8 hours will complete the two curves required.

17. The value of the machine will be very great in any work where the whole tide-curve is of service, and will be of great value in engineering works in which a foreknowledge of the tides is necessary, such as in constructing the foundations of quay walls, embankments, dock sills, &c., the whole time during which the work can be prosecuted being seen at a glance for every tide.

NOTES REGARDING ACCESSORIES REQUIRED FOR THE TIDE-PREDICTING MACHINE.

18. This should be reeled on stout strawboard centres of 2 inches internal diameter and not less than $\frac{1}{4}$ inch in thickness. The rolls should contain from 800-850 continuous feet of paper, which is sufficient for 4 curves on the 6-inch or 8 curves on the 3-inch scale. The outside diameter of the roll should not exceed 8 inches, therefore if thick paper is used it may be necessary to make rolls large enough to run off only 3 curves on the large scale, (about 600 feet of paper). The width of the paper should not be more than 22 inches nor less than $21\frac{7}{8}$ inches. The last supply was obtained through H. M. Stationery Office and previously from Messrs John Dickenson, Old Bailey, London.

Paper.

19. The wire originally used was brass wire 0·006 inch diameter. The wire used in England prior to the shipment of the machine to India was phosphor-bronze 0·005 inch diameter. This wire was tried in India but was found very brittle and easily snapped. It was moreover difficult to put on the machine, for the same reason. A very strong woven fishing line was tried but proved too extensible for the purpose.

Wire.

A satisfactory wire has since been obtained for the machine, which is in use at present.

20. Cord. The counterpoise cord is silk eye-glass cord.

21. The old pens with glass points were supplied by Messrs Légé.

Pens.

As these glass points were continually breaking new brass points were made locally, which have given far finer lines and cleaner curves.

22. Ink made from red Eosine powder dissolved in water is satisfactory, about one table spoon of glycerine being mixed with each pint of solution. Too much glycerine causes a thick slow drying trace.

Ink.

23. These scales which were supplied by the National Physical Laboratory, Teddington, England are 12.025 inches in length (2 days) divided into 2 equal parts of 24 hours. Each hour is divided into 10 parts measuring 6 minutes. It would have been better had these been 6 parts of 10 minutes. Also the scales were made 12·025 inches in length instead of 12 inches to allow of the use of a rather thicker paper, which, when stretched round the recording drum of the machine had a slightly larger circumference than paper of the normal thickness. The scale in use is of boxwood with ivory inlaid scale glued to the edges of the wood. A finer edge on plain boxwood would be an advantage. As the method of measuring times direct from the height chart is not much used now, there is no necessity for replacing this scale.

Scales for time measurement direct on height chart.

24. The scale used in England was a metal scale 24 inches in length (8 days) divided into 16 parts marked alternately A and M, (afternoon and morning), each 'A and M' is divided into 12 parts each equal to 1 hour and again into subdivisions of ½ an hour.

Scale for measuring times for riverain ports.

Two set squares were also necessary for use with the above scale.

Instead of this scale and the set squares which were used in England celluloid scales have been prepared on the 24 hours system, each consisting of 2 scales jointed at mean sea level. The horizontal scale is set in proper position to read the height and time corrections on the vertical scale (which itself consists of 2 scales actually set at 6 hrs. apart by scale). The reasons and use of this scale are explained in paras 110 and 129-132 of Chapter I.

25. Twelve wooden scales 1 foot long have been made for this purpose with central portion 5·51 inches equivalent to 6 hours in time with primary divisions to show single hours, one end being divided into secondary

Scale for measuring chrono-sheet.

divisions to show 10 minutes, with subdivisions to 2 minutes. The length 5·51 was adopted instead of 5·50 to allow for the normal thickness of the paper, which, when stretched on the chronograph drum has a slightly larger circumference than the drum itself.

Scales for height measurement.

26. For height measurements the following scales are used ½, ¾, 1, 1½, 2, 3 and 4 inches to 1 foot, divided into equivalents of feet and inches.

The following list shows the scales used for the various ports :—

½ inch. Bhāvnagar.

¾ inch. Mergui.

1 inch. Karāchi, Okha Point, Port Albert Victor, Bombay Apollo Bandar.

1½ inch. (Maskat), Porbandar, False Point, Akyab, Diamond Island, Bassein.

2 inch. Suez, (Perim), Aden, Bushire, Marmagao, Kārwār, Cocanāda, Vizagapatam, Port Blair.

3 inch. Basrah, Beypore, (Minicoy), Madras, Elephant Point.

4 inch. Cochin, Tuticorin, Pāmban Pass, Galle, Colombo, Trincomalee, Negapatam, Dublat, Diamond Harbour, Kidderpore, Chittagong, Rangoon, Amherst, Moulmein.

Forms.

27. No. 1 Tid Pred. Computation form for setting and checking machine for height or height and time combined.

No. 2 Tid Pred. Computation form for setting and checking machine for time only, (Chrono-method).

No. 3 Tid Pred. Computation form of height and times for Riverain port predictions.

No. 4 Tid Pred. Form for entering tide-tables for open sea-ports for press.

No. 5 Tid Pred.	do	do	for Hooghly R. ports.
No. 6 Tid Pred.	do	do	for Bassein R. ports.
No. 7 Tid Pred.	do	do	for Rangoon R. ports.
No. 8 Tid Pred.	do	do	for Moulmein R. ports.

NOTES ON TIDE MACHINE AND TIDE PREDICTING.

Erection.

28. Before attempting to lift the top portion of the machine, the dials and main drive shafts should be removed, in order to lighten it sufficiently for four coolies to lift.

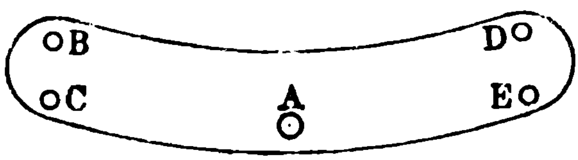
Two sound wooden poles 6 feet long 3" diam. should be lashed firmly to the frame, and their ends tightly secured to the top dial bracket, so that when these poles are raised there is no danger of over-turning.

The top portion should be raised from the floor on to the base of the machine in two stages ; first from the floor on to two high stools, then the coolies can get their shoulders under the poles and carry the top portion straight forward on to the base.

Two holding down bolts should be put in at once, but, before all four can be fitted, it will probably be found necessary to slightly loosen the bolts which attach the brass plate to the top frame.

Put on the main drive shafts and dial plates. Before tightening up the dial plate, remove all the worm shafts, so that the crank spindles can be swung round freely by hand.

The dial plate is fixed by five bolts thus :—



There is a certain amount of play in the bolts B, C, D, E, so that the dial plate can be rocked slightly about the bolt A.

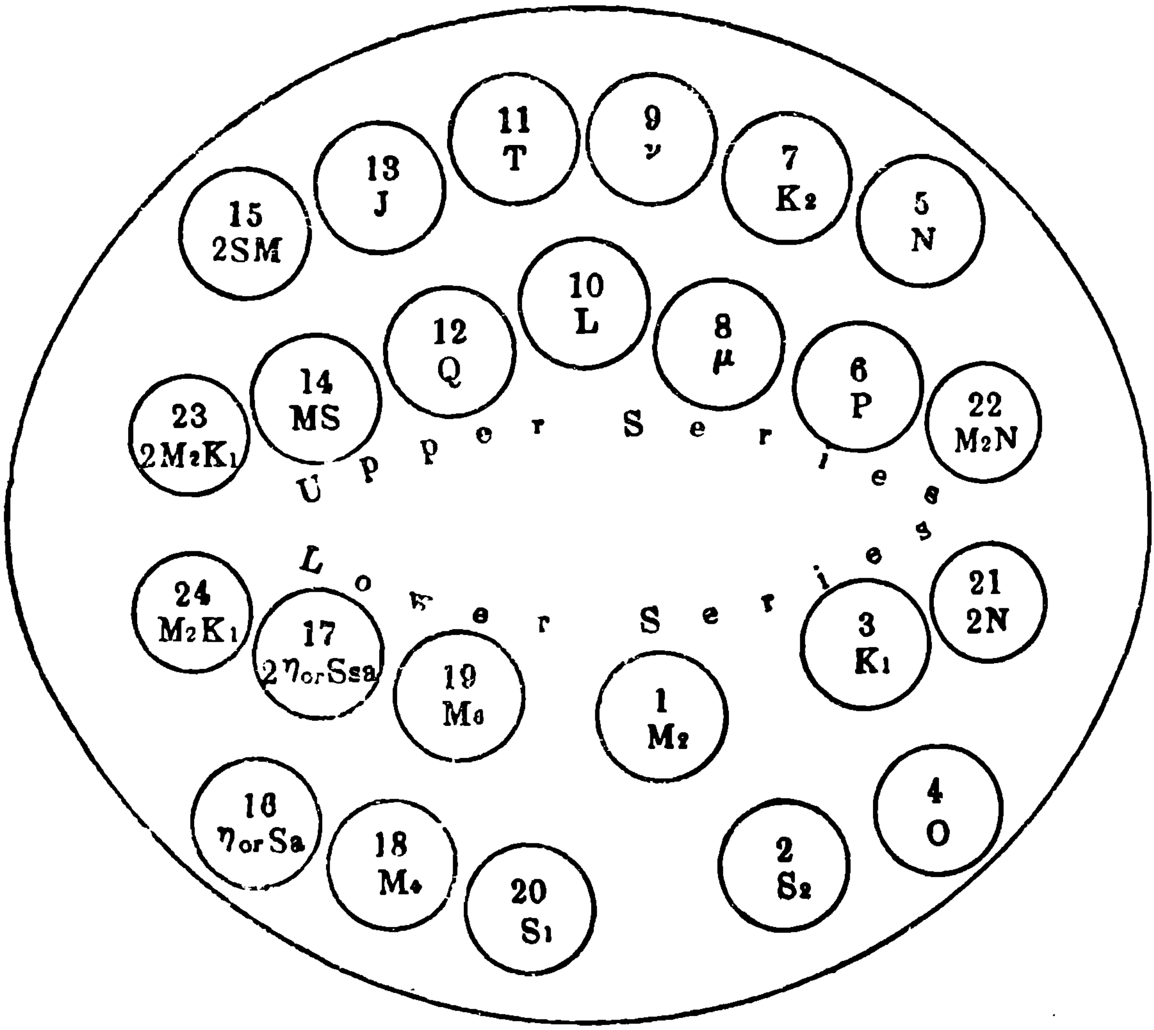
Make the bolt A fairly tight, then find the position for the dial plate in which all the crank spindles will rotate quite freely in their bearings, and then clamp A, B, C, D, E, so as to fix the dial plate in this position.

Put back the worm shafts into position.

29. TABLE AND DIAGRAM OF COMPONENTS :—The following table shows the names of the Tides, with their symbols and numbers inscribed on the tide-predicting machine.

Symbol	Name of Tide	Numbered on machine
M_2	Principal or mean lunar (semi-diurnal) ...	1
S_2	Principal or mean solar (semi-diurnal) ..	2
K_1	Luni-solar declinational (diurnal) ...	3
O	Larger lunar ,, ,, ...	4
N	Larger lunar elliptic (semi-diurnal) ...	5
P	Larger solar declinational (diurnal) ...	6
K_2	Luni-solar declinational (semi-diurnal) ...	7
μ	Lunar 'variational' (semi-diurnal) ...	8
ν	Larger lunar 'evectional' (semi-diurnal)	9
L	Smaller elliptic (semi-diurnal) ...	10
T	Larger solar elliptic (semi-diurnal) ...	11
Q	Larger lunar declinational elliptic (diurnal)	12
J	Supplementary lunar declinational elliptic (diurnal) ...	13
MS	Compound luni-solar (quarter-diurnal) ...	14
2SM	Compound luni-solar (semi-diurnal) ...	15
η or Sa	Solar (annual) elliptic ..	16
2η or Ssa	Solar (semi-annual) declinational ...	17
M_4 & M_6	Two mean lunar over-tides of the semi-diurnal tide ...	18 & 19
S_1	Mean solar (diurnal) ...	20
2N	Second order lunar elliptic (semi-diurnal)	21
M_2N	Compound lunar (quarter-diurnal) ...	22
$2M_2K_1$ & M_2K_1	Two compound lunar (ter-diurnal) tides ...	23 & 24

DIAGRAM :—The following diagram shows a back view of the machine with symbols and numbers of each tidal component.



30. *See that, when the milled heads on the cranks are downwards, the dial pointers are vertically upwards.* See that all gears, worms etc. are marked with the number of the component which they are working (see Table on previous page).

Points to be observed before starting the machine.

See that the worm gears are well in the teeth of the wheels they drive, otherwise the teeth are likely to get destroyed, also that there is not much play in the worm shaft; that all shafts are perfectly true and all bearings well oiled.

There must be a little play in the worm shafts or their bearings will run hot.

31. Set all components to read 90° or 270° on the dials and clamp. The fixed dial pointers should read 0°.

Setting verniers and scales.

Taking each component in turn, loosen the clamping screws on the sliding rod and bring the top line of the scale on the rod in exact alignment with the top line of the fixed vernier. In the case of S_1, one of the centre long lines of the scale on the sliding rod should be set; otherwise this rod may hit the pen guide.

Then set the numbered aluminium scales to zero.

32. Set all components to 90° or 270°.

Winding on wire.

Wind about 60 feet of wire carefully on to a fishing reel with a fairly strong check action.

Fix the reel upright on the floor below the pen guide. Open the back of the pen guide and draw the wire through it; attach a weight to the free end of the wire, pass the wire over the pulleys of M_2, S_2, K_1 etc. in rotation keeping a steady strain on it all the time; at any stop allow the weight to hang freely keeping the wire in tension.

When the M_6 pulley has been passed, carry the wire straight down to about 12 inches from the floor and let the weight hang free.

Hold the wire near the reel firmly and cut.

Keeping the weight hanging free, so as to keep the wire in tension, attach the other end by a loop to one of the hooks on top of the pen-box.

There are three hooks on the pen-box or moving pen to which the wire can be attached. It has been found in practice that the central hook serves all purposes. However in case of deficient or excessive contact of the pen with the paper, (or the wheel with the copper contact in the chronograph method), the wire may be attached to the back or forward hook to improve matters.

Pull steadily on the weighted end of the wire until the pen is about 6 inches above the bottom of the pen guide.

The pen will now maintain the tension in the wire. Hold the wire firmly near the weight and cut off the weight.

Keeping the pen hanging free about 4 inches below the middle of the pen guide, pass the free end of the wire under the pulley of S_1 and through the hole on the rim of the milled screw on the brass plate and fix it. Wind up the wire on the screw until the pen is at the middle of the pen guide and clamp.

Great care must be taken to keep the wire always in tension or it will kink or spring off the pulleys.

The machine is now ready for setting.

SETTING AND RUNNING THE MACHINE

33. The method usually adopted for predicting times of tides is to obtain a chronograph chart for times by the method 2 described hereafter and then to obtain heights independently by method 1, but, as method 1 was adopted in England for both time and height combined, and is the method for which the machine was originally intended, this method is described first.

34. *For height or height and time diagram combined.*

Method 1.

First clean the two fixed pens and the moving pen and fill with ink.

If the ink runs too freely, put pins in the airholes on top of the pens.

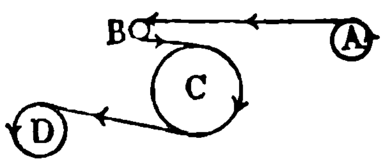
The moving pen should be suspended from one of its hooks, (generally the central one), so as to have the pen pressing sufficiently against the paper on the drum.

35. Put the drums in position with the paper roll on the drum A on the motor side of the machine.

Putting on the paper.

See that the points on the big drum C run freely in the two grooves on the roller B.

Wind the paper as in the diagram below :—



36. Set the upper of the two fixed pens so as to make a line about 11 to $11\frac{1}{2}$ inches from the bottom of the drum; this is the mean-sea-level line.

Setting upper fixed pen to mean-sea-level line.

37. In the case of Riverain ports the lower or datum-line pen is not set.

Setting lower or datum-line pen.

There are altogether 9 Riverain ports viz :— Basrah, Dublat, Diamond Harbour, Kidderpore, Chittagong, Amherst, Moulmein, Elephant Point, and Rangoon.

In the case of all other ports, set the lower or datum-line pen to the measured distance to scale of the datum below mean-sea-level for the port in question which is published in the tide-tables and is also given in table XXI at the end of chapter I. It should also be entered in the Form 1 Tid. Pred. as A_0.

Setting moving pen to mean-sea-level.

38. Set all dial pointers without clamping to 90° or 270° and verify that all verniers and scales are at zero. Then, by twisting the milled screw to which the fixed wire is attached, move the pen until the moving pen coincides with the mean-sea-level.

The last movement of the pen should be a rising one, since, when the wire is slackened, it does not always set tightly at the bottom of the grooves on the pulley wheels and so the pen does not fall as much as it should.

Data for setting.

39. The amplitudes and phases of the angles for setting on the machine are those obtained from the Form 1 Tid. Pred.

In the case of Diurnal charts for Riverain ports only 8 components are set, the amplitudes of the remaining 16 being set to zero, and the dials corresponding being left unclamped.

Setting amplitudes.

40. Set all dial pointers without clamping vertically upwards to 0° or 180°. The fixed pointers on the dials should be at 0°, so that the milled heads of all the cranks are downwards.

Taking the amplitude of each component in turn by twisting the milled head attached to the micrometer box on its crank, bring the pointer to the correct number of millimetres of amplitude up or down on the aluminium scale, then set the first decimal place by means of the vernier, note the position of the pointer on the milled head and use the graduations on the milled head to set the 2nd decimal place. (One small division on the milled head $=0\cdot025$ millimetres).

Check on position of pen.

41. Move the drum C towards the drum D so as to draw a fresh portion of paper around the drum. The moving and the 2 fixed pens will all trace lines on the paper.

The distance between the mean-sea-level line and the trace of the moving pen can be measured off by means of a pair of compasses and read off from a diagonal scale and compared against the pen-height calculated in the Form 1 Tid. Pred. (Height-Sheet). Care should be taken that there is no wire slackness when this test is made. An error of less than $0\cdot05$ inch can be accepted. Wire slackness can be detected by turning all the dial pointers back to 90° or 270°, when the moving pen should again coincide with the mean-sea-level line. If not, the process of setting the pen on the mean-sea-level line should be repeated, and when the dial pointers are again turned to 90° or 270°, the pen height should check. In running the machine it is found that the wire slackness is taken up, and this can be verified by the final pen-check afterwards described.

It may be here remarked that the movement of the pen is equal to double the amplitude, owing to the wire round the pulleys. The factors given allow for this and give half *i.e.*—in the computation the pen height is obtained by dividing the result of the summation by 12·7 instead of 25·4, the number of millimetres per inch.

Setting Dials.

42. The dial pointers are now set to the phase angles and clamped—these phase angles are obtained by subtracting ζ from 360°, in order to avoid the machine tracing a reverse curve, vide para 3. These dials should ordinarily be all clamped as tight as possible by hand. If a spanner is used, it should be very lightly applied, as the dials tend to tighten up when the machine runs forward, and the teeth of the wheels are liable to get damaged while working and especially when unclamping.

The phase angles are checked over after clamping, and the position of the pen, checked as before, should give the value R_2 within the same limits as mentioned above. A further check can be applied by checking the values R M Cos (360° − ζ) on the opposite side of the machine in case of gross error.

Setting to Standard Time.

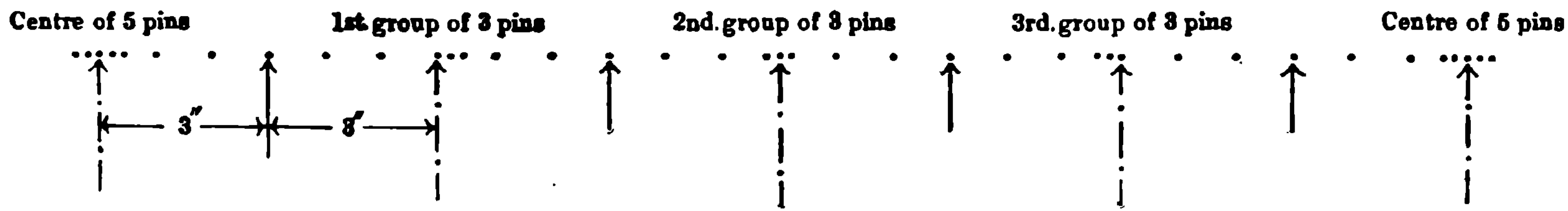
43. The above settings set the machine for local mean noon on 28th December of the year preceding that for which the height predictions are required; the big pointer on the date dial should be set to this date.

If the tide-table is in standard time, the machine must be run forward or back by hand until the correct value for S_2 is obtained (see table XXI, Chap. I). If the machine has to be run back, this should be done smoothly and slowly to avoid loosening any of the components.

Setting the drum.

44. Having set the machine to standard time as above, set the drum for time as follows:—The recording drum is fitted with brass pins at equidistant intervals to form the time indications on the paper by perforations:—vide diagram opposite. It will be seen that starting with the central pin of a group of 5 pins we pass over in succession 5 single pins, then to the first of a group of three pins, then 5 single pins and to the centre of a group of 3

DIAGRAM TO ILLUSTRATE THE GROUPING OF PINS ON THE RECORDING DRUM OF THE TIDE-PREDICTING MACHINE AND THE PERFORATIONS CAUSED THEREBY ON THE DIAGRAM.



To face page 16.

pins, then 5 single pins and to the last of a group of 3 pins, then 5 single pins and come on again to the centre of a group of 5 pins. It is immaterial whether we take the noon line on the 1st., 2nd. or 3rd. group of three, provided it is the correct one in sequence of the punctures but it is better to make sure of the starting position, by setting on the centre of a group of 5 pin-punctures.

Move the big drum, which should be unclamped, by rotating the drum D in a counter clockwise direction, until the middle point of the 5 punctures on the drum comes under the wire, by lifting the lid of the pen slide and looking to see that the wire is immediately above the central puncture. Parallax may be avoided by looking through the tips of the thumbs placed almost together centrally in front of the wire. Clamp the big drum by tightening the nut on the spindle underneath. Do not use too much force in tightening up, as the spindle is liable to get broken. The nut has a reverse action and will tighten up as soon as the machine runs. Write on the diagram paper the following information :—

Name of Port.

Scale.

Date and time of 1st setting (whether standard or local time).

Engaging the gear

45. There are two notches on the wheel in which the gear lever engages, one marked R which gives 3 inches of paper for the day, the other, not marked, which gives 6 inches of paper to the day. The mid-position between these notches is neutral.

The gear clutch frees the gear and enables the paper to pass through freely.

The notch marked R, giving 3 inches of paper is used for the diurnal charts of all Riverain ports (except Bassein), and for the tidal curves of all open seaports of which only the heights are required, the times having been already determined by the chronograph method mentioned in paras 4 and 5.

If however combined heights and times are required to be read for open seaports, (or Bassein). from the one chart, the notch giving the 6-inch scale must be used. Engage the gear suitable for the port and purpose required by engaging the gear lever in the appropriate notch.

Running the machine.

46. Before starting, oil all working parts of the machine thoroughly. To start the machine, put the motor switch to the "start" position. If the motor does not

turn, assist it to start by turning the horizontal spur-wheel on the machine in a counter clockwise direction.

As soon as the machine is running at full speed put the switch over to the "load" position.

Run the machine for 24 hours exactly, this is done by watching the S_2 dial pointer which does 2 revolutions per day and returns to the same position at midnight and noon The current should be switched off when S_2 indicates midnight and the machine will come to rest shortly before the pointer of this dial reaches the noon position, as originally set, to which it can be moved by moving the spur-wheel of the machine forward by hand.

The wire should now be exactly over the noon point on the drum 24 hours from the starting noon point. If not, unclamp the drum and reset allowing for the lag and try again for another 24 hours. It is a case of trial and error but there should be no difficulty in getting it right in the 3 days between 28th. December and 1st January, from which the prediction commences, after which the machine may be allowed to run on. While running, it may be advisable to occasionally enter a date on the diagram from the pointer on the date dial.

After 369 days stop the machine a short time before local or standard mean noon 1st January (31st December in leap years) whichever is earlier. This is done by watching the S_2 pointer which does 2 revolutions per day. It should be shut off when the S_2 pointer is about 180° from its final reading on the last day.

(1) Procedure supposing local mean noon is the earlier.

47. The machine has to be brought up to local mean noon on the date above mentioned in order to check pen height R_3 and phase angles ζ after 369 days motion against those computed in form 1 Tid. Pred. This is effected by moving the spur-wheel of the machine forward by hand till the S_2 dial pointer reads the value ζ' for this particular component on form 1 Tid. Pred.

The phase angles are now all checked against the computed values given on the form 1 Tid. Pred. They should all check within half a degree of the correct value. An error up to half a degree may occur owing to play in the gears or in the pointers.

In order to obtain a trace of the local mean noon line and to determine the point to which the pen-height check has to be taken, the cord between the pulleys M_2 N & 2 N is gently pulled outwards, care being taken that the cord settles back in the grooves. This has the effect of raising the cord and tracing the noon line on the diagram. The intersection of this line with the curve is the point to which the pen-height above mean-sea-level is measured by means of compasses when the curve is completed.

The machine is again run forward by hand as before till the S_2 dial pointer reads the value of ζ' for this component for standard time, the cord being drawn out as previously to mark the standard noon. The standard or local time line (as the case may be according to which was adopted in setting up) should agree with the line drawn between the punctures representing noon on the diagram.

(2) Procedure supposing standard time is the earlier.

48. It is obvious that the process has merely to be reversed *i.e.*, the machine has to be stopped and the standard noon line marked first and then it has again to be run on by hand to the local mean noon to mark this latter time and to obtain the checks of pen height and phase angles.

Conclusion of curve.

49. The machine is now allowed to run on by means of the motor for a day or so before taking off the curve. In leap years the dial, though indicating January 1st, is only at noon 31st December and it is essential at least to run till midnight of the 1st January is past, so as to ensure that no high or low-water curves are missed up to the termination of the year's prediction. It is as well however, whether predicting for leap years or otherwise, to run the machine for a day or so extra to form a check against the succeeding year's prediction, if required, and enable smoothing to be carried out.

The machine takes about 4 hours to run off a year's curve for one port, whether a chronograph chart for time, or height and time.

Changing paper while running a curve.

50. If it should be necessary to change a roll in the middle of running a tide, stop the machine when about 18 inches of the old paper still remains clear of the big drum C (vide para 35).

Put on the new roll, cut the free end of the paper straight and pass outside the old paper round cylinder B and between the old paper and drum C.

Now run the machine and press the free edge of the new paper keeping it vertical between the old paper and drum C until it is gripped by the time points and carried round.

When it emerges between C and D pull gently on it and pass round D until it is gripped there. Care should be taken that the new paper is correctly dated, and name of port, scale, etc. inserted.

51. *Setting and running the machine for time with the chronograph.*

Method 2.

This method has been introduced with a view to obtaining more accurate measurements of the predicted times of high and low-waters for open seaports.

52. The chronograph drum has first to be carefully levelled up, aligned and its spindle attached to the spindle of the S_1 dial and the connecting screw tightened up.

Levelling and attaching Chrono-Drum.

The drum should then be run to see if it runs smoothly.

53. Put the paper carefully round the Chrono-Drum, seeing that the overlap is well turned in and the paper fits tightly when the paper is pasted down and the clips are put on.

Fixing paper on the Chrono-Drum.

54. The contact-wheel should now be substituted for the ink-pen on the pen-box, and the latter suspended by its central, (or other), hook, vide para 32.

Fitting contact-wheel.

The little contact strip with projecting copper contact, which is attached to a small strip of wood for the purpose of insulating, may now be placed in position and clamped on the line corresponding, to the mean-sea-level *viz*, about 11 or 11½ inches from the bottom of the pen guide. This clamping should not be done too tightly as there is a danger of the pen-box jamming in the guide when running. It should however be just sufficiently tight to give a good contact without checking the free motion of the pen.

55. Electrical connections should now be made as shown in the diagram opposite. One connection in the circuit should be left open till actually required for use in order to save current.

Electrical connections.

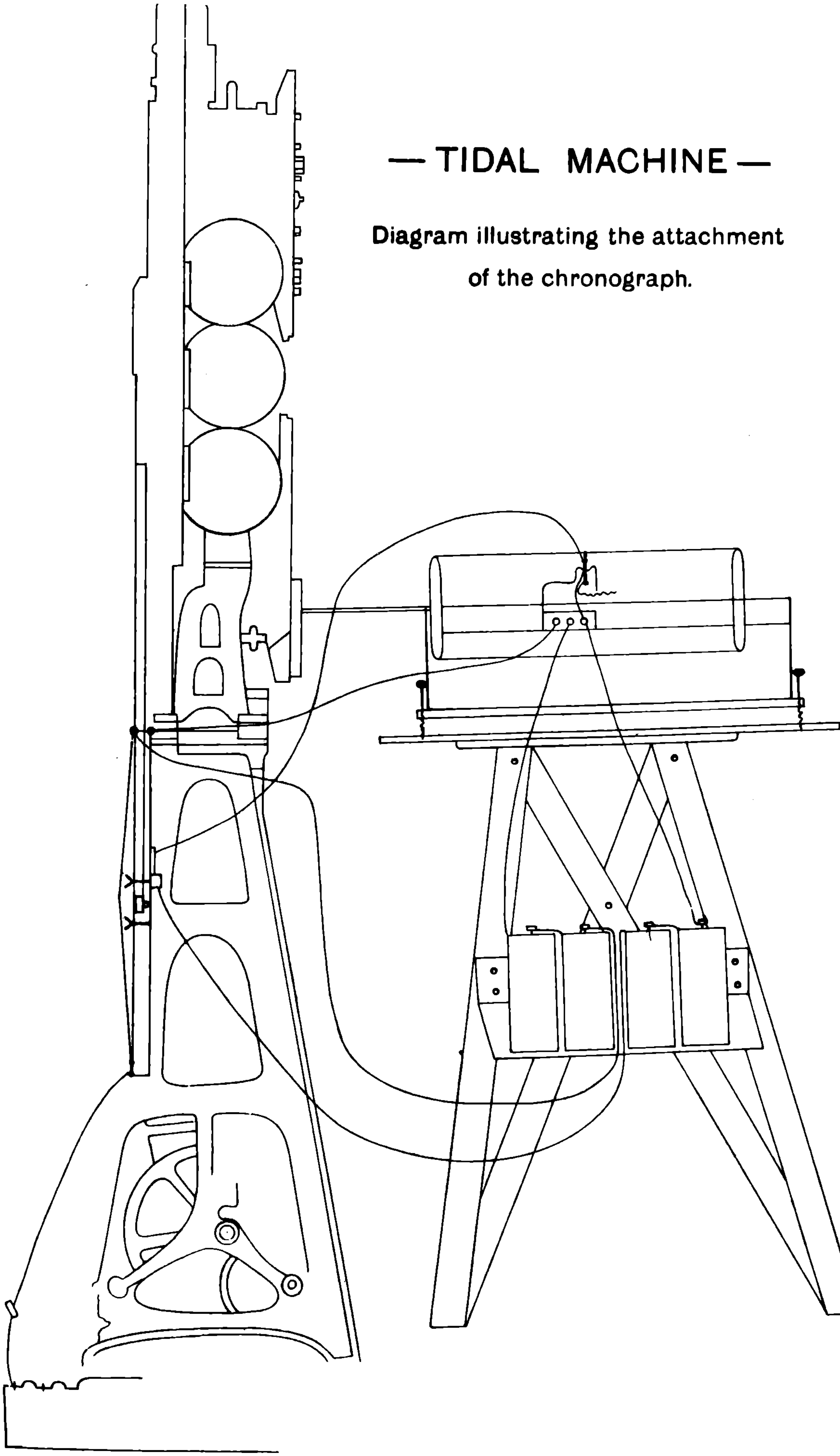
56. Unclamp all dials and set all dial pointers to 90° or 270°. The amplitude scales, both main and vernier, should now be checked to see that they read zero.

Setting contact-wheel on pen-box to zero.

Now swing the M_2 dial pointer slowly from 0° down towards 270°, counterclockwise. If the contact wheel is in correct adjustment, a click should be heard on the relay the instant the M_2 dial pointer reaches 270°. If the click does not occur precisely at this point, the wire to which the pen-box is attached, must be lengthened or shortened by unclamping and turning the milled screw to which the fixed end of the wire is attached, so as to raise or lower the contact wheel until satisfactory contact is obtained. Clamp tentatively and test contact by moving the M_2 dial

— TIDAL MACHINE —

Diagram illustrating the attachment of the chronograph.



pointer as before, counter clockwise from 0° to 270° and again clockwise from 0° to 90°, seeing that the click occurs exactly at 270° and 90° respectively. Repeat the adjustment until perfect. The object in swinging the dial pointer counter clockwise and clockwise as above, is to ensure that the same point of the wheel comes in contact with the copper strip in each case. Consequently, when the machine is run, the first contact, which is recorded when the pen-box is running downwards, is made by the same point of the wheel as the last contact, when the pen-box is running upwards. Now when the pen-box is running downwards, high water is being recorded, so the commencement or the break on the chrono-drum represents the time of high-water, and when the pen-box is running upwards, low-water is being recorded, so that the end of the break on the chrono-drum represents the time of low-water, and these are the points to which measurements are afterwards made. It is important to see that the low-water contacts are not dragged *i.e.*—that the breaks on the chrono diagram terminate the moment the low-water contact is completed. For this purpose it is advisable to make the contacts as light as possible and to have a fairly strong spring on the pen on the chrono-drum to pull the pen back immediately the contact terminates.

Setting amplitudes.

57. The amplitudes should now be set as explained in para 40 but to the values given in Form 2 Tid. Pred. The pen height should then be checked in the ordinary manner or by measuring the height of the contact-wheel above the contact strip with a pair of compasses gripping them with a piece of cloth so as to avoid an electric shock, and comparing this with the computed value of R_1 from Form 2 Tid. Pred. by measurement on a diagonal scale.

Setting dial pointers.

58. The dial pointers are now set as explained in para 42 but to the values given in Form 2 Tid. Pred. Check the height of the contact wheel, as above, against the computed value of R_2.

Setting to Standard time.

59. If time is required in standard time, move the machine by hand till the S_2 dial pointer gives the correct reading for standard time as in para 43.

Setting the date dial.

60. Set the date dial to read 28th December before running the machine.

Preliminaries to running.

61. Close chrono-circuit, ink up chrono-pen, press wires adjoining M_2 component so as to cause a contact between contact wheel and contact strip in order to record a click for position of noon on chrono-sheet, this

also tests both electrical circuits on the machine and the chrono. Mark name of port, date and time (standard or local).

Running the machine

62. Start the machine watching the S_2 dial pointer which does 2 revolutions per day, and returns to the same position at midnight and noon, the current being switched off when S_2 indicates midnight. The machine will then come to rest shortly before the pointer of this dial reaches the noon position as originally set, to which it can be moved by turning the spur-wheel of the machine forward by hand.

The tapper is then pressed so as to record the 2nd noon position, repeat this operation until all lag is taken up, and the noon position on successive days remains unchanged.

After this the machine can be allowed to run on. Mark the high and low-tide breaks for a few days only on the chrono diagram—high-tide, falling pen, with red chalk—low-tide, rising pen with blue chalk. The remaining breaks can be chalked up in continuation of those already marked after the diagram is removed.

A man may be then left standing alongside to see that the recording fountain pen is inking properly and to look after the machine generally before it reaches the end of a diagram.

Before changing or removing a diagram the machine must be stopped and run up by hand as before till S_2 reaches the correct noon position as described previously, a mark being made by pressing the wires adjoining the M_2 component together so as to cause a contact between the contact wheel and contact strip and thereby to record a click correspoding to the final noon line.

The diagram can then be removed by releasing the clips and cutting the paper along the 0 hour line on the chrono-drum. Should it be necessary to revolve the drum in order to do this, the reading of the S_1 dial must be carefully noted before it is unclamped and the S_1 dial revolved in a clockwise direction to avoid backlash when restarting. The new paper can then be put on and the S_1 dial pointer brought back and clamped to its previous reading.

Write on the new diagram the name of the port and date and carry on as before, marking the noon positions (standard or local as required) for the 1st, 2nd or 3rd day, till all the lag of the drum is taken up and again before removing the sheet.

When the whole year has been run, the checking of the dial readings of the machine after 369 days at local mean noon is made as in para 47, the check of the height of the contact wheel corresponding to the pen check, being carried out as described previously. The machine is then run up to midnight of 1st January (31st Dec. in leap years) and the final chart removed.

Setting dial pointers in the midd'e of the year in case of accidental stoppage.

63. In the event of any accidental stoppage of the machine when running *e.g.* :—on account of a snapped wire, missed contacts, or other irregularity, it is convenient to be able to set the dial pointers for the commencement of any particular month, rather than to re-run the whole tidal curve or Chrono chart from the very commencement of the year. The table XXII at the end of Chapter I, which is based on the speeds of the tides, will be found very useful for this purpose, as the angles given in this table have only to be added to or subtracted from the original phase angles $(360° - \zeta)$ or $(360° - \zeta + 90°)$ on the Forms 1 and 2 Tid. Pred. respectively to obtain the appropriate setting of the dial pointers for the first of every month, the amplitudes set on the front of the machine remaining unchanged.

64. Two other methods of running the machine, which are sometimes necessary, are included in the abstract of the ordinary methods of running the machine given in the following table, but as they are both similar to Method 1, there is no need to describe them separately in detail.

ABSTRACT OF PROCEDURE ADOPTED

Method 1. To obtain a Height chart, or a Height and Time chart combined.	Method 2. To obtain a Chrono Sheet.
Suspend pen-box with pen, by central or other hook.	Suspend pen-box by central or other hook, and see the contact-wheel makes proper contact.
Put paper on drum, fill pens with ink, set mean-sea-level and datum-line pens.	Put paper on chrono-drum, connect up electric circuits leaving chrono-circuit open.
Set all dial-pointers to 90° or 270°, see that the fixed pointers are all at 0°; check that amplitudes read zero; set pen to mean-sea-level line.	Proceed as in Method 1 for setting dial-pointers and checking amplitudes. Close chrono-circuit and set pen to give contact the instant M_2 pointer when moved clockwise from 0° to 90°, reaches 90°, or counter clockwise from 0° to 270°, reaches 270°.
Turn dial-pointers to 0° or 180°, so that the screw-heads of all the micrometer boxes in front, point downwards. Set amplitudes from 1 Tid. Pred. and check position of pen. (All pen checks should agree within 0·05 inch).	Open chrono-circuit and proceed as in Method 1, except that amplitudes are set up from 2. Tid. Pred. Check position of pen. (All pen checks should agree within 0·05 inch).
Set dial-pointers from 1 Tid. Pred., clamp, then check dial-pointers and height of pen.	Set dial-pointers from 2 Tid. Pred., clamp, then check dial-pointers and height of pen.
If standard time is required, move the machine by hand till S_2 gives the correct reading.	Proceed as in Method 1.
Set date dial to read 28th December. Clamp up drum with wire over the centre point of the group of 5 points on the drum. Mark name of port, date, time and scale of chart. Run on for 24 hours and check position of drum, correcting if necessary, finally drawing the vertical noon line by pulling the wire between M_2N & 2N to one side and releasing the wire. See that all lag is taken up within the three days before the commencement of the year's predictions, correcting the position of the drum each day if necessary, as before. NOTE:—For height only, the 3-inch scale, marked R on the gear wheel, is used. For height and time, the 6-inch scale.	Set date dial to read 28th Dec., close chrono-circuit, press cords adjoining M_2 dial together for position of noon on chrono-sheet, mark name of port, date and time on sheet. Run on for 24 hours and press cords adjoining M_2 dial for position of 2nd. noon, and repeat until the noon position on successive days remains unchanged. Mark high and low-tide breaks with red and blue chalk. High-tide, falling pen, red—low-tide, rising pen, blue.
Check dial-pointers and height of pen after 369 days at L. M. N. from 1 Tid. Pred. Run the machine on for a day or so and then remove the chart.	Mark noon position at end and beginning of each sheet, and write date and name of port on each sheet, check dial-pointers after 369 days at L.M.N. from 2 Tid. Pred., and run on machine for a day or so, then remove chrono-sheet.

IN RUNNING THE MACHINE

Method 3. To obtain a Diurnal Chart for a Riverain Port.	Method 4. To read Heights direct.
As in Method 1.	As in Method 1.
As in Method 1, except that no Datum-line is required.	Fix the wooden scale for measuring on the pen-guide by propping up the lid of the latter on, say, another wooden ruler, before tying the measuring scale in position.
Proceed as in Method 1.	Proceed as in Method 1 for setting dial-pointers and checking amplitudes. Set pen by the wooden scale to give the value A_0 from 1 Tid. Pred. (Distance between M.S.L. and datum for the particular port: this is also given in the Tide-tables and in table XXI at the end of Chap. I). The pen is not actually visible but a point on the back of the pen-box opposite it is measured to.
Proceed as in Method 1. The 16 amplitudes not used, should all be set to zero.	Proceed as in Method 1. In checking the pen height, subtract A_0 from the height of the pen by the scale and multiply the result by the factor C (vide table xxi). The value so obtained should agree with R_2 on the Form 1 Tid. Pred. within 0·05 inch.
Proceed as in Method 1. The 16 dial-pointers not used, need not be set and should be left unclamped.	Proceed as in Method 1.
Proceed as in Method 1.	Proceed as in Method 1.
Proceed as in Method 1. NOTE:—For a Diurnal chart the 3-inch scale marked R on the gear wheel is used.	Set date dial to read 28th December. Run the machine, checking the 1st. few height readings against those published in the previous years' Tide-tables, recording from 1st. January onwards in the form for the purpose. A reading has to be recorded by eye on an average every 7 seconds. At the end of a month it is advisable to stop the machine and check the differences of alternate tides, if any error is detected, it can be corrected by interpolation. *N.B.—This method is tiresome to the observer and is not recommended.*
Proceed as in Method 1.	Check dial-pointers and height of pen after 369 days at L. M. N. from 1 Tid. Pred. Run on machine for a day or so, recording the heights as far as required.

65. Using a sharp hard pencil (4 H) and a good straight edge, rule lines 6 inches apart between the noon or midnight perforations.

Ruling up the diagram for height or height and time combined.

The arrangement of perforations is as shown in the diagram facing page 16 Chapter III :—

On the small scale each arrow indicates a noon perforation on the diagram.

On the large scale the chain dotted arrows mark noon and the firm line arrows midnight.

In both cases the latter are the best ones to join up.

These lines have to be ruled in the case of diurnal charts for riverain ports, and in all cases where times have to be measured on the charts; but if the charts have been made solely for the purpose of measuring heights the chart should be dated at the proper perforations in order to facilitate entry of the heights according to date. It is not absolutely necessary to rule lines in this case, though sufficient care should have been taken in running the chart for the timings to be approximately correct. Though these times are not actually required for record, they serve as a rough check against those obtained from the chrono-sheet.

Diurnal Charts are run for all Riverain ports (except Bassein) on the small scale and these require the time lines ruled up.

Measuring the diagram times should not as a rule be measured from the small 3-inch scale diagrams, run for height only, as times from these small diagrams are liable to be in error because the scale is only graduated to 30 minutes and can only be estimated to about 15 mins. These can be read, if required, by the metal scale marked A and M vide para 24.

66. Times on the large or 6-inch diagrams are measured from the midnight lines using the ivory edged scale vide para 23, which are graduated to 6 minutes but which can be estimated with care to within 3 minutes.

Reading of times.

67. After the times have been read and recorded on the appropriate form (4 to 8 Tid. Pred.) either from the 6-inch height and time diagram as above, or from the chrono-sheet, the heights are read from the 3 or 6-inch height diagram, using the correct scale for the port in question, vide para 26.

Reading of heights.

68. When both heights and times are read from the one diagram smoothing of the readings is not absolutely necessary, though this was usually carried out in England by grading the intervals, rising to

maximum and lowering to minimum. Corrections, if applied, should never exceed 5 minutes. The fact of the heights and times increasing and decreasing by graded intervals is of great assistance when reading proofs of tide-tables, as, by taking out alternate differences, errors can be easily detected.

The results of predicted times and heights for open sea ports have in the past been found to agree well with actuals, about 75 per cent of predictions being within 10 minutes in time and 90 per cent within 4 inches in height.

Ruling up chrono sheet.

69. Using a sharp hard pencil (4 H), rule up the noon line and mark it 12 hours.

With dividers obtain the 4th. part of one complete line on the sheet, which is about 22 inches long. This represents 6 hours in time, step this off each way from the noon line at top and bottom of the sheet, and rule up marking the lines respectively 18 hours, 0 hours, 6 hours.

This can also be done on the chrono-sheet while it is on the drum by making use of the S_2 dial pointer, the machine being moved by hand and stopped at every 30 degrees from the first noon or midnight line, and the recording pen of the chronograph made to rule the successive hour lines.

Notes on the chrono sheet.

70. Note on the blank space on the left of the sheet:—(in red chalk) "Measure high-tides to left of break"; (in blue chalk) "Measure low-tides to right of break": line up high-tides with red chalk and low-tides with blue chalk in continuation of those marked when running the machine.

Enter dates at intervals along the noon line and edges of the sheet. (N.B. The date changes as the line crosses the 0 hour or midnight line).

Roll over the chart and put the extremities of the lines together and mark any breaks which occur at or near this junction, as otherwise breaks may be incorrectly read, or even missed, at these points.

Chrono sheet for ports with complex tides.

71. When the times of high and low-water are predicted by means of the chronograph sheet for certain ports which have complex tides, it will be sometimes necessary to refer to the height chart corresponding, to interpret the chronograph sheet correctly, and to ascertain which is the lowest or highest point reached by a tide, and in some cases which is the lowest of successive low-waters on the same day. The following ports in particular have such peculiarities :—

Suez	Bhāvnagar	Galle
Aden	Kārwār	Trincomalee
Perim	Porbandar	Colombo
Port Albert Victor	Tuticorin	Pāmban Pass

If such peculiar tides occur, mark those breaks on the chrono-sheet which are to be measured and those which are to be ignored after examination of the height chart.

If, as at Aden, there are occasions when there is no inferior high- nor low-water, there will be no break formed on the chrono-sheet at these times. The remark "No inferior high- nor low-water" should be written on the line on the chrono-sheet. In publishing the heights and times of such tides as these, when there is slack water, it is customary to print them in a special type and mark them with an asterisk to denote the peculiar nature of the tides.

If contact has failed at any time and there are no breaks where they should be, interpolate the markings by means of a pair of compasses, after comparison of the breaks before and after the missed contact, gradually increasing or reducing the interval as the case may demand. It is not advisable to interpolate too many missed intervals and it is better to rework the chart if too many contacts have been missed.

Reading the Chrono-sheet.

72. The high and low-waters should be read in the actual sequence in which they occur and commencing each day with the tide that immediately follows midnight, and not by the slipshod method of reading first all high-waters and then all low-waters, as mistakes in recording are apt to arise.

The times on the chrono-sheet can be read with ease to the nearest minute, and the result should be correct to within two minutes. The greatest probability of error is at neap tides.

To prevent mistakes in recording, the reader should call out the individual figures of the minutes *e.g.* the time 2 hours 50 minutes should be called out "two hours five nought". If he said "fifty" the recorder might put down "fifteen".

From the chrono-sheet for a port the times are read and entered up in their correct places on one or other of the forms 4 to 8 Tid Pred, selecting the one applicable to the port in question and the heights are afterwards entered in their proper places from the height chart.

73. The Diurnal charts for Riverain ports (which have been run with the 8 diurnal components $2M_2K_1$, J, Q, P, M_2K_1, S_1, K_1 and O only), is utilised for correcting the values of height and time obtained by direct computation for the semi-diurnal tide, so as to include the effect of the diurnal tide as explained in paras 129 to 132, Chap. I. The diurnal charts are therefore passed to the Riverain computers after having been run on the machine, and ruled up and dated as explained in para 65.

Disposal of Diurnal charts.

74. In addition to the methods of using the tide-predicting machine already described, a new device has been originated by Dr. J. de Graaff Hunter, M.A., Sc. D., F. Inst. P., whereby the height of water *at any time* in tidal prediction can be electrically registered. This has already been referred to in para 5. A full description of the method is given below:—

Prediction by multiple contact.

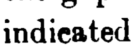
The tide-predicting machine ordinarily traces a height-time curve on a roll of paper some 22 inches wide, by means of a pen carried by a wire which passes over and under the wheels of the 24 "components." The time scale generally employed was 6 inches = 24 hours. It was found troublesome to the personnel available to read off times of high and low water from the curve with sufficient precision. There is naturally some vagueness as to the precise moment of maximum or minimum; but though this does not necessarily cause any serious flaw in the resulting tide-tables, it renders the ordinary system of checking proofs (and the measurements and copy from which they have resulted), by the method of reading the differences of times of alternate high-waters, much more troublesome. On this account it was at first thought desirable to run off a second curve—the differential with regard to time of the height-time curve—whose ordinate vanishes at the time of high or low-water. This was soon made more convenient by dispensing with tracing the curve, and causing the pen to break an electric contact when crossing the zero-line, this event being recorded on a chronograph-drum actuated by the S_1 component. It was then seen that a similar arrangement with *multiple* contacts corresponding to successive values of ordinate could conveniently be made to record the main character of the ordinary height-time curve. This arrangement indicates the times the predicted height of water reaches the various selected heights, e.g. with a tide range of 15 feet, it may be conveniently arranged to show the times of the water reaching *every whole foot of height above datum throughout the whole range.* It was thought that this information would be of greater value to mariners than the bare statement of height and time of high and low waters, with no information regarding water heights at intervening times.

Multiple contact chart.

75. The arrangement now to be described was made with this object in view. As will be seen from the diagram opposite, which shows part of a chart for Bombay, the time of water reaching each foot is shown by a notch in the day line. The notches are upwards for ascending water, and downwards for descending water; while a level near to mean water level is indicated by a break in the line.

The diagram shows the Height of water at any time at Bombay, 1925; also Times and Heights of all High-Waters and Low-Waters. Local mean time (correction to Indian Standard Time 5½ hours E. of Greenwich is + 39 minutes). Heights refer to datum of soundings on charts.

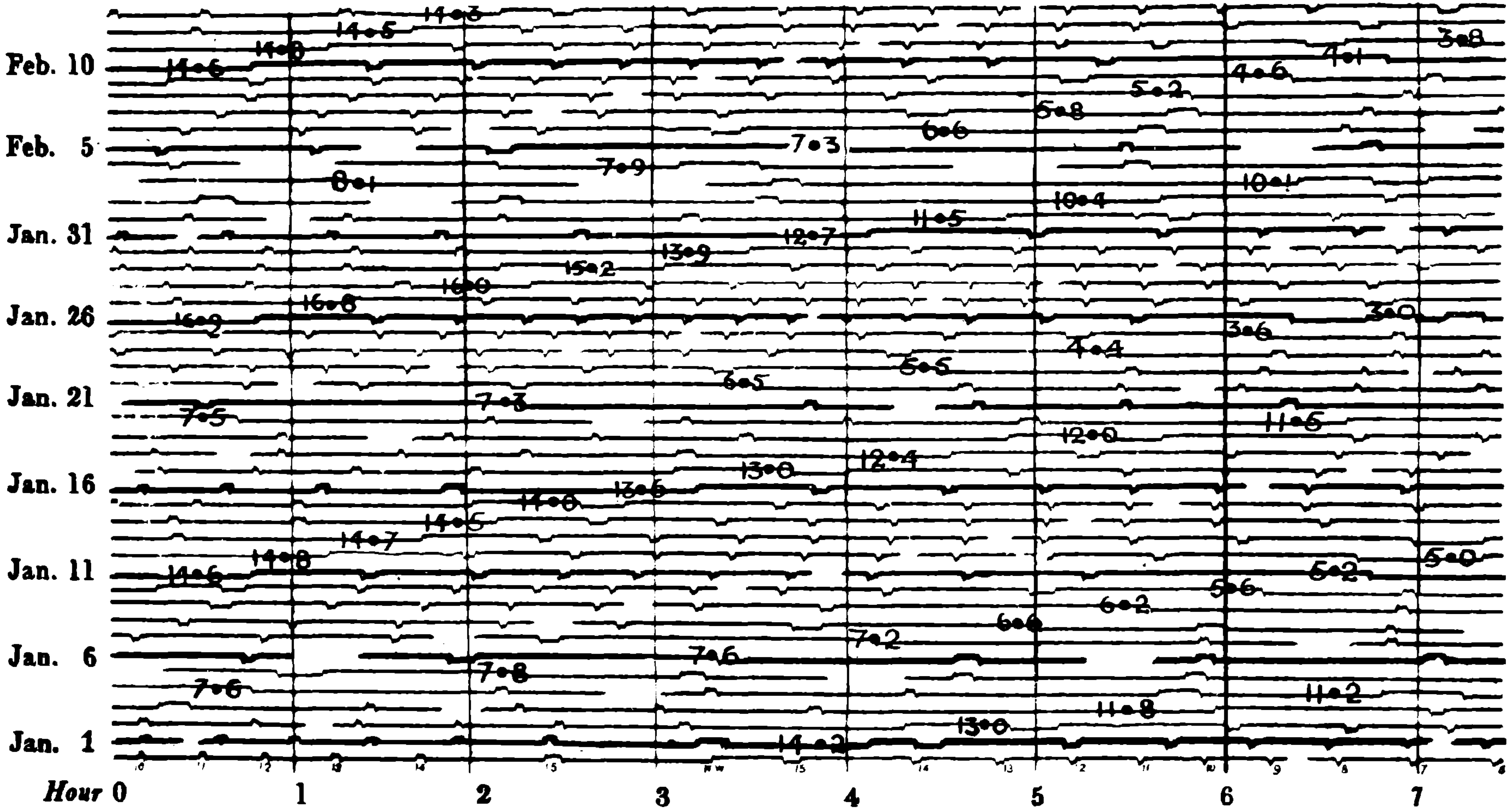
Explanation. Each horizontal line represents one day, and the date of every fifth line is given. The vertical lines represent hours from 0 to 24 (civil time).

The horizontal lines have notches which indicate the time at which the water is 1, 2, 3,......feet above datum. When the notches are upwards  the water is rising: when they are downwards  the water is falling. There are gaps in the horizontal line when the water is 8·5 feet above datum (about mean-sea-level), accordingly the height of an upward notch to the right of a gap is 9 feet above datum and an upward notch to the left of the gap is 8 feet above datum: while for downward notches the height of one to the right of the gap is 8 feet and of one to the left of the gap is 9 feet. The height indicated by any notch may be found by counting from the nearest gap 9, 10, 11 ....feet for rising water and 8, 7, 6.........feet for falling water. The process is illustrated in the bottom line of the chart for the day 31 Dec. 1924.

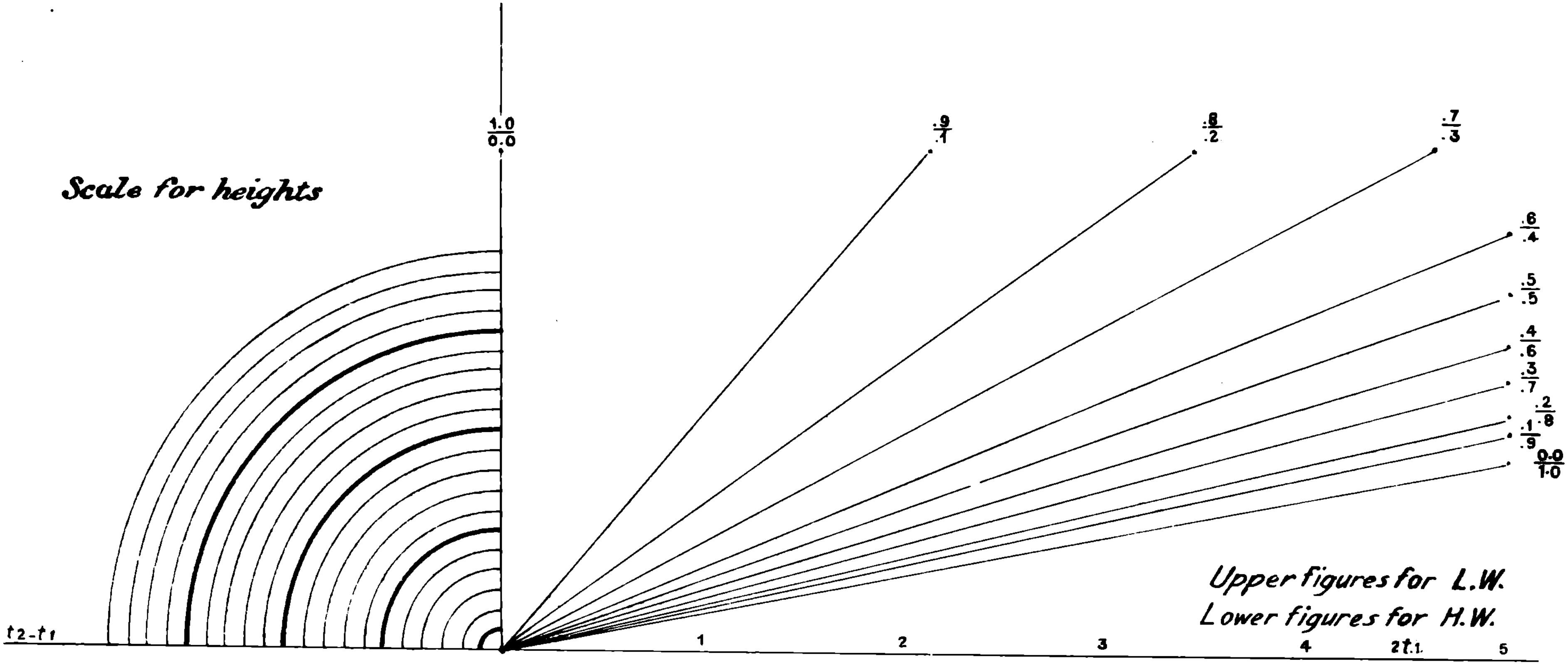
76. *Examples.*

(*i*) What is the height of water on January 26 at 3 hours 30 minutes?

Follow horizontal line for January 26 to meet vertical line for 3 hours. Here the notches are downward, indicating falling water and the height of the water at $3^h\ 30^m$ is between 9 and 10 feet: by estimation it is seen to be 9·8 feet.



Part of Chronograph Chart for Bombay 1925.



Scale for heights
1.0
0.0
.9
.1
.8
.2
.7
.3
.6
.4
.5
.5
.4
.6
.3
.7
.2
.8
.1
.9
0.0
1.0
t2-t1
1
2
3
4
2t1
5
Upper figures for L.W.
Lower figures for H.W.

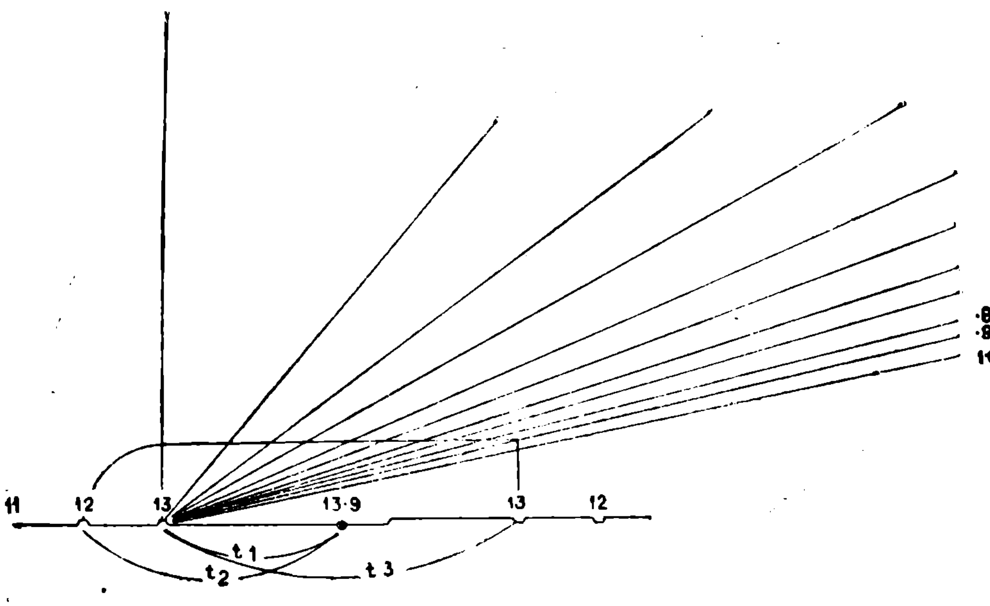
(*ii*) When will the water height be 10 feet on February 10?

Following line for February 10 there is seen to occur—

at 03h.	12m.	on falling tide	
10	14	rising ,,	read off complete chart
14	22	falling ,,	
22	12	rising ,,	

(*iii*) What are the time and height of first high water on Jan. 30?

It will be noticed on the chart facing p. 30 that the heights of high and low waters are entered in figures to one decimal of a foot. The position of the decimal point indicates the time of the occurrence. The method of ascertaining these times and heights is described with reference to the diagram below and scale opposite.



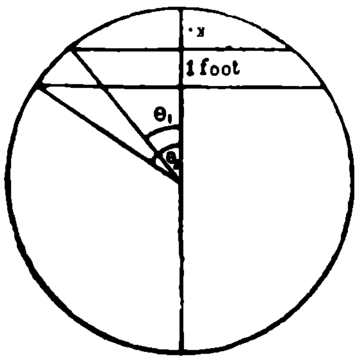
First note the change of tide and bisect the distance (t_3 or 2 t_1) between the ticks on either side of the change and put a dot, which is the point sought.

Next, to calculate the height put the black zero line of the scale (sample opposite) vertically on the tick just to the left of the dot and trace along the curve from the next left tick and then along the horizontal line until it cuts the vertical line from the tick just to the right of the dot (*vide* diagram). From the position of this point between the lines of the rays, the decimal part of the height may be read off—lower figures for high water and upper figures for low waters. A reference to the scale opposite will make this clear.

The height is thus ascertained to be 13·9 feet as shown by the printed figure and measurement (or estimation by eye) of the position of the decimal point from the adjacent time line shows that this occurs at $03^h\ 12^m$.

77. The principle on which the scale for making these measurements is based is as follows* :—

If the tide be considered as a simple harmonic tide at phase angles θ_1, θ_2 corresponding to times t_1, t_2 reckoned from high-water, $2r$ being the height of H. W. – height of L. W. and A the height of M.S.L. above datum = ½ (height of H. W. + height of L. W.) :—*vide* figure below, in which r is the radius of the circle, and its centre A feet above datum.



Then the heights at times t_1, t_2 are $A + r \cos\theta_1$, $A + r \cos\theta_2$ respectively.

As these differ by 1 foot

$$r(\cos\theta_1 - \cos\theta_2) = 1 \text{ ft.}$$

Now the tide rises a further distance (say) $\cdot x$ ft. above the last complete foot to its maximum.

$$\text{Therefore } r - r\cos\theta_1 = \cdot x \text{ ft.}$$

Now if $\cdot x$ were as great as a whole foot we would have the relation

$$1 - \cos\theta_1 = \cos\theta_1 - \cos\theta_2$$
$$\text{or } 2\cos\theta_1 = 1 + \cos\theta_2$$

* This principle is also applicable to finding the height of water at any time when the time of H. W. and L. W. is given, as, in this case,

$$\text{Height} = A + r\cos h \text{ or } A - r\cos h'$$

$$\text{where } h = 180° \times \frac{\text{Interval from H.W.}}{\text{Duration of rise or fall}}$$

$$h' = 180° \times \frac{\text{Interval from L.W.}}{\text{Duration of rise or fall}}$$

whence by expanding

$$2-\theta_1^2 = 2 - \frac{\theta_2^2}{1\cdot 2}$$

$$\text{whence } \frac{\theta_2}{\theta_1} = \frac{t_2}{t_1} = \sqrt{2} \text{ or roughly } \frac{7}{5}$$

$$\text{or } \frac{t_2-t_1}{2t_1} = \frac{1}{5}.$$

Similarly the relations between $\frac{t_2-t_1}{2t_1}$ may be found if $\cdot x$ is $\cdot 9, \cdot 8, \cdot 7, \cdot 6,$............$\cdot 1$ ft. and the angle of inclination of the lines representing $\cdot 9, \cdot 8, \cdot 7$...... .....$\cdot 1$ ft determined for purposes of constructing the scale.

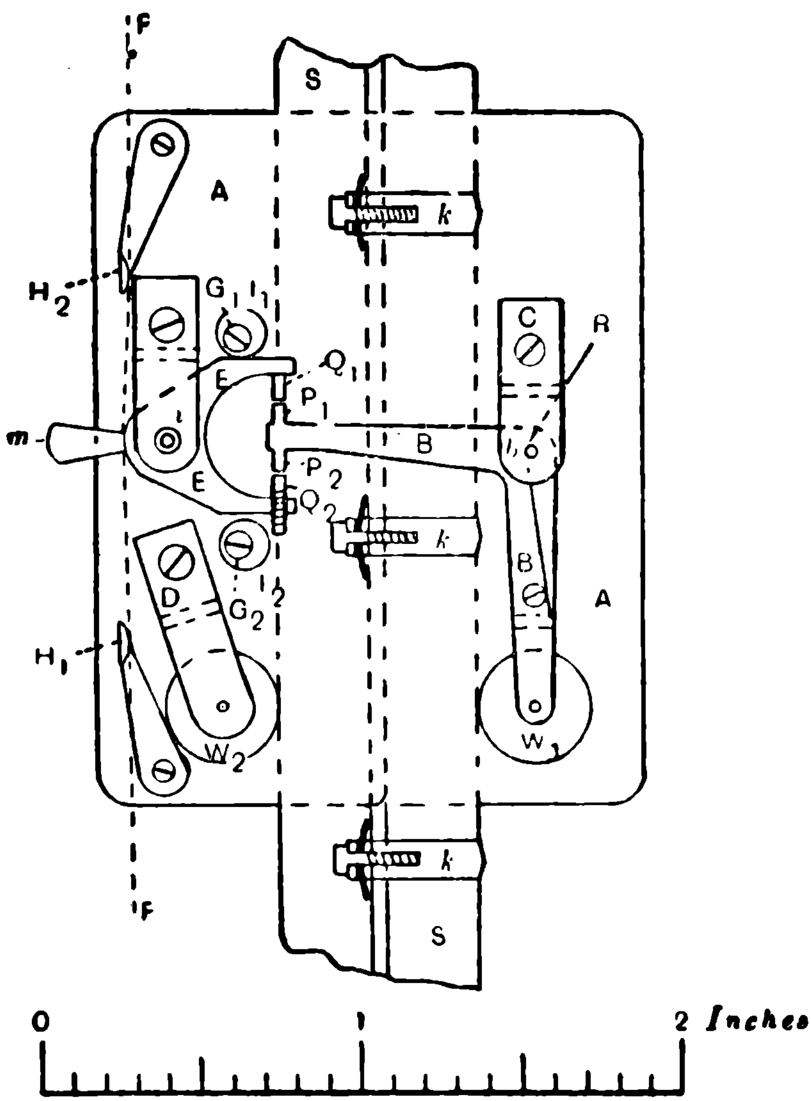
Difficulties in multiple contact system.

78. The chief difficulties encountered in this method of chronograph registration of multiple contacts were in making the contacts certain in action, while keeping friction forces very small, especially for those curves where the duration of contact was very small owing to rapid fall or rise of the curve. In this connection it may be noted that the wire which actuates the tide-curve pen is essentially of small diameter ($\cdot$005 inch was generally used) and is 32 feet long. A small variation in tension of this wire extends it visibly, while if a thicker wire be used inaccuracy arises owing to this not being sufficiently flexible to fit close to all the wheels over which it passes. As regards duration of contact it is to be mentioned that the chrono-paper passes at the rate of about 23 mm. per second, and that a whole year's prediction is dealt with in about 2½ hours.

How contacts are arranged.

79. In the figure shown on page 34, AA is a brass plate which is attached to the back of the pen of the tide machine, with which it moves up and down when the machine is set in motion. BB is a bell-crank lever provided with a pivot *b* carried by the bracket C and the plate AA. The horizontal arm of BB carries two platinum points P_1, P_2 near its left end and the vertical arm carries at its lower extremity the wheel W_1. There is a second wheel W_2 whose pivots are carried by the plate AA and the bracket D. These two wheels W_1, W_2 roll on the edges of a fixed brass strip SS, being maintained in contact by the pressure of a spring R.

The strip SS is shown in dotted lines, being in front of the plate AA and the horizontal arm B; it is of accurately uniform breadth, and is slotted at intervals of an inch for the pieces *k,k*. It is made of T-section to secure rigidity, and the pieces *k*, *k* are adjusted laterally by screws working in the vertical member of the T. The pieces *k*, *k* are slightly pointed as shown, and can be made to project a small amount (actually about 0·3 mm.) beyond the right edge of the strip SS. As the plate AA is carried up and down with the pen, the wheel W_1 rides over the points of the pieces *k*, *k*, and rocks the bell-crank lever, causing the platinum points P_1, P_2 to move slightly up and down.



The part EE bears two platinum points Q_1, Q_2 in its upper and lower jaws, Q_2 being adjustable in the direction of its length. This part has a pivot *i* with insulating bush, and is balanced by the counterweight *m*. Its motion is restricted by the insulators I_1, I_2 mounted excentrically on the two screws G_1, G_2. These screws are sufficiently tight-fitting to maintain their position, and by turning them to suitable positions the rotation of EE is regulated as desired. EE is actuated by the friction of the fixed wire FF, which lies in a groove cut in EE and also bears on the two pallets H_1, H_2. These pallets are borne on arms which can be rotated, and by this means the drag of the wire on EE can be adjusted; this can also be done by modifying the tension of the wire FF. The wire FF is attached to the frame of the machine at the top of the pen slide by means of an insulator and passes through another insulator at the lower end of the slide, being kept in tension by an attached weight. It makes metallic contact with EE and forms a portion of the chronograph electric circuit.

It will be seen that when the plate AA is ascending the upper jaw is pressed against the insulator I_1 by virtue of the drag of the wire FF, while when AA is descending the lower jaw of EE presses against I_2. The following events occur when AA is set in motion:

(1) AA ascending; E is pressed against I_1. Platinum points P_1, Q_1 and P_2, Q_2 are *not* in contact until wheel W_1 rides over one of the projections *k*,*k*, when P_2 *makes* contact with Q_2.

(2) AA descending; E is pressed towards I_2, but is held slightly away from it by the platinum point Q_1 bearing on P_1. When W_1 rides over one of the projections *k*,*k*, P_1 separates from Q_1 and E presses on I_2.

The terminals of the chronograph pen circuit are joined to the strip SS and the wire FF respectively. It is obvious that the effect of W_1 riding over a projection *k* is to *make circuit* in the case of AA rising; and to *break circuit* in the case of AA falling. This is represented on the chart by __⊓__ and ‾‾⊔‾‾ respectively. A change of direction of motion of AA, which corresponds to high or low water, is indicated by __⌈‾‾ and ‾‾⌉__; but owing to variation in tension, and hence in length, of wire, the actual time of this event is somewhat displaced. In order to provide a reference point, frequently repeated on the chrono-sheet, an additional electromagnet has been introduced, which lifts the chrono-pen from the paper when contact is established, making the indication —— —— on the sheet. This is arranged for by the provision of a springy contact placed near mean water level, against which the wheel W_1 presses in passing.

The chronograph is of the drum variety, and its shaft can be coupled to the square shanks of anyone of the 24 tidal components. Hitherto the component used has been S_1, so that the lines of the chrono-sheet correspond to 24 hours.

Research Fountain Pen.

80. It may be of interest to state that a "Research Fountain Pen*" has been used with excellent results for the chronograph. This is a very light pen and has a very steady and ready flow of ink. Inking begins with very light pressure and there is no blotting or leaking. Ordinary fountain pen ink has been used; when photographic reproduction is intended a little lamp-black is mixed with the ink.

* Supplied by Mr. A. Munro, 65 Preston Road, Winson Green, Birmingham.

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A

B

B.—(*Continued*).

C

C.—(*Continued*).

D

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E.—(*Continued*).

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R.—(*Continued*).

S

S.—(*Continued*).

S.—(*Continued*).

T

T.—(*Continued*).

Geod. Br. P.O.—1926—300.